Taking Student Understanding to New Dimensions: Investigating the Mean and Median

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Mean and Median

In R^1

In R^n

The Evolver Program

Mean and Median in R^1

- Standard definitions (adapted from *Go Math!*) : When n = number of data items

  mean = sum of the data items divided by n

  median = middle value when data are written in non-decreasing order. If n is even, take the mean of the two middle values.

- Textbook approaches:
  - -Lead to computational facility
  - -Generally lack emphasis on conceptual understanding

Student Understanding

- “My sense is that students’ understanding of the mean and median is generally deficient. For example, when asked about the median, most can tell me that it is the ‘middle,’ but surprisingly few can tell me what percent of a population falls above or below the median. As far as an explanation of the mean goes, the best I can usually get is ‘the average,’ which of course begs the question.” (Randall Roeser, Iroquois Middle School)

- “When students come to my class, they know what mean and median are. I do not think they all fully understand what these measures are explaining.” (Shana DeRocco, Niskayuna High School)

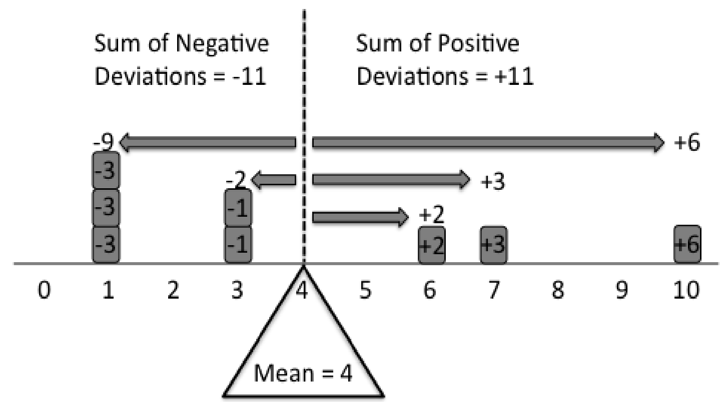
Conceptualizations of Mean and Median

Median

- Splits the data set into two nearly equal parts
- Minimizes the sum of absolute deviations (Euclidean distances)
- Equilibrium under weight and string forces (Evolver)

Mean

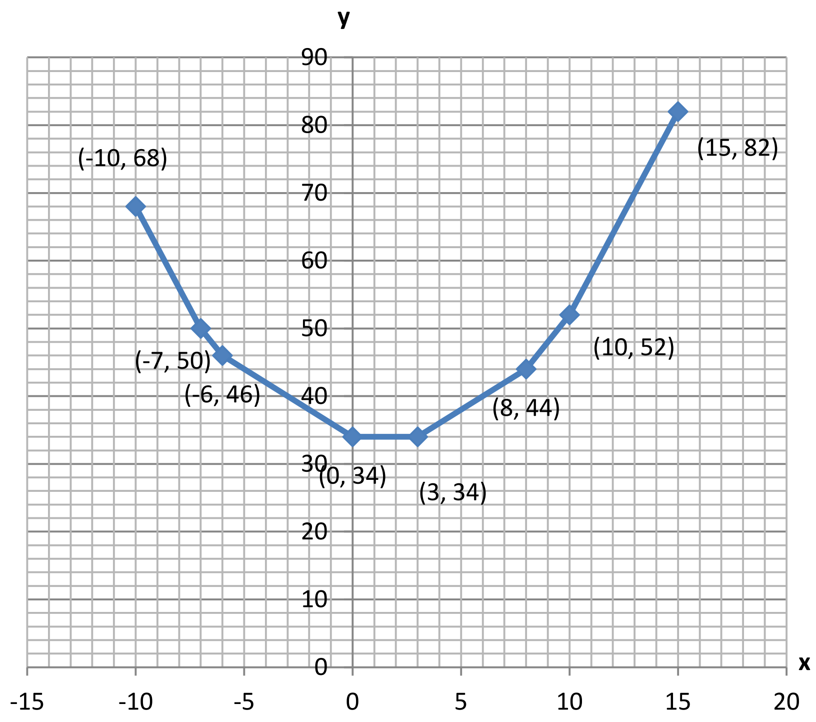
- Balance point (fulcrum representation)



- Equal sharing
- Minimizes sum of square deviations (Euclidean distances)
- Equilibrium under rubber band forces (Evolver)

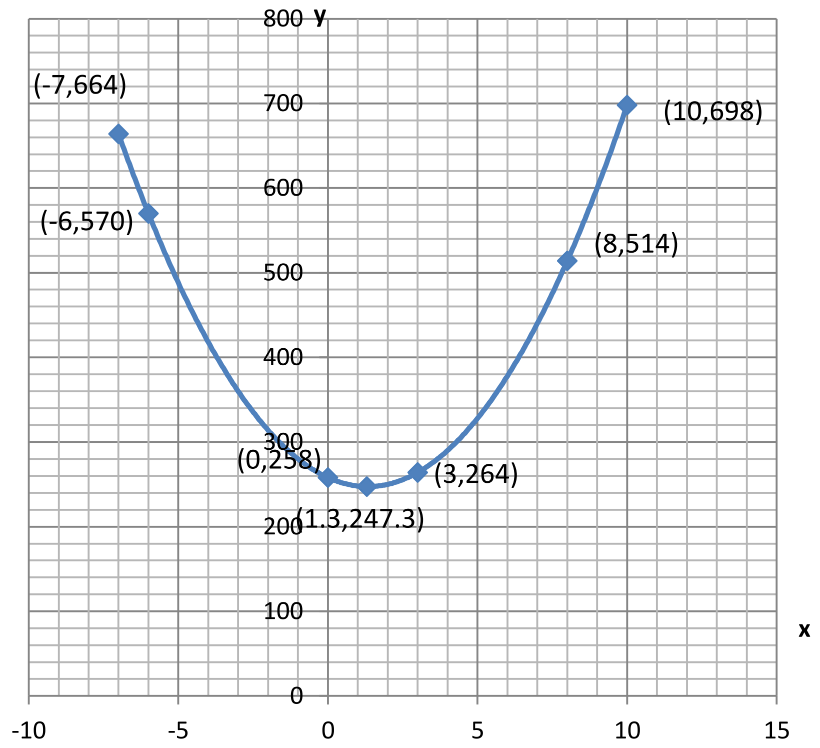
Considering the data set {-7, -6, 0, 3, 8, 10}

The median (1.5) minimizes the sum of absolute deviations



$$f(x) = \sum_{i=1}^{n} \|x_i - x\|$$

The mean (1.3) minimizes the sum of square deviations



$$f(x) = \sum_{i=1}^{n} \|x_i - x\|^2$$

Mean and Median in Higher Dimensions

- Only one method of finding the mean in R^n
- Median depends on *order*
- Multiple ways to generalize the median into R^n

- We will briefly discuss one such generalization of the median: the Mediancentre.

The Mediancentre

- Based on minimization role of R^1 median
- For a data set in R^n, mediancentre = point that minimizes the sum of absolute deviations (Euclidean distances)
- Not always unique!

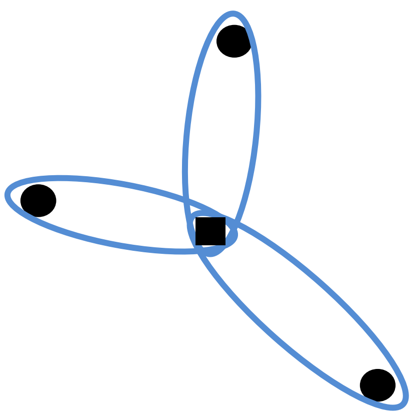
  -NOT UNIQUE for any even-numbered data set in R^1 such that two middle points are different

  -UNIQUE for any odd-numbered collinear data set *or* any non-collinear data set

The Evolver Program: Visualizing Mean and Mediancentre as Equilibria

Mean

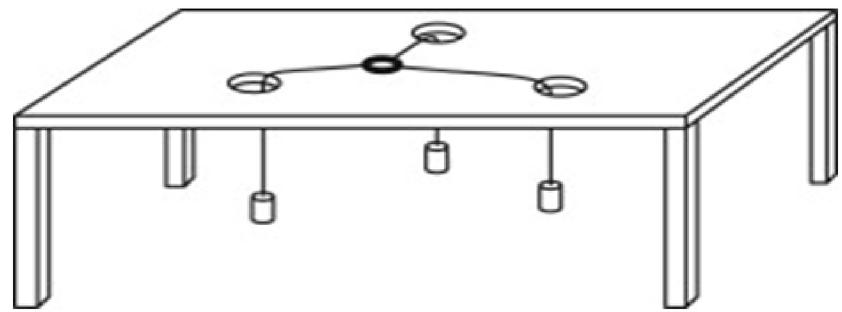
- "Ideal" Rubber Bands (force proportional to amount of stretch and rest length equal to zero)



- **Theorem:** A central, moveable point connected to a set of points via ideal rubber bands will come to rest at the mean location of the set of points.

Mediancentre

- Weights and Strings (fixed force)



- **Theorem:** A central, moveable point connected to a set of points via weights and strings will be at equilibrium if and only if it is also located at a mediancentre of the set of points.