Pentagrids and Penrose Tilings

Stacy Mowry, Shriya Shukla

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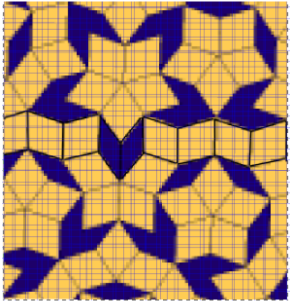
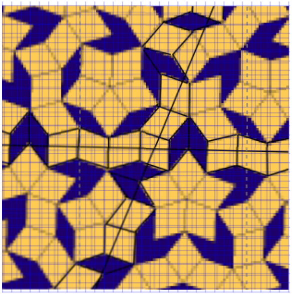
DeBrujin's Insight





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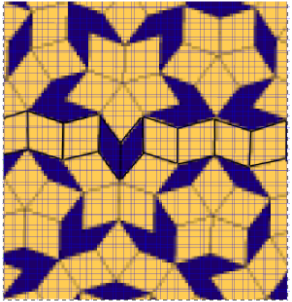

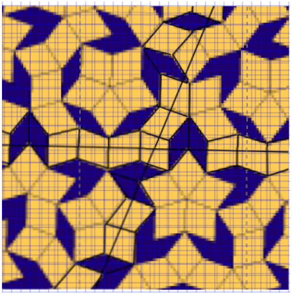

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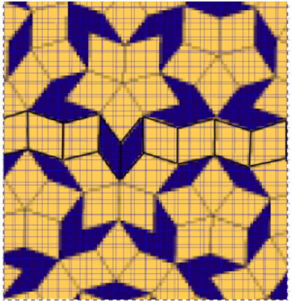

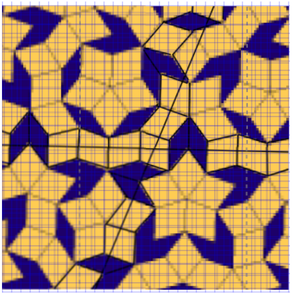

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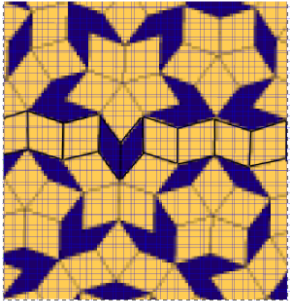
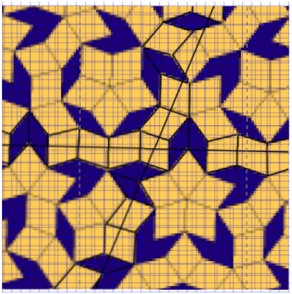
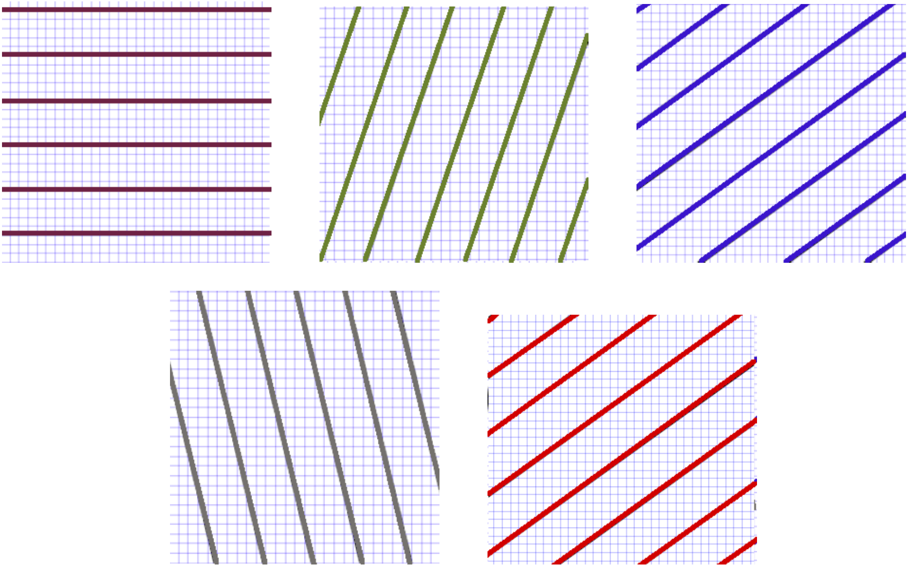


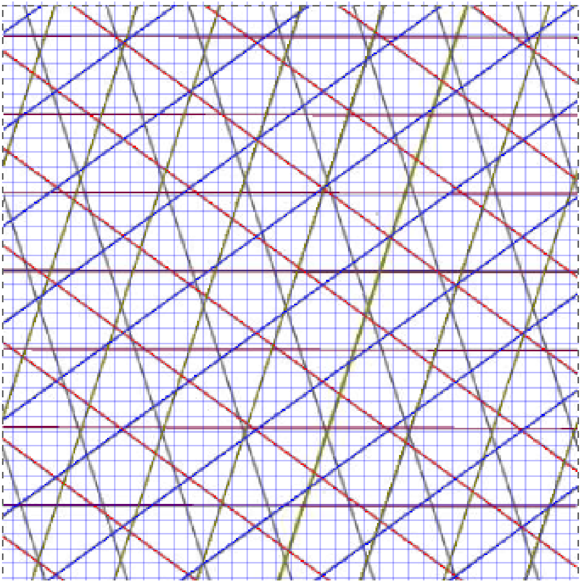


- Intersections of 2 ’ribbons’ in the Penrose tilings used to represent a particular rhomb.
- There exist five different orientations for the rhombii.
- Hence it is possible to find a relation between the pentagrid and Penrose tilings.

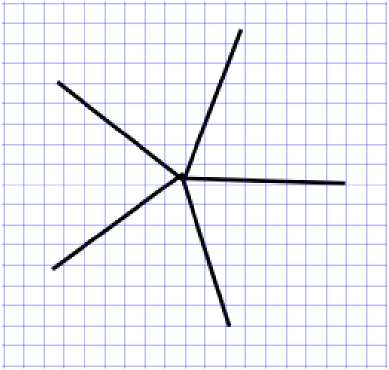
What is a Pentagrid?

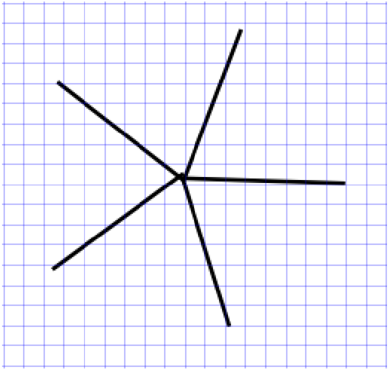


Full Pentagrid



DeBrujin's Conditions





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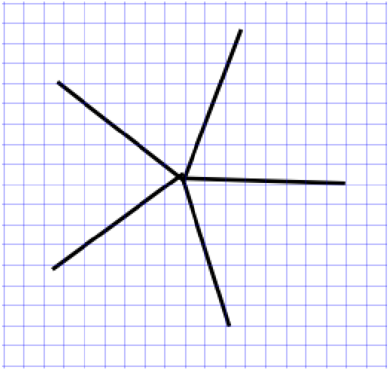

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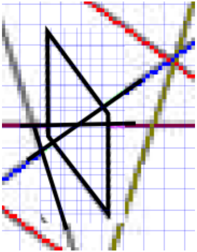
- $\sum_{j=0}^{4} \lambda_j = 0$

Explaining the dual



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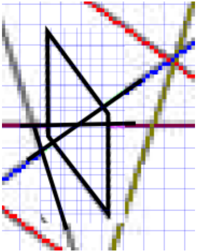

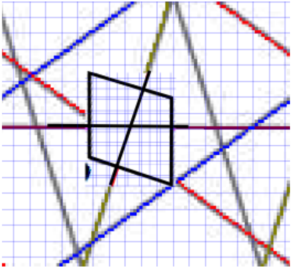

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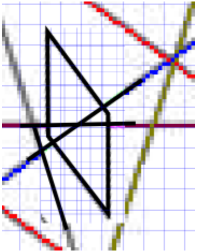

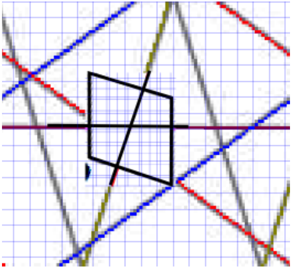

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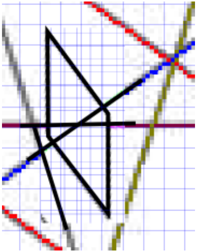
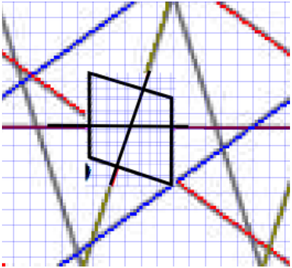
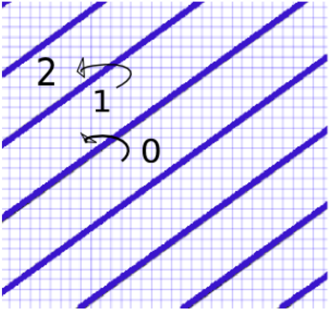
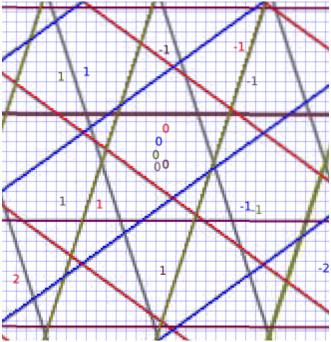




- The dual here is a 1-1 mapping between the intersection of ribbons and the rhombi in the Penrose.
- Each section around the intersection of two ribbons is called a space and the four spaces represent the four vertices of the rhombus they map to.

Labeling the Pentagrid





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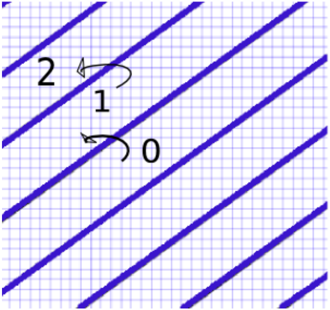

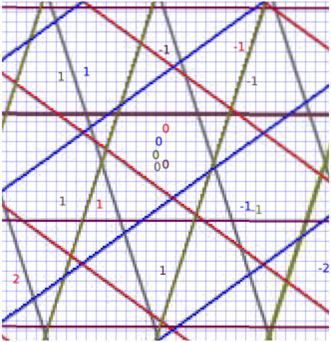

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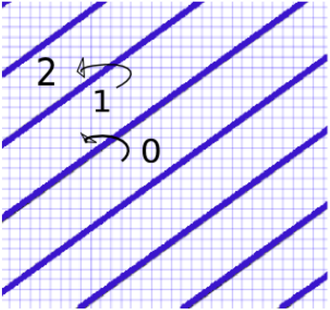

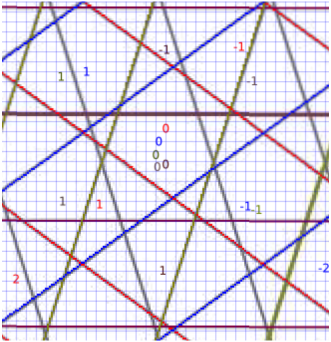

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- Second, we label each of the five grids either clockwise or counter-clockwise.

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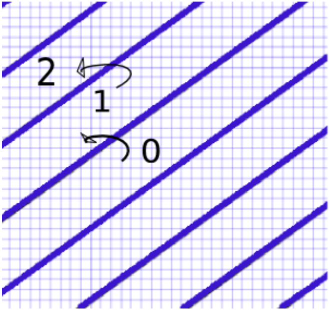

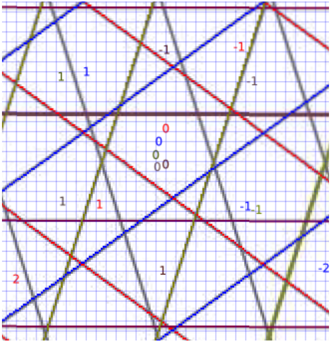

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- Each space becomes a five-tuple, which later is mapped to a two-dimensional point that marks a vertex of a rhomb.

Introducing DeBrujin's Formula

Given the five-tuple, $(K_0, K_1, K_2, K_3, K_4)$, associated with a space, the following formula is used to determine the vertex (a_i, b_i) which it corresponds to in the two-dimensional plane

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$$(a_i, b_i) = \sum_{j=0}^{4} K_j(cos\frac{2\pi j}{5}, sin\frac{2\pi j}{5})$$

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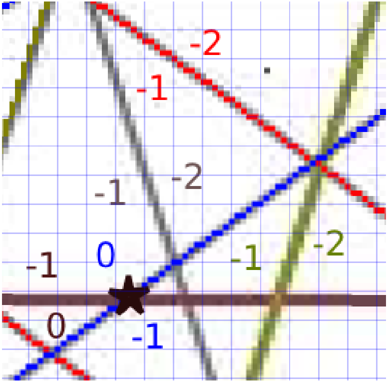

Example:Mapping from five dimensions to two dimensions

- Using DeBrujin's formula i.e., $(a_i, b_i) = \sum_{j=0}^{4} K_j(cos\frac{2\pi j}{5}, sin\frac{2\pi j}{5})$,we get the following four vertices : $(0, 0)$, $(0.309, 0.951)$, $(.5, .363)$ and $(-.21, .608)$.

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- For instance, $(-1, -1, -1, -1, -1)$ maps to -1$(cos0, sin0)$ - $(cos\frac{2\pi}{5}, sin\frac{2\pi}{5})$ - $(cos\frac{4\pi}{5}, sin\frac{4\pi}{5})$ - $(cos\frac{6\pi}{5}, sin\frac{6\pi}{5})$ - $(cos\frac{8\pi}{5}, sin\frac{8\pi}{5}) = (0, 0)$

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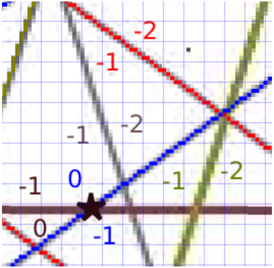

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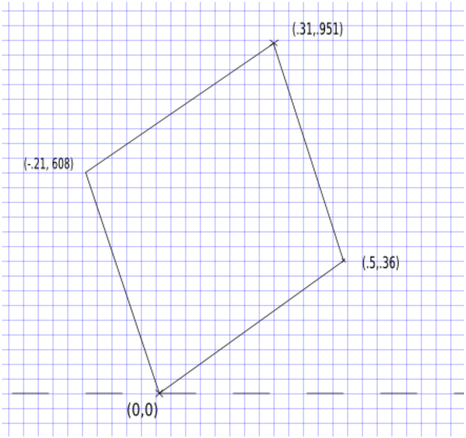

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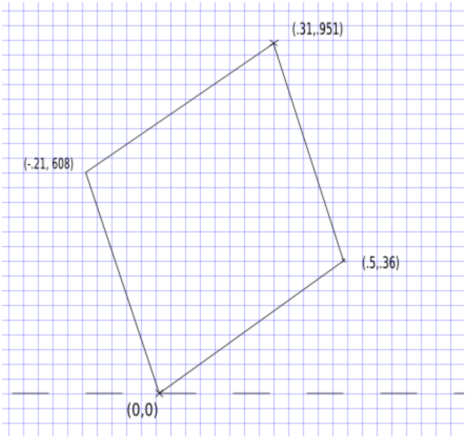
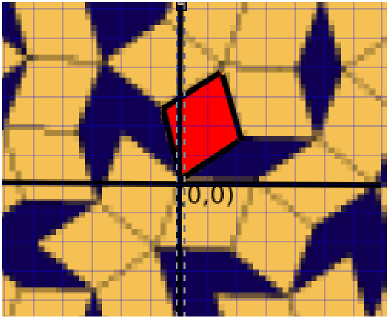
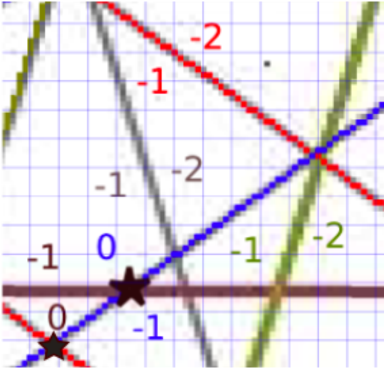




Next step : A different rhomb



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