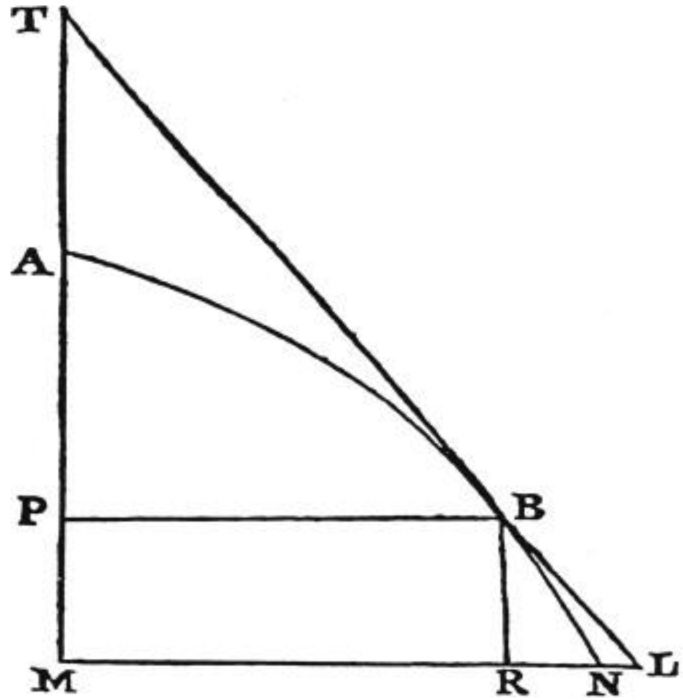
Berkeley's Philosophy of Mathematics

Adam Fix



Bishop George Berkeley

Bishop George Berkeley

- Born in Ireland on March 12th, 1685
- A theologian, philosopher, scientist, and mathematician
  - Best known for his empiricist and immaterialist philosophies

THE

ANALYST;

OR, A

DISCOURSE

Addreſſed to an

Infidel MATHEMATICIAN.

WHEREIN

It is examined whether the Object, Principles, and Inferences of the modern Analyſis are more diſtinctly conceived, or more evidently deduced, than Religious Myſteries and Points of Faith.

By the AUTHOR of *The Minute Philoſopher*.

Firſt caſt out the beam out of thine own Eye; and then ſhalt thou ſee clearly to caſt out the mote out of thy brother's eye. S. Matt. c. vii. v. 5.

LONDON:
Printed for J. TONSON in the *Strand*. 1734.

The Analyst, 1734

- Berkeley addressed problems of rigor in both Newton's "method of fluxions" and Leibniz's "method of differentiation"
- These two methods today are better known as Calculus
- *The Analyst* criticizes the theoretical underpinnings of the new calculus

Critique of Calculus

Let $y = x^2$ be a function of position versus time

Let *h* be an <u>infinitesimal</u>.

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

Critique of Calculus

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} 2x + h$$

"Ghosts of departed quantities"

$$\boxed{\frac{dy}{dx} = 2x}$$

Aftermath

- Berkeley pointed out these problem of calculus, but offered no solution
- James Jurin and Colin Maclaurin (two early Newtonians) devised an alternate derivation based on limits, but still not rigorously defined
- Continental mathematicians (the Bernoulli family) used infinitesimals without addressing Berkeley at all

(ϵ, δ)–definition of limit

- The more formal definition, pioneered by Cauchy, Bolzano in 1817, formalized by Weierstrass
  - Jurin’s work hints at this, doesn’t fully develop it

(ϵ, δ)–definition of limit

The derivative of *f(x)* equals *f'(x)* if and only if

$$\forall \varepsilon > 0 \exists \delta > 0 : \forall x (0 < |x - c| < \delta \Rightarrow \left| \frac{f(x) - f(c)}{x - c} - f'(c) \right| < \varepsilon$$

(ϵ, δ)–definition of limit

- Given $y = x^2$, we hypothesize that $\dfrac{dy}{dx} = 2x$
- Given ϵ > 0, we need to find δ > 0 such that

$$0 < \left| x - c \right| < \delta \longrightarrow \left| \frac{x^2 - c^2}{x - c} - 2c \right| < \varepsilon$$

$$\frac{x^2 - c^2}{x - c} - 2c = \frac{(x+c)(x-c)}{x-c} - 2c = x - c$$

(ϵ, δ)–definition of limit

Simplifying, we find that

$$0 < |x - c| < \delta \longrightarrow |x - c| < \varepsilon$$

Pick ϵ = δ, and…done

(ϵ, δ)–definition of limit

- Avoids infinitesimals altogether
  - No statement about infinitely small numbers
- Instead invokes values that can be made *arbitrarily* small (key difference!)

A word about infinities

- Berkeley rejected both infinitesimals and actual infinity
- Believed in Aristotelian “potential infinity”
- The (ϵ, δ)-definition re-derived calculus using only potential infinities
- In 1960 Abraham Robinson laid out rigorous infinitesimal calculus in *Non-Standard Analysis*

Historical Significance

- 1st Opinion
  - Berkeley re-emphasizes rigor in mathematics
    - He did exactly what a good mathematician should do
    - Placed burden of proof back on those making claims—just like in any other science

Historical Significance

- 2nd Opinion
  - *The Analyst* hindered British mathematics by emphasizing philosophy
    - British mathematicians prided themselves on rigor and a proper geometric approach
    - Berkeley’s demand for rigor and rejection of infinitesimals caused Britain to fall behind continental Europe
    - Cauchy, Weierstrass, Euler, Gauss were all European

Historical Significance

- My Opinion
  - Confusion and controversy lie at the beginning of many sciences, mathematics included
  - Science and mathematics are not the sole property of England, or any other nation
  - Berkeley started a legitimate debate, and the whole field of mathematics benefited as a result

Philosophical Significance

- *The Analyst* attacks the mechanical view of nature established by Newton's laws of motion
  - The "infidel mathematician" believes that calculus and Newton's laws imply a "watchmaker God"

Philosophical Significance

Berkeley argued that calculus, and Newton's laws of physics, lack theoretical bases

- Denied the mechanical model of nature
- Reemphasized a present and active God, not a passive watchmaker
- Therefore, *The Analyst* also carried a criticism of the religious views Newtonian physics helped inspire

Prelude to Formalism

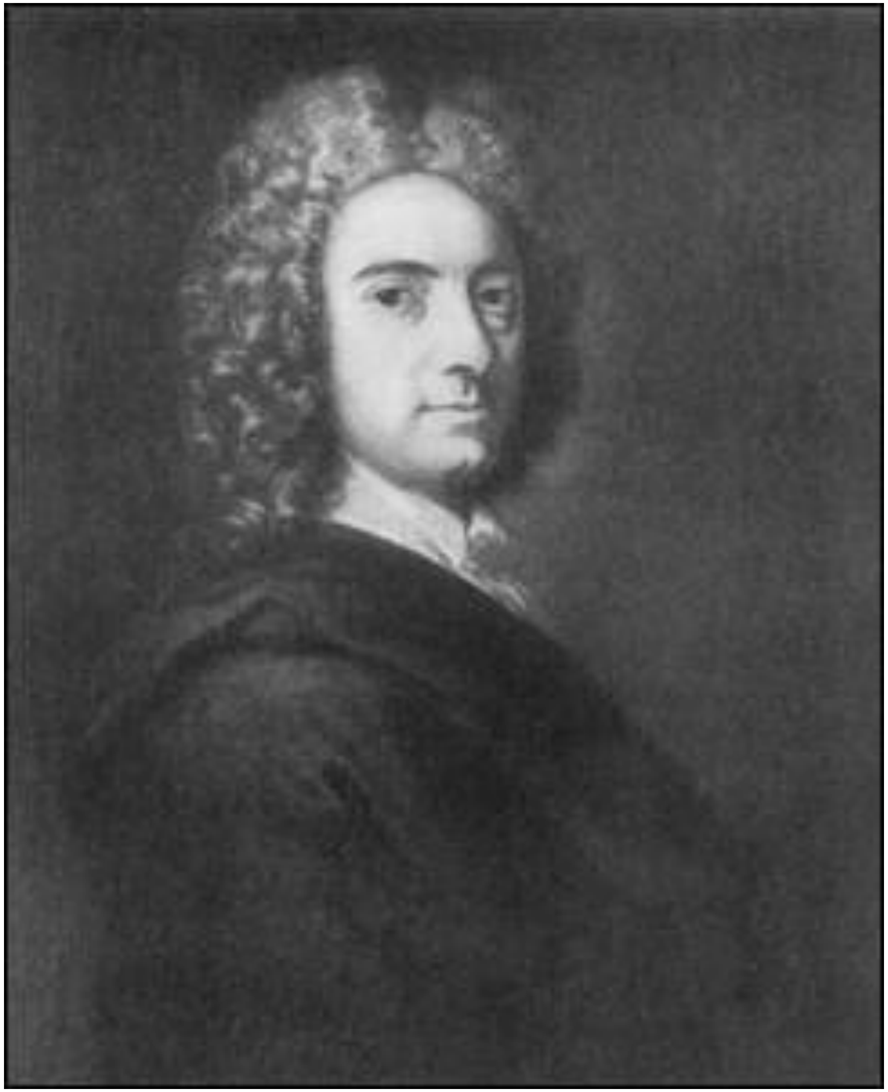
- All mathematical knowledge derives purely from certain rules and axioms
- Mathematics is not about nature
- Mathematics' relation to nature is entirely beside the point

Prelude to Formalism

- Berkeley knew calculus worked, but did not conclude that nature was fundamentally mathematical
  - His numbers were symbols, “signs”, a kind of language
- Mathematics is used to interpret God’s word, nothing more
- Berkeley’s mathematics contained no meaning in of itself—the textbook definition of formalism

Conclusion

- Berkeley played an integral (haha) role in 18^{th} century calculus
  - Questioned both the method of the new mathematics and its philosophical ramifications
- In *The Analyst* Berkeley provides
  - A criticism of calculus that inspired the (ϵ, δ)-definition
  - A critique of Newtonian mechanical philosophy
  - A prelude to formalism



The End