From Robinson to Taylor-Socolar

Mazi Kazemi and Axel Yung

Vassar College

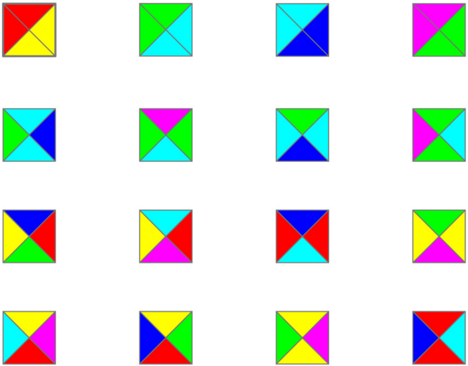
4/6/13

Introdction to Aperiodicity

- Periodic Tiling: Plane can be tiled by vector translations of prototiles.
    - Infinite chessboard
- Non-periodic: Opposite of periodic
    - Infinite chessboard, but switch around 2 squares at random.
- Aperiodic Tile Set: Set of prototiles that only form non-periodic tilings
- Question: Does an aperiodic prototile set exist?

Introduction to Aperiodicity (cont)

- Wang [1961] first proposes no aperiodic prototile set exists
  - "Decidability"
- Example:



[1]

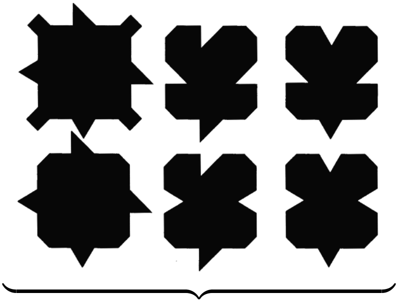
[1] http://grahamshawcross.com/2012/10/12/wang-tiles-and-aperiodic-tiling/usersgrahamshawcrossdocumentsblog_draftswang-tiles-and-aper-6/

Introduction to Aperiodicity (cont)

- 1966: Berger, Wang's student, uses Turing machine to show that Wang tiles must tile aperiodically.
    - 20,000+ tiles in first set
- Penrose [1974] narrows it down to 2 tiles
- "Einstein" problem
    - Ein = one, stein = rock
    - Can the plane be tiled aperiodically by only one tile?
- Taylor and Socolar [2010] find the solution... Maybe? (Next talk!)

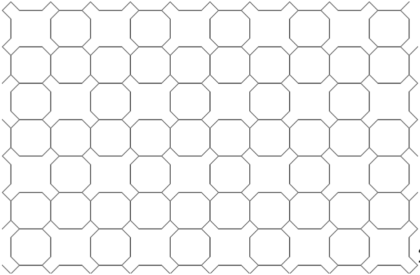
The Robinson Tiles

- Robinson [1971] described set of six tiles
- Tile plane aperiodically
-



$(a), (b), (c)$
$(d), (e), (f)$

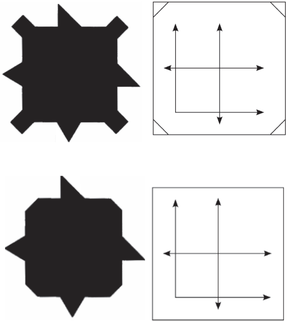
Corners

- Notice the difference of tile (a). Call it "cornered," others "cornerless"
- Following "Tilings and Patterns" (Grünbaum and Shephard, 1987), first consider simplified tile set
  - Only cornered and cornerless squares (ignoring the sides)
-  [2]
- Note that the cornered tiles must alternate every row or column and cannot be "back-to-back"

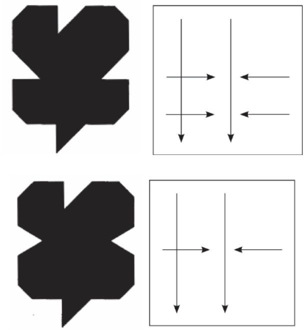
[2] http://fac-web.spsu.edu/math/tiling/21.html

Alternate Representation

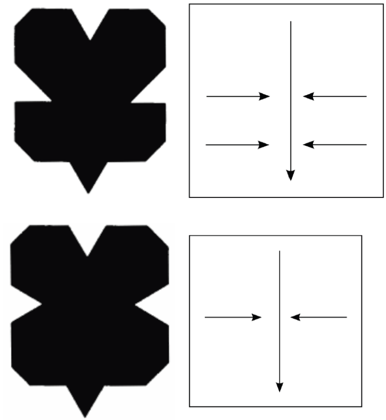
- For visual/analytic clarity,



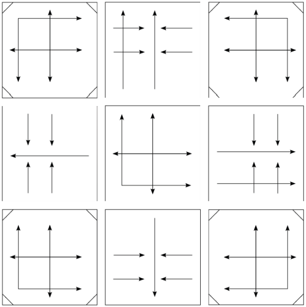
Alternate Representation (cont)

- 

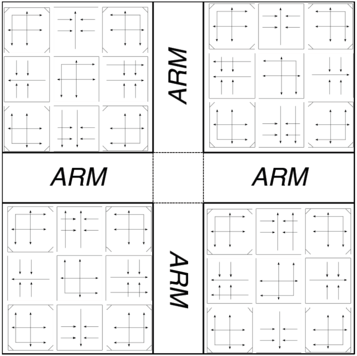
Alternate Representation (cont)

- 

3x3 Block

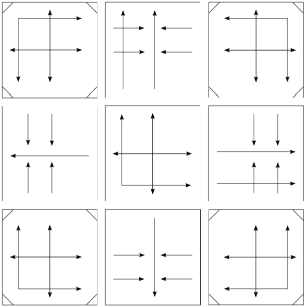
- Consider the 3×3 block
- 
- Cornered... Corners
- Uncornered cross in the middle

7x7 Block

- 

- Notice the three 3×3 blocks separated by two "fault" lines
- Can extend again to 15×15 etc... Using the same technique with "faults"

Proof of Aperiodicity

- Reconsider 3×3 block
- 
- Crosses are 2 squares apart
- In general, for any n, in a $(2^n - 1) \times (2^n - 1)$ block, crosses will be 2^n apart.
- However, this means no symmetry through a smaller distance than 2^n.
- Make 2^n arbitrarily large -> no symmetry -> Non-periodic

Relation to Taylor-Socolar

- Key is the ever expanding "block"
- Next talk will reveal an ever expanding "triangle" created by Taylor-Socolar
- Thus, they both have aperiodicity emerge from same source.

References

- Berger, R. (1966). "The undecidability of the domino problem", Memoirs Amer. Math. Soc. 66 (1966)
- Grünbaum, Branko; Shephard, G. C. (1987), Tilings and Patterns, New York: W. H. Freeman
- Penrose, Roger (1974), "The role of aesthetics in pure and applied mathematical research", Bulletin of the Institute of Mathematics and its Applications 10: 266ff.
- Robinson, R.M. (1971), "Undecidability and non-periodicity for tilings of the plane", Inventiones Mathematicae 12 (3): 177–190
- Socolar, Joshua E. S., and Taylor, Joan M. (2010), "An Aperiodic Haxgonal Tile," . arXiv:1003.4279. Bibcode:2010arXiv1003.4279S.
- Wang, Hao (January 1961). "Proving theorems by pattern recognition—II", Bell System Tech. Journal 40(1):1–41