A Graph Theoretic Model for Music Information Retrieval

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Bates College

April 6, 2013

Presentation Outline

The Graph Model

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- Converts a musical theme in a query (audio or score fragment) into its representative graph.
- Representative graph is compared with the graphs of musical themes in a database.
- Comparison is done after filtering and then using a similarity function.

The result then is a list of graphs containing the matching theme and its variations by the addition of new notes between pairs of notes of the query graph.

A Musical Theme and its Representative Graph

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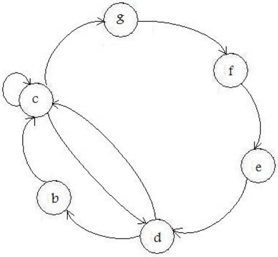
A melodic theme is a musical phrase that is comprised of one complete recognizable melody.



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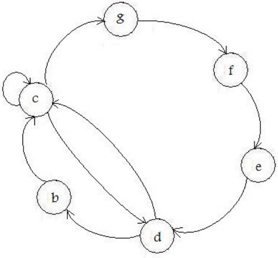

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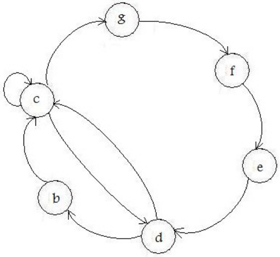

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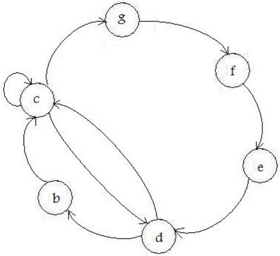

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- The last note is sent back to the first note using a directed edge.

Themes and Graphs

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A trail with the same starting and ending vertex is called a *circuit*.

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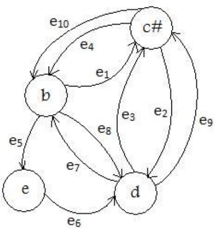

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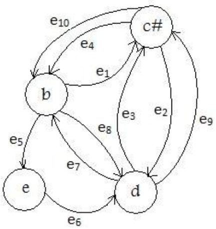

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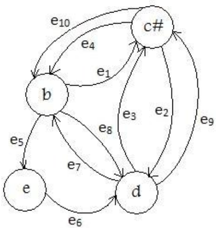

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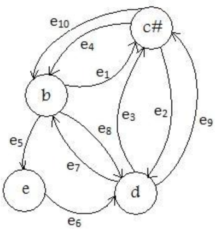

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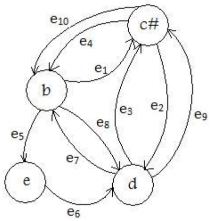

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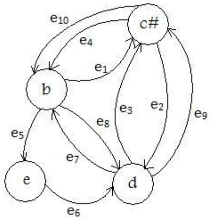

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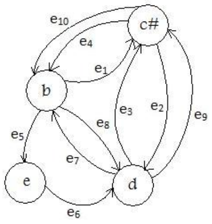

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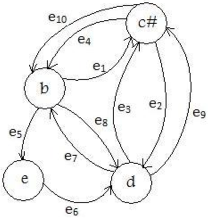

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All graphs that are constructed from a musical theme are Eulerian.

Themes having same representative graphs

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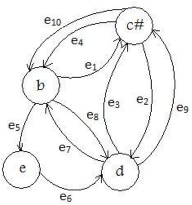

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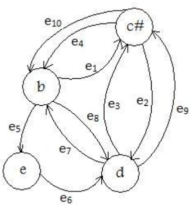


This is just a different Eulerian circuit of that same graph.

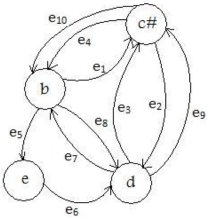
$$de_7be_8de_3c\#e_2de_9c\#e_{10}be_1c\#e_4be_5ee_6d$$

The Eulerian circuit corresponding to the Pathetique theme is

$$be_1c\#e_2de_3c\#e_4be_5ee_6de_7be_8de_9c\#e_{10}b$$

Musical Themes and Graphs

However, not every Eulerian circuit of a graph gives a different musical theme. For instance,



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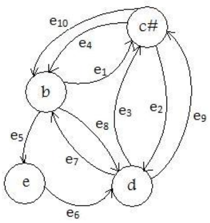

These two different Eulerian circuits of this graph correspond to this same melodic theme

$de_7be_8de_3c\#e_2de_9c\#e_{10}be_1c\#e_4be_5ee_6d$

$de_7be_8de_9c\#e_2de_3c\#e_4be_1c\#e_{10}be_5ee_6d$

Presentation Outline

Transitive Closure

The transitive closure of order i of graph P is the graph P^i such that $V(P^i) = V(P)$ and $E(P^i)$ contains all edges of P and edges joining any pair of vertices which have a trail of i edges between them.

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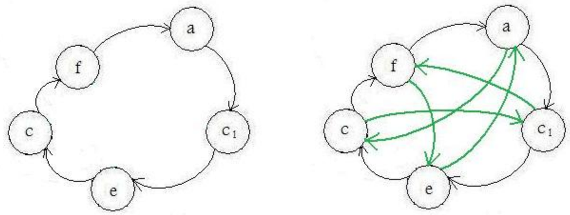
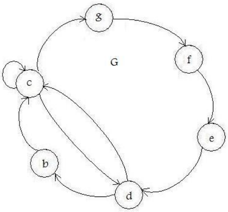
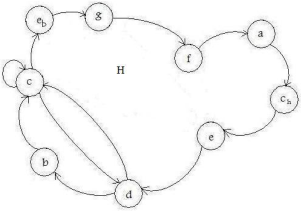
For example, the graph on the right below represents P^3 of the graph P given on the left



Difference Graph

Difference Graph, $H \backslash G$ is obtained by deleting those edges from H that are in common with G.





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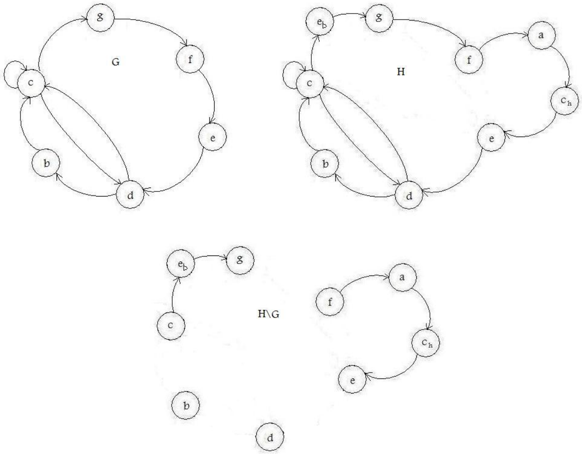

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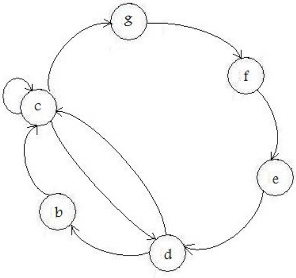
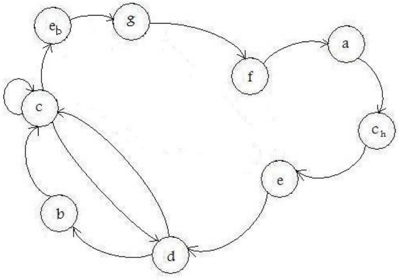
$$\sigma(G, H) = \frac{|G| - |G \backslash (H \backslash G)^2|}{|G|} + \frac{|G| - |G \backslash (H \backslash G)^3|}{|G|} + \frac{|G| - |G \backslash (H \backslash G)^4|}{|G|}$$

Consider this theme and its graph A



A variation of this theme and its graph B is



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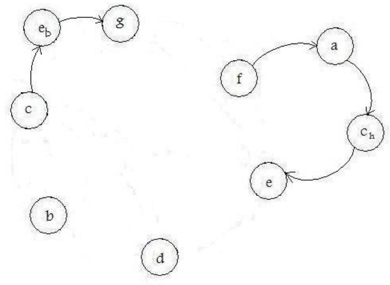

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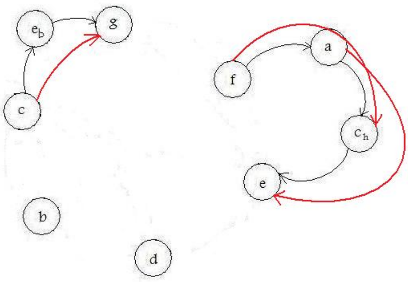

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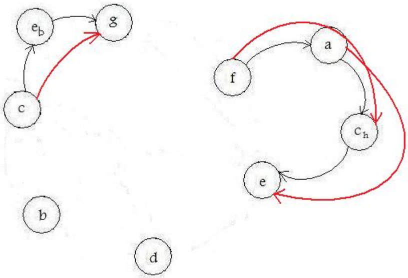
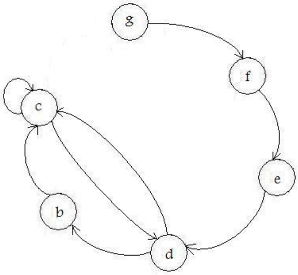


$$A\backslash(B\backslash A)^2$$

$$|A \backslash (B \backslash A)^2| = 8$$



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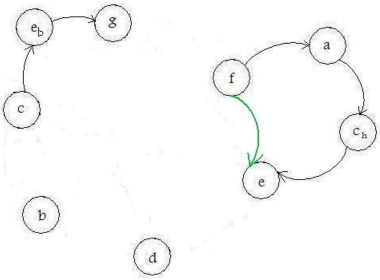

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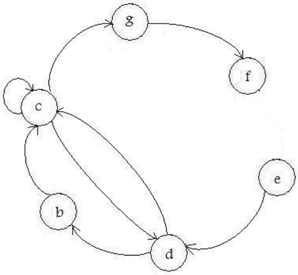

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Greater this fraction for a graph H, the more different it is from the query graph. Hence, the results should be given in increasing order of this fraction.

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- It only allows for new notes between pairs of notes in the variation of the theme.
- The list of results is large because of the possibility of one graph representing more than one theme. The maximum number of themes represented by a graph is equal to the total number of Eulerian circuits in the graph.

Further Work

This procedure has been implemented and tested on Maple.

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- Understanding the similarity function proposed by Pinto and Haus.
- Figuring out a way to represent rhythm in a graph.
- How to make the results more precise?
- How to get from the graphs to the musical themes?

Thank you for your attention today.