# A Solution to the Einstein Problem: The Single-Tile Aperiodic Tiling Set

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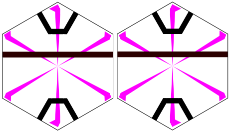
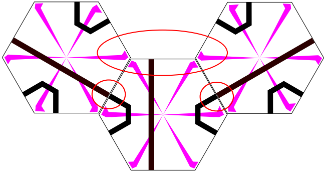
# Introduction to the Taylor-Socolar Tile

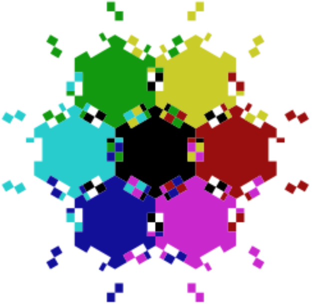


Reflections of the tile are allowed, and matching rules enforced by decorations as shown:



# Introduction to the Taylor-Socolar Tile

The second version of the tile's matching rules are enforced by shape alone:



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However, this tile is no longer a connected set.

[1]Socolar, Joshua E. S. and Joan M. Taylor, "An Aperiodic Hexagonal Tile", Journal of Combinatorial Theory, Series A 118 (2011) pp2207-2231

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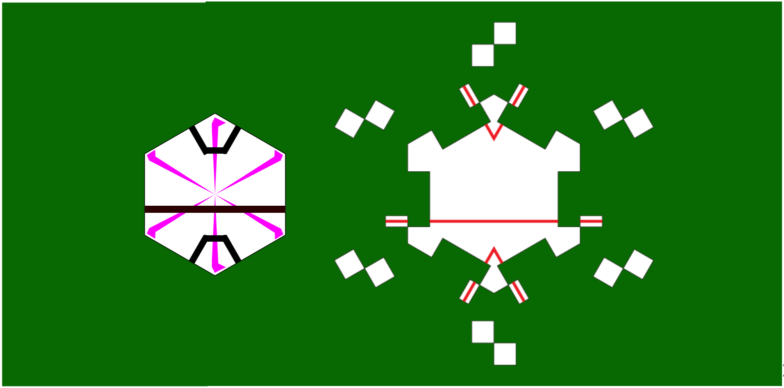

[2]

[2] Harris, Edmund "Socolar and Taylor's Aperiodic Tile". *Maxwell's Demon, Vain Attempts to Construct Order*. April 1, 2010. http://maxwelldemon.com/2010/04/01socolar_taylor_aperiodic_tile/

# Controversy

- The perfect aperiodic tile would:

  1) Have matching rules enforced by shape alone

  2) Be a simply connected set

  3) Tile the plane with rotations alone, and not reflections

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- The perfect aperiodic tile would:

  1) Have matching rules enforced by shape alone

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  3) Tile the plane with rotations alone, and not reflections

- It's important to note that 1) and 2) together create a strong condition for the allowed matching rules. Together, they limit the matching rules to local, pairwise matches.

# Controversy

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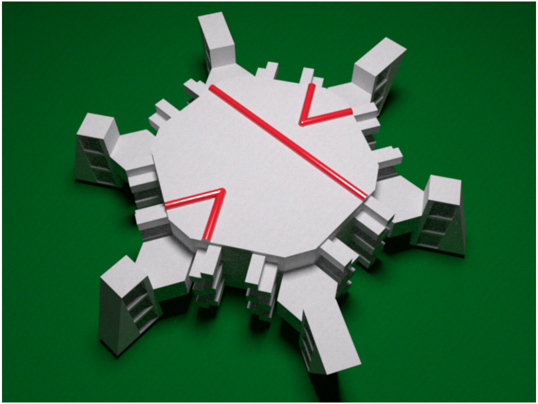
- Obviously, the TS tile breaks either 1) or 2), depending on which version of the tile we are examining.
- Taylor and Socolar's defense: the "shape alone" condition can be broken in a manner that isn't too objectionable.
- Relaxing 1) and instead requiring that all matching rules enforce a pairwise interaction, that still seems acceptable.

# Controversy

- It is possible however to make a 3D version of the tile which tiles the 2D plane aperiodically, is simply connected, and forces the tiling by shape alone.



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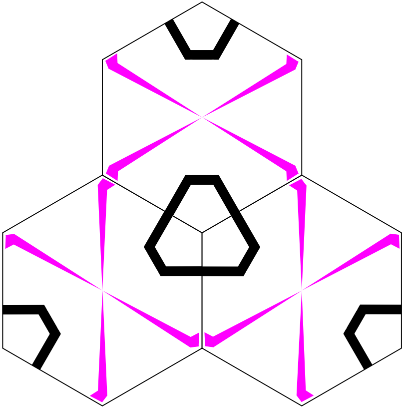
[3] Harris, Edmund "Socolar and Taylor's Aperiodic Tile". *Maxwell's Demon, Vain Attempts to Construct Order*. April 1, 2010.

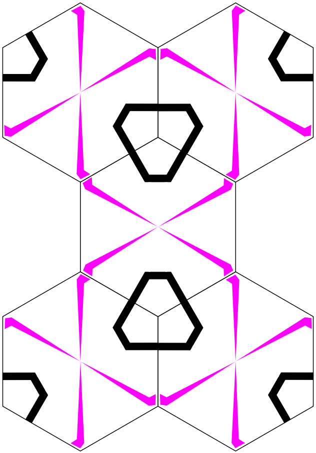
# Proof of Aperiodicity

The tile's matching rules quickly imply that adjacent tiles must form small triangles made from the black lines at the top and bottom of the tile:



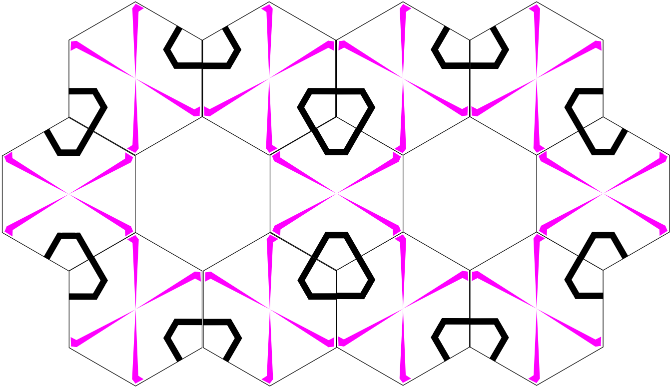
# Proof of Aperiodicity

Further, the flags ensure that every small black triangle must have another small triangle across from it like so:



# Proof of Aperiodicity

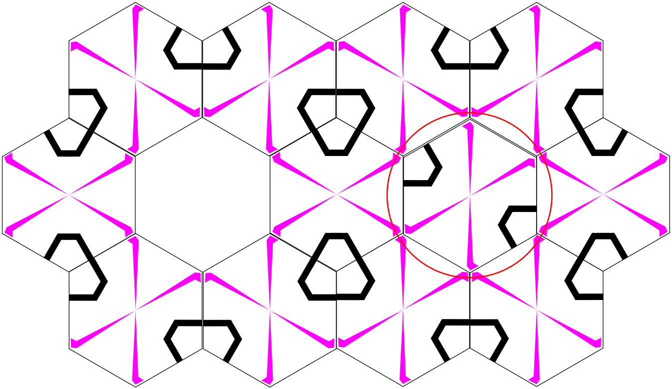
This forces the tiling to form honeycomb like lattices:



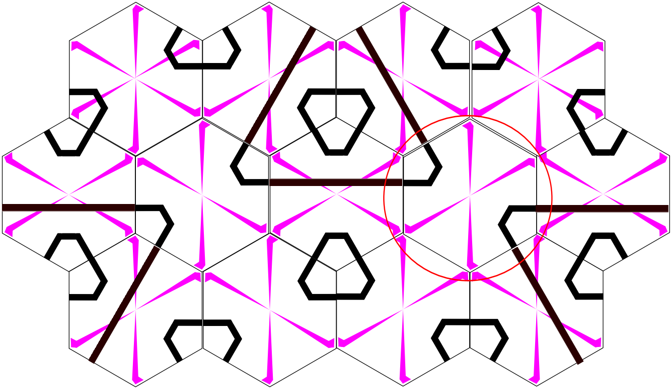
These are 2 level 1 lattices. Note that the center of every lattice is a free tile.

# Proof of Aperiodicity

Suppose we fill the right free tile with a tile that is rotated 60 degrees counter clockwise from a vertical orientation like so:

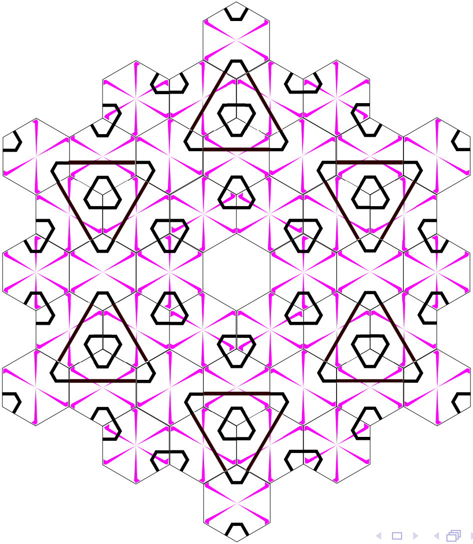


Then the matching rules immediately constrain the nearby tiles (shown here by adding the additional decorations) and start to form larger triangles:



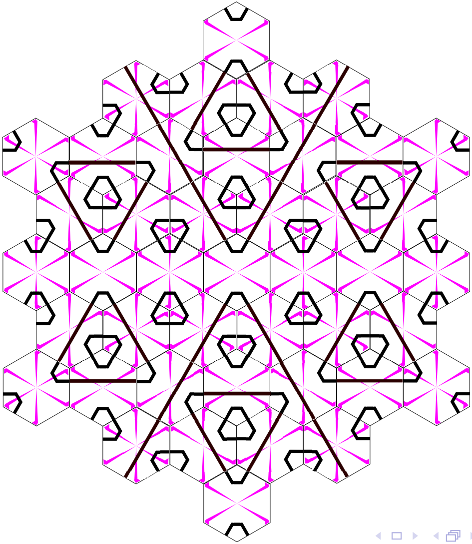
# Proof of Aperiodicity

The flags again force a large triangle across from every large triangle. This gives us a level 2 lattice:



# Proof of Aperiodicity

The center of a level two lattice is a free tile as well. Suppose we choose a vertically oriented tile for the center of this lattice:



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- Applying the previous argument iteratively, we know that we will have a free tile at the center of a level n lattice, which will create bigger triangles which forces a level n+1 lattice.
- Since there is no biggest triangle nor lattice, any tiling of the TS tile will be non periodic. Then, the tile forms an aperiodic tiling set.