# Techniques for Fractal Terrain Generation

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# Outline

# Connecting Fractal Geometry and Terrain

- What is fractal geometry?
- Benoit Mandlebrot and the length of the British coastline. (Stanger)

Figure: The coastline of Britain has a fractal dimension

# Why are Fractals useful in Generating Landscapes?

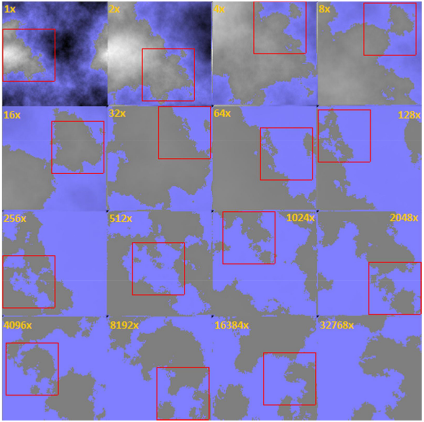



Figure: Fractals are self-similar, and have a non-integer dimension

# Midpoint Displacement Method

- Relatively straightforward
- Realistic features
- Begin with a line between two points.
- The midpoint of this line is then displaced by some random amount in a vertical direction.
- The midpoints of these two new line segments are then displaced by a random amount in the vertical direction.
- Repeat until you have reached desired level of detail.

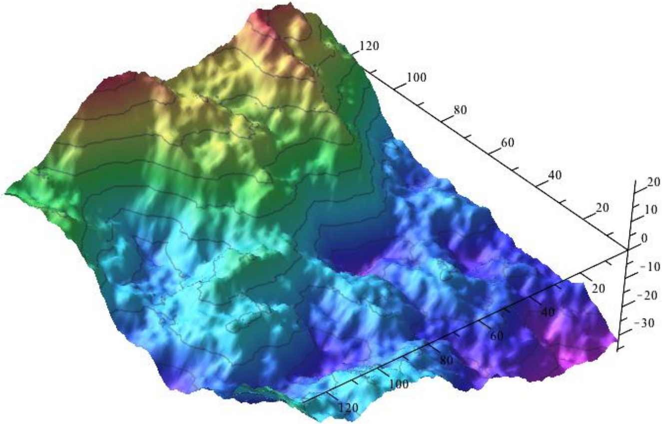
# Midpoint Displacement (cont.)

- Must be a square with sides of length $2^N + 1$, so that each sub-square has an exact middle lying on a gridpoint.
- $k, k' \in \{1, 3, \cdots, 2^{N-n} - 1\}$
- $m \in \{0, 2, \cdots, 2^{N-n}\}$
- $n < N$ and $m \in \mathbb{Z}$.
- H is a smoothing parameter
- A is an amplitude parameter
- R is uniform random variable between -1 and 1.

# Midpoint Displacement (cont.)

- Given a heightfield, $T = \begin{bmatrix} T_{0,0} & \cdots & T_{0,2^N,} \\ \vdots & \ddots & \vdots \\ T_{2^N,0} & \cdots & T_{2^N,2^N} \end{bmatrix}$
- Recurrence relations for midpoints:
- along columns
  $T_{k\cdot 2^n, j\cdot 2^n} = \frac{T_{(k+1)\cdot 2^n, j\cdot 2^n} + T_{(k-1)\cdot 2^n, j\cdot 2^n}}{2} + A \cdot R \cdot 2^{-H\cdot n},$
- along rows
  $T_{j\cdot 2^n, k\cdot 2^n} = \frac{T_{j\cdot 2^n, (k+1)\cdot 2^n} + T_{j\cdot 2^n, (k-1)\cdot 2^n}}{2} + A \cdot R \cdot 2^{-H\cdot n},$
- and between corners
  $T_{k\cdot 2^n, k'\cdot 2^n} = \left(\frac{1}{4}\right) T_{(k-1)\cdot 2^n, (k'-1)\cdot 2^n} + \left(\frac{1}{4}\right) T_{(k-1)\cdot 2^n, (k'+1)\cdot 2^n} +$
  $\left(\frac{1}{4}\right) T_{(k+1)\cdot 2^n, (k'-1)\cdot 2^n} + \left(\frac{1}{4}\right) T_{(k+1)\cdot 2^n, (k'+1)\cdot 2^n} + A \cdot R \cdot 2^{-H\cdot n}$

# Terrain Generated Using Midpoint Displacement

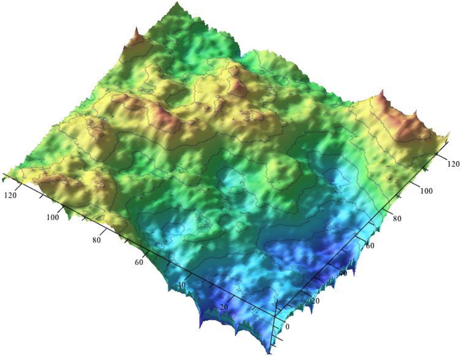


# Diamond Square Algorthim

- Variations of the Midpoint Displacement method
- Midpoint Displacement sometimes leaves square-shaped artifacts in the terrain.
- The Diamonds and Squares method attempts to alleviate this by alternating calculated values to square and diamond patterned midpoints.

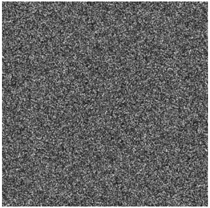
# Benefits of using the Diamond Square Algorithm and Midpoint Displacement Algorithm

- Both the Midpoint Displacement and the Diamond Square Algorithm run in linear time compared to the Fourier transformation which runs in NlogN time.



# How the Fast Fourier Transform Works in Generating Fractal Landscape

- Not an iterative process
- Begin using a random Gaussian noise
- This is a two dimensional NxM grid of discrete random values.
- Apply the fast Fourier transform (FFT)



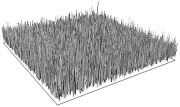
# Fast Fourier Transform

- The Fast Fourier Transform performs the discrete Fourier transform (DFT).
- A two dimensional discrete Fourier transform for an *N*x*M* grid in *x* and *y* is

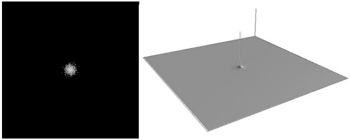
$$F(u,v) = 1/NM \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y) e^{-2\pi i (xu/N + yu/M)}$$

# How the Fast Fourier Transform Works in Generating Fractal Landscape (cont.)

- The FFT decomposes the random noise into the sum of the sine and cosine functions and converts magnitudes into frequency domain



- We scale these frequencies using a frequency filter of the form $1/f^r$.



# How the Fast Fourier Transform Works in Generating Fractal Landscape (cont.)

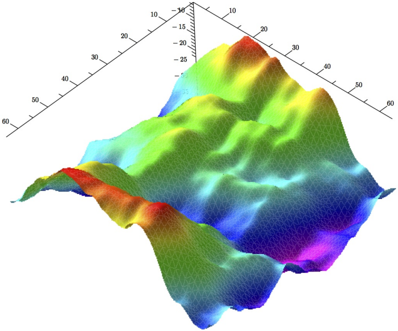
- Apply an inverse fast Fourier transform

$$f(x,y) = \sum_{u=0}^{N-1} \sum_{v=0}^{M-1} F(u,v) e^{2\pi i(xu/N + yv/M)}$$

in order to generate a fractal landscape by summing the sine and cosine waves at different frequencies.

# Benefits of Terrain Generation Using The Fast Fourier Transform

- Fractal landscape with smooth rolling features rather than ridges and peaks.
- Terrain can be tiled using this method.



# Multifractal Technique

- Very new technique
- The multi fractal technique uses four parameters to create a generated landscape:
  - $\alpha$
  - $C_1$
  - $S$
  - $H$

# How the Multifractal Technique Works in Generating Fractal Landscape

- There are four different stages in creating a multi-fractal generated terrain:
  - The first stage is to generate a Levy noise field with a certain $\alpha$

$$S_n = \frac{x_1 + x_2 + \ldots + x_n}{n^{\frac{1}{\alpha}}}$$

  - The second stage is to filter the output of the first stage to get a multi-scaling behavior as defined by the characteristic function:

$$K(q) = \frac{C_1}{\alpha - 1}\left(q^{\alpha} - q\right)$$

    which results in

$$\Gamma_{\lambda}$$

# How the Multifractal Technique Works in Generating Fractal Landscape (cont.)

- The third stage is to exponentiate and normalize the output of stage two

$$\epsilon_\lambda = N_1 * e^{\Gamma_\lambda}$$

- The fourth and final stage is to fractal integrate the multi-fractal field

$$\frac{1}{f^\beta}$$

where

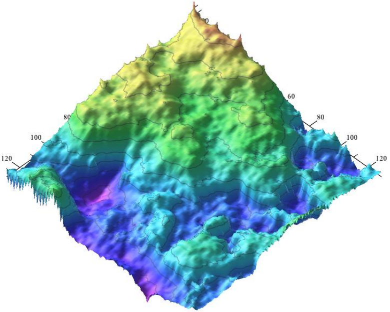
$$\beta = 2H + 1$$

# Conclusion

- The standard method seems to be Diamond Square algorithm, as it produces fairly realistic landscape with little effort.
- The fast Fourier transform allows for flatter terrain generation and the ability to tile
- Newest method is the Multifractal Technique which uses images of real terrain to generate terrain with very accurate features

# Outlook

- Multiplication Technique
  - Creates a landscape that looks more realistic
  - Done by multiplying two terrains created by the methods above
  - Multiplication will result in a new landscape with smoother valleys and rougher peaks.



# Applications

- Video Games
- Movies and Television Shows

- http://planetside.co.uk/galleries/terragen2-gallery

Thank You

# For Further Reading

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