Optimizing Blackjack Playing Strategy with “Lucky Bucks”

Danny Guo

Williams College

Rules of Blackjack

- Player is pit against dealer (“the house”)
- Player places bet first and is dealt two cards face up, while dealer is dealt one card face up and the other face down
- Card values
  - Face cards and tens are worth 10
  - Aces are worth 1 and 11 (soft hands)
  - Other cards are worth face value
- Objective is to beat dealer by having a higher card value that does not exceed 21

Rules of Blackjack

- Blackjack occurs when player's original cards are an ace and ten-card
  - Payoff is 3-to-2
  - Should dealer also have blackjack, player loses
- If player and dealer have the same total, a "push" occurs and player's bet is returned
- Player can take four actions after being dealt
  - Stand: not receive additional cards
  - Hit: receive additional card with option to hit or stand
  - Double down: doubling bet and receiving only one additional card
  - Splitting: when dealt pair of identical cards, player can split the cards and play them as separate hands

Dealer’s strategy

- The house employs a set strategy requiring the dealer to reach a card value of 17 or more
- Dealer card value outcomes are 17 through 21 or bust
- House may apply various alterations to increase their advantage

What are “Lucky Bucks”?

- Promotional coupons that offer additional return on winning hands without downside penalty
- Typically offers return 3-to-2 or 2-to-1
- Eg) Player wagers $1 lucky buck
  - Winning hands result in payoff of $2
  - Losing hands result only in a loss of $1
  - Blackjack results in payoff of $3
  - Winning on double down results in payoff of $4

Methodology

- Examine expected value of all actions under any given circumstance through recursive programming
- Assumption that cards are dealt from infinite deck, giving constant probability of any card being drawn equal to 1/13

Dealer Probabilities

Hard Hands	17	18	19	20	21	Bust
4	.130	.126	.121	.116	.111	.394
5	.122	.122	.118	.113	.113	.416
6	.165	.106	.106	.102	.102	.423
7	.369	.138	.079	.079	.079	.262
8	.129	.359	.129	.069	.069	.245
9	.120	.120	.351	.120	.120	.228
10	.121	.121	.121	.371	.371	.230
11	.111	.111	.111	.111	.111	.212
12	.103	.103	.103	.103	.103	.483
13	.096	.096	.096	.096	.096	.520
14	.089	.089	.089	.089	.089	.554
15	.083	.083	.083	.083	.083	.586
16	.077	.077	.077	.077	.077	.615
17	1	0	0	0	0	0
18	0	1	0	0	0	0
19	0	0	1	0	0	0
20	0	0	0	1	0	0
21	0	0	0	0	1	0

Soft Hands	17	18	19	20	21	Bust
S1	.189	.189	.189	.189	.078	.167
S2	.151	.151	.151	.151	.151	.245
S3	.146	.146	.146	.146	.146	.272
S4	.140	.140	.140	.140	.140	.300
S5	.135	.135	.135	.135	.135	.327
S6	.129	.129	.129	.129	.129	.354
S7	1	0	0	0	0	0
S8	0	1	0	0	0	0
S9	0	0	1	0	0	0
S10	0	0	0	1	0	0
S11	0	0	0	0	1	0

Optimal Play Strategies

Dealer Hands

Player Hands (hard)	Payback Ratio = 1										Payback Ratio = 1.5										Payback Ratio = 2									
	A	2	3	4	5	6	7	8	9	0	A	2	3	4	5	6	7	8	9	0	A	2	3	4	5	6	7	8	9	0
2	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
3	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D	D	H	H	H	H
4	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D	D	H	H	H	H
5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D	D	H	H	H	H
6	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D	D	D	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D	H	H	H	H	H	H	D	D	D	D	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H	H	D	D	D	D	D	H	H	H	H	H	D	D	D	D	D	D	H	H	H
9	H	H	D	D	D	D	H	H	H	H	D	D	D	D	D	D	D	D	D	D	H	D	D	D	D	D	D	D	D	D
10	H	D	D	D	D	D	D	D	D	H	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
11	H	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
12	H	H	H	S	S	S	H	H	H	H	H	H	H	S	S	S	H	H	H	H	H	D	D	D	D	D	D	H	H	H
13	H	S	S	S	S	S	H	H	H	H	H	S	S	S	S	S	H	H	H	H	H	S	S	S	S	S	D	H	H	H
14	H	S	S	S	S	S	H	H	H	H	H	S	S	S	S	S	H	H	H	H	H	S	S	S	S	S	H	H	H	H
15	H	S	S	S	S	S	H	H	H	H	H	S	S	S	S	S	H	H	H	H	H	S	S	S	S	S	H	H	H	H
16	H	S	S	S	S	S	H	H	H	H	H	S	S	S	S	S	H	H	H	S	H	S	S	S	S	S	H	H	H	S
17	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
18	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
19	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
20	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
21	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S

Legend

H	Hit
S	Stand
D	Double Down
P	Split

Optimal Play Strategies

Dealer Hands

Player Hands (soft)

Payback Ratio = 1

	A	2	3	4	5	6	7	8	9	0
2	H	H	H	H	H	H	H	H	H	H
3	H	H	H	H	H	D	H	H	H	H
4	H	H	H	H	D	D	H	H	H	H
5	H	H	H	D	D	D	H	H	H	H
6	H	H	D	D	D	D	H	H	H	H
7	H	S	D	D	D	D	S	S	H	H
8	S	S	S	S	S	S	S	S	S	S
9	S	S	S	S	S	S	S	S	S	S
10	S	S	S	S	S	S	S	S	S	S

Payback Ratio = 1.5

	A	2	3	4	5	6	7	8	9	0
2	H	D	D	D	D	D	H	H	H	H
3	H	D	D	D	D	D	H	H	H	H
4	H	D	D	D	D	D	H	H	H	H
5	H	D	D	D	D	D	D	H	H	H
6	H	D	D	D	D	D	D	D	H	H
7	H	D	D	D	D	D	D	D	H	H
8	S	D	D	D	D	D	S	S	S	S
9	S	S	S	S	D	D	S	S	S	S
10	S	S	S	S	S	S	S	S	S	S

Payback Ratio = 2

	A	2	3	4	5	6	7	8	9	0
2	H	D	D	D	D	D	H	H	H	H
3	H	D	D	D	D	D	D	H	H	H
4	H	D	D	D	D	D	D	D	H	H
5	H	D	D	D	D	D	D	D	D	D
6	H	D	D	D	D	D	D	D	D	D
7	D	D	D	D	D	D	D	D	D	D
8	S	D	D	D	D	D	D	S	S	D
9	S	D	D	D	D	D	S	S	S	S
10	S	S	S	S	S	S	S	S	S	S

Legend

H	Hit
S	Stand
D	Double Down
P	Split

Player Hands (pairs)

Payback Ratio = 1

	A	2	3	4	5	6	7	8	9	0
A	P	P	P	P	P	P	P	P	P	P
2	H	H	H	P	P	P	P	H	H	H
3	H	H	H	P	P	P	P	H	H	H
4	H	H	H	H	D	D	H	H	H	H
5	H	D	D	D	D	D	D	D	D	H
6	H	H	P	P	P	P	H	H	H	H
7	H	P	P	P	P	P	P	H	H	H
8	P	P	P	P	P	P	P	P	P	P
9	S	P	P	P	P	P	S	P	P	S
10	S	S	S	S	S	S	S	S	S	S

Payback Ratio = 1.5

	A	2	3	4	5	6	7	8	9	0
A	P	P	P	P	P	P	P	P	P	P
2	P	P	P	P	P	P	P	P	P	H
3	P	P	P	P	P	P	P	P	P	H
4	H	D	D	D	P	D	H	H	H	H
5	D	D	D	D	D	D	D	D	D	D
6	H	P	P	P	P	P	P	P	P	P
7	H	P	P	P	P	P	P	P	P	P
8	P	P	P	P	P	P	P	P	P	P
9	P	P	P	P	P	P	P	P	P	P
10	S	S	S	S	S	S	S	S	S	S

Payback Ratio = 2

	A	2	3	4	5	6	7	8	9	0
A	P	P	P	P	P	P	P	P	P	P
2	P	P	P	P	P	P	P	P	P	P
3	P	P	P	P	P	P	P	P	P	P
4	P	P	P	P	P	D	P	P	P	H
5	D	D	D	D	D	D	D	D	D	D
6	P	P	P	P	P	P	P	P	P	P
7	P	P	P	P	P	P	P	P	P	P
8	P	P	P	P	P	P	P	P	P	P
9	P	P	P	P	P	P	P	P	P	P
10	S	P	P	P	P	P	S	S	S	

Implications

- As payback ratio increases, player's strategies become more aggressive to maximize expected gain
  - Expect player to revise optimal plays from hits or stands to double downs or splits
- Curious change from stand to hit
  - Case when player has 16 and dealer has 10

References

- Benjamin, A.T., & Huggins, E. (1993). Optimal blackjack strategy with "lucky bucks". Journal of Undergraduate Mathematics and Its Applications, 14(4): 309-318.