Counting number of edges, thickness, and chromatic number of k-visibility graphs

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Albany Area Math Circle

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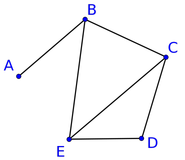
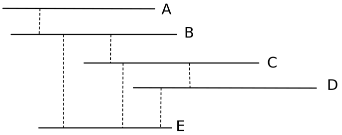
Bar Visibility Graphs

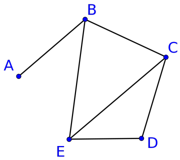


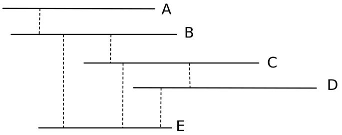
Bar Visibility Graph



Bar Visibility Representation

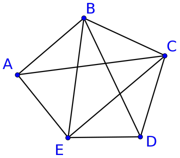
Bar Visibility Graphs

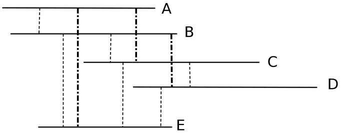




Bar Visibility Graph

Bar Visibility Representation





Bar 1-Visibility Graph

Bar 1-Visibility Representation

Thickness and Chromatic Number

Definition

The *thickness* $\Theta(G)$ of a graph G is the least number of colors needed to color the edges of G so that no two edges with the same color intersect.

Definition

The *chromatic number* $\chi(G)$ of a graph G is the least number of colors needed to color the vertices of G so that no two vertices with the same color are adjacent.

Upper Bound on Thickness of Bar k-Visibility Graphs

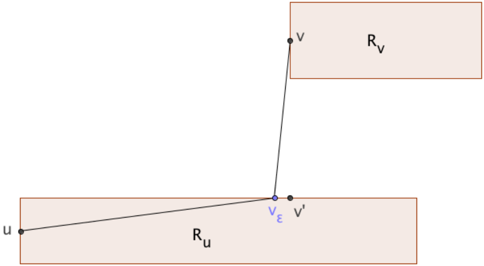
Theorem

$\Theta(G_k) \leq 6k$ for all bar k-visibility graphs G_k.

- Great improvement over old quadratic bound of $18k^2 - 2k$ found by Dean *et al.* (2005).
- Found with method used to bound thickness of semi bar 1-visibility graphs, found by Felsner and Massow (2008).
- Not tight: $\Theta(G_1) \leq 4$ proven by Dean *et al.* (2005).
- There exist G_k with $\Theta(G_k) \geq k + 1$.
- Maximal thickness grows at $O(k)$.

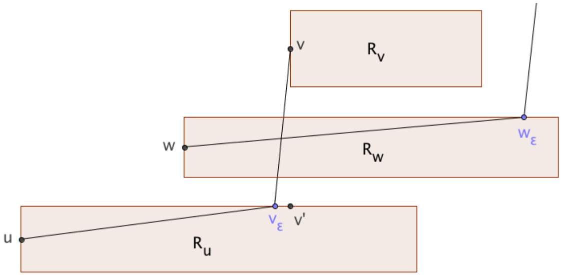
Proof of Upper Bound

- Based on bound of $\chi(G_k) = 6k + 6$ by Dean *et al.* (2005).
- Method: construct graph based on representation. Thicken bars to rectangles. Assume no two vertices have same x-coordinate.
- Use one-bend edges.

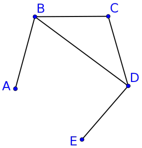


Proof of Upper Bound

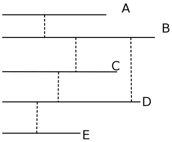
- No two horizontal or two vertical segments intersect.
- Color edges based on vertex-coloring of G_{k-1}.
- Intersecting edges intersect in rectangle of horizontal segment, thus left endpoints of the edges must have different colors when considering $(k-1)$-visibility.



Semi Bar Visibility Graphs

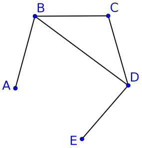


Semi Bar Visibility Graph

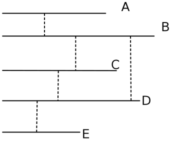


Semi Bar Visibility Representation

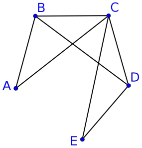
Semi Bar Visibility Graphs



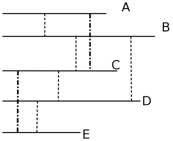
Semi Bar Visibility Graph



Semi Bar Visibility Representation



Semi Bar 1-Visibility Graph



Semi Bar 1-Visibility Representation

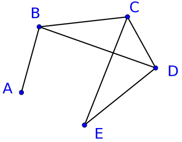
Upper Bound on Thickness of Semi Bar k-Visibility Graphs

Theorem

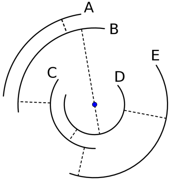
$\Theta(G_k) \leq 2k$ for all semi bar k-visibility graphs G_k.

- Better than bound found using $\chi(G_k) \leq 2k + 3$, found by Felsner and Massow (2008)
- Proof based on how many one-edges cross any given bar.
- There exist G_k with $\Theta(G_k) \geq \left\lceil \frac{2}{3}(k+1) \right\rceil$
- Maximal thickness grows at $O(k)$.

Arc Visibility Graphs

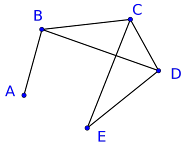


Arc Visibility Graph



Arc Visibility Representation

Arc Visibility Graphs



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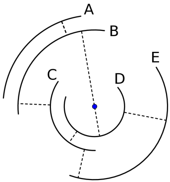
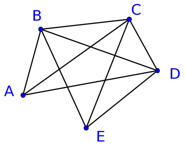
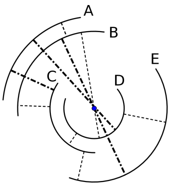


Arc Visibility Representation



Arc 1-Visibility Graph



Arc 1-Visibility Representation

Number of Edges, Chromatic Number

Theorem

Arc k-visibility graphs with n vertices have at most $(k+1)(3n-k-2)$ edges.

- Found by considering endpoints of arcs

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Theorem

$\chi(G_k) \leq 6k+6$ for all arc k-visibility graphs G_k.

- Bounded by maximum number of edges

Upper Bound on Thickness of Rectangle k-Visibility Graphs

Theorem

$\Theta(G_k) \leq 12k$ for all rectangle k-visibility graphs G_k.

- Double the upper bound for bar k-visibility graphs.

Conclusion

What Did We Do?

- Improved bounds on thickness of bar k-visibility graphs, created bound on thickness of semi bar k-visibility graphs
- Placed bounds on number of edges and chromatic number of arc k-visibility graphs
- Found bound on thickness of rectangle k-visibility graphs

Conclusion

Future work:

- Tighten bounds for bar, semi bar, arc, rectangle k-visibility graphs

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Bar 1-Visibility Representation of K_8