BLOCK ISOPERIMETRIC PROBLEMS

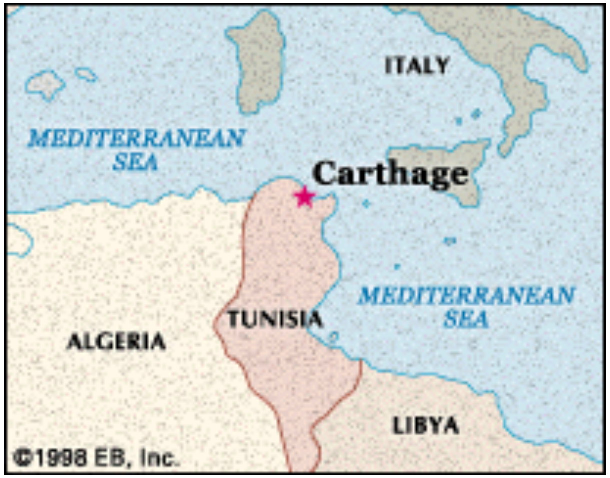
What do math and Legos™ have in common? Whales.

Our Research Problem

Using Legos™, what is the smallest perimeter that will enclose a given area?

HISTORICAL MOTIVATION

Queen Dido


ITALY
MEDITERRANEAN SEA
Carthage
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LIBYA
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Previous Work

$$MA(P) = (\frac{P}{2} - (\lceil\frac{P}{4}\rceil + 1))(\lceil\frac{P}{4}\rceil - 1)$$

- Maximum area is always square or pronic.
- Perimeter is always even.
- Conclusion: A square or a square-like rectangle has the greatest amount of area.

Relations Between The Projects

- They maximized area given a specified perimeter. We wanted to minimize perimeter given a specified area.

- Research Question: Given a specified area, what is the most efficient way to enclose that given area?

Definitions!

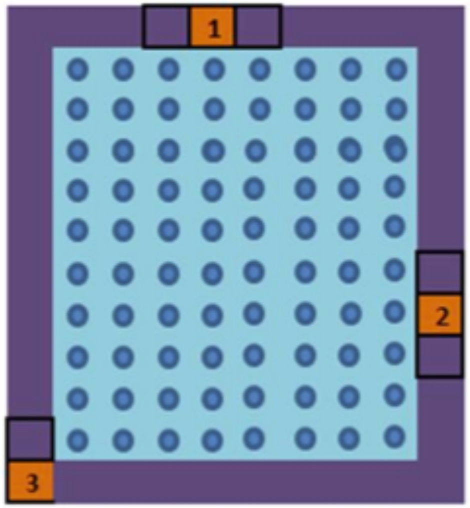
- Lego™ Curve: a single continuous arrangement of 1x1 blocks such that each block has exactly two neighbors. *

- Neighbor: a block that occupies the space adjacent to another block in one of the four cardinal directions along the grid. *
- * Referenced from work of: Bower, Espinoza, Green, Roca, and Townsend

Improper Lego™ Curves

Definitions!

- Perimeter: the number of blocks used to create a Lego™ curve. (Reminder: Perimeters are always even)



- Area: the number of studs enclosed by a Lego™ curve.

~~Our Research Problem~~

~~Background Information~~

Whale Theorem Proof

Hydropronic Sequence

Future Work

WHALE THEOREM

The Whale Theorem

Using Legos™, what is the smallest perimeter needed to enclose a specified area?

$$P(A) = 2\lceil 2\sqrt{A}\rceil + 4$$

~~Our Research Problem~~

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Whale Theorem Proof

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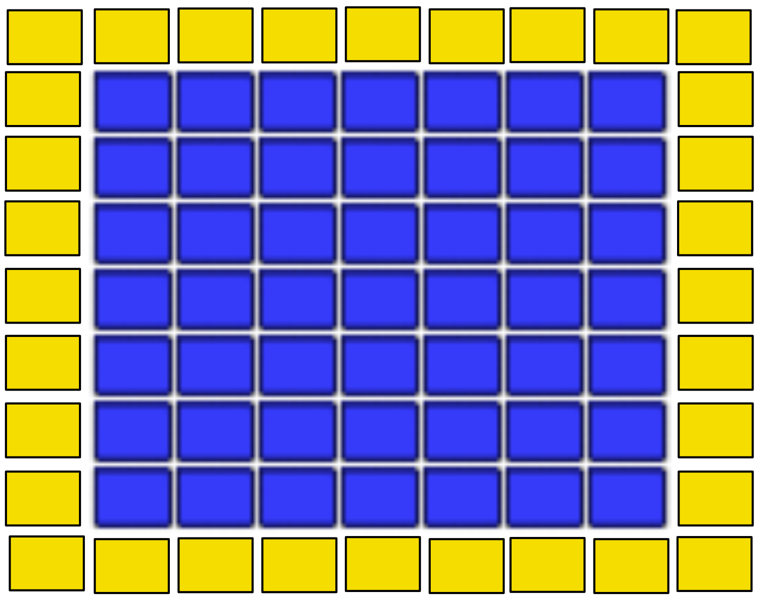
PROOF

Arrangement of Blocks

Area: 45
Next Square or Pronic: 49



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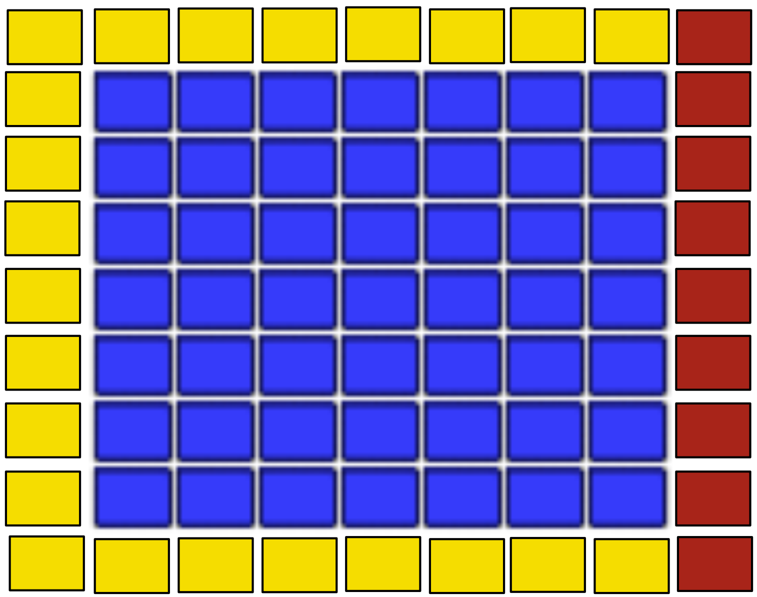

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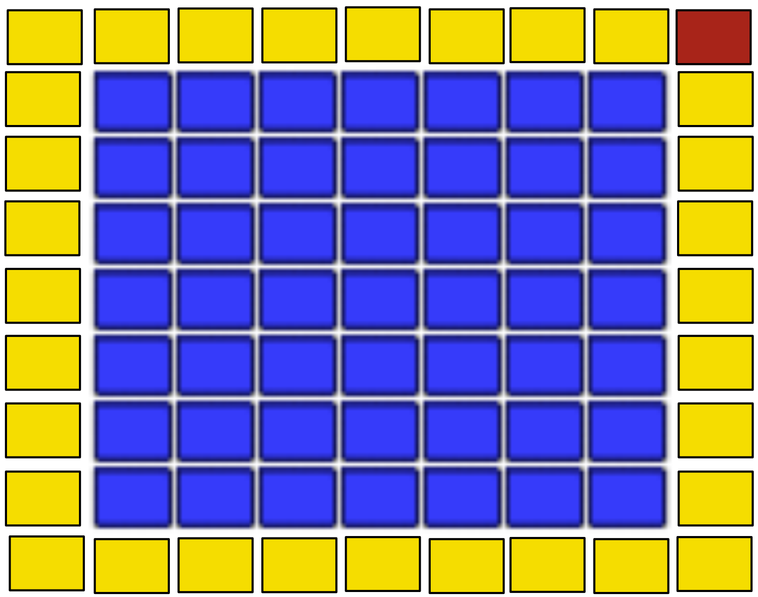

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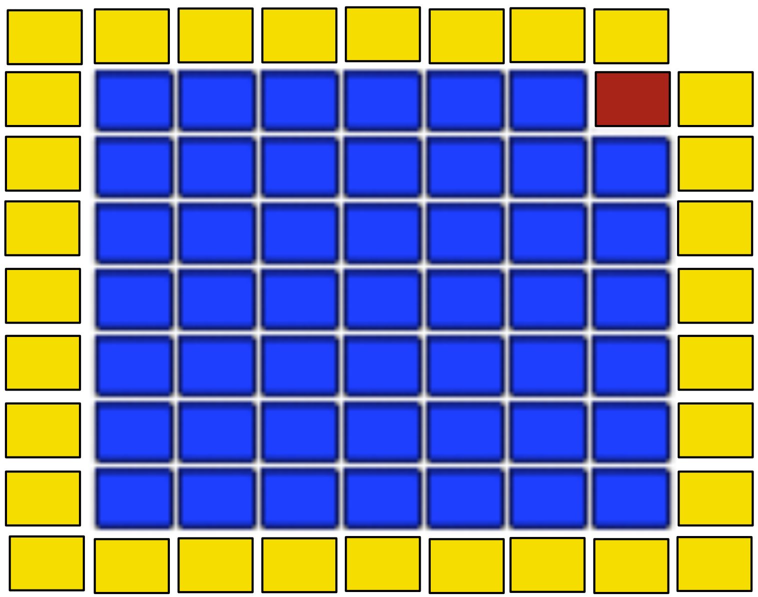

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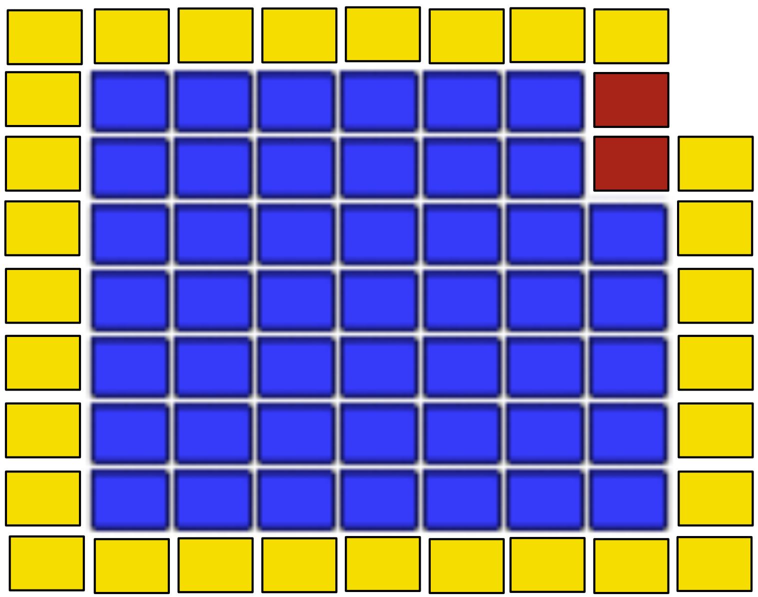

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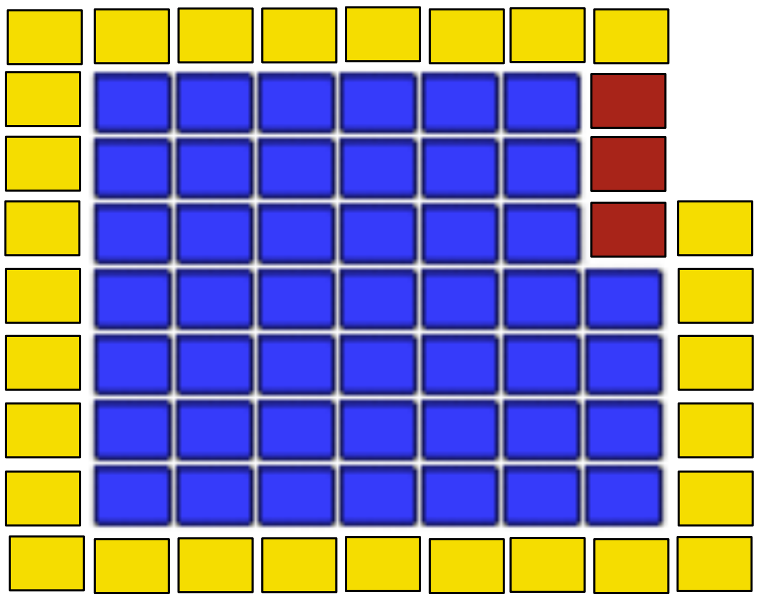

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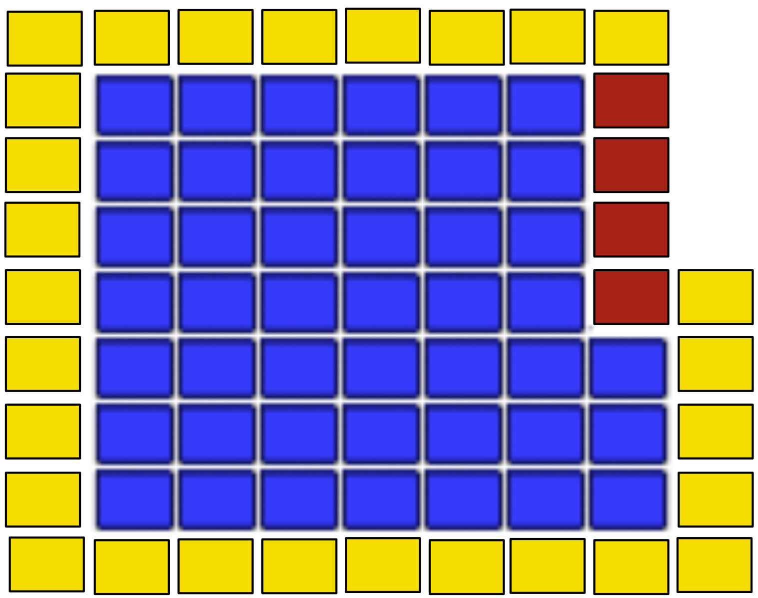

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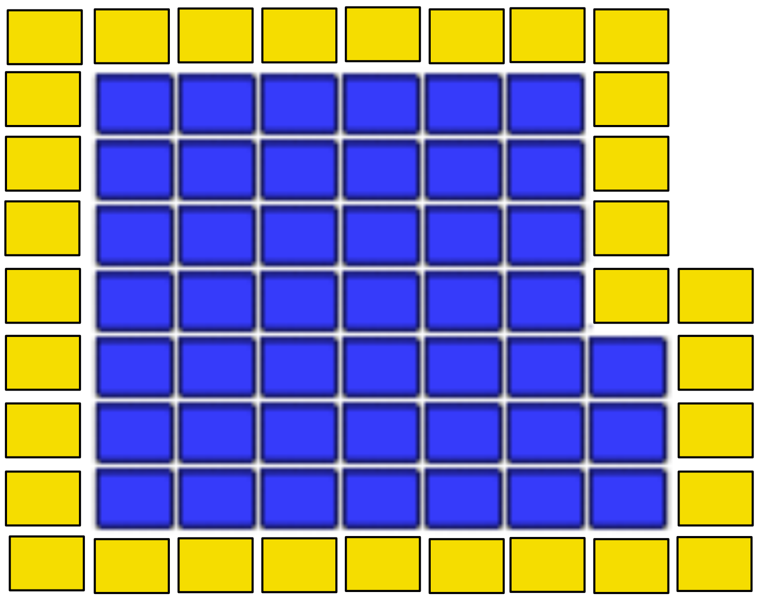


Non Square & Non Pronic Formula

- In order to find the next possible square or pronic use this formula:

$$A^* = \left\lceil \frac{A}{\lceil \sqrt{A} \rceil} \right\rceil \lceil \sqrt{A} \rceil$$

- Examples:
  - A = 8
  - A = 2012

Examples

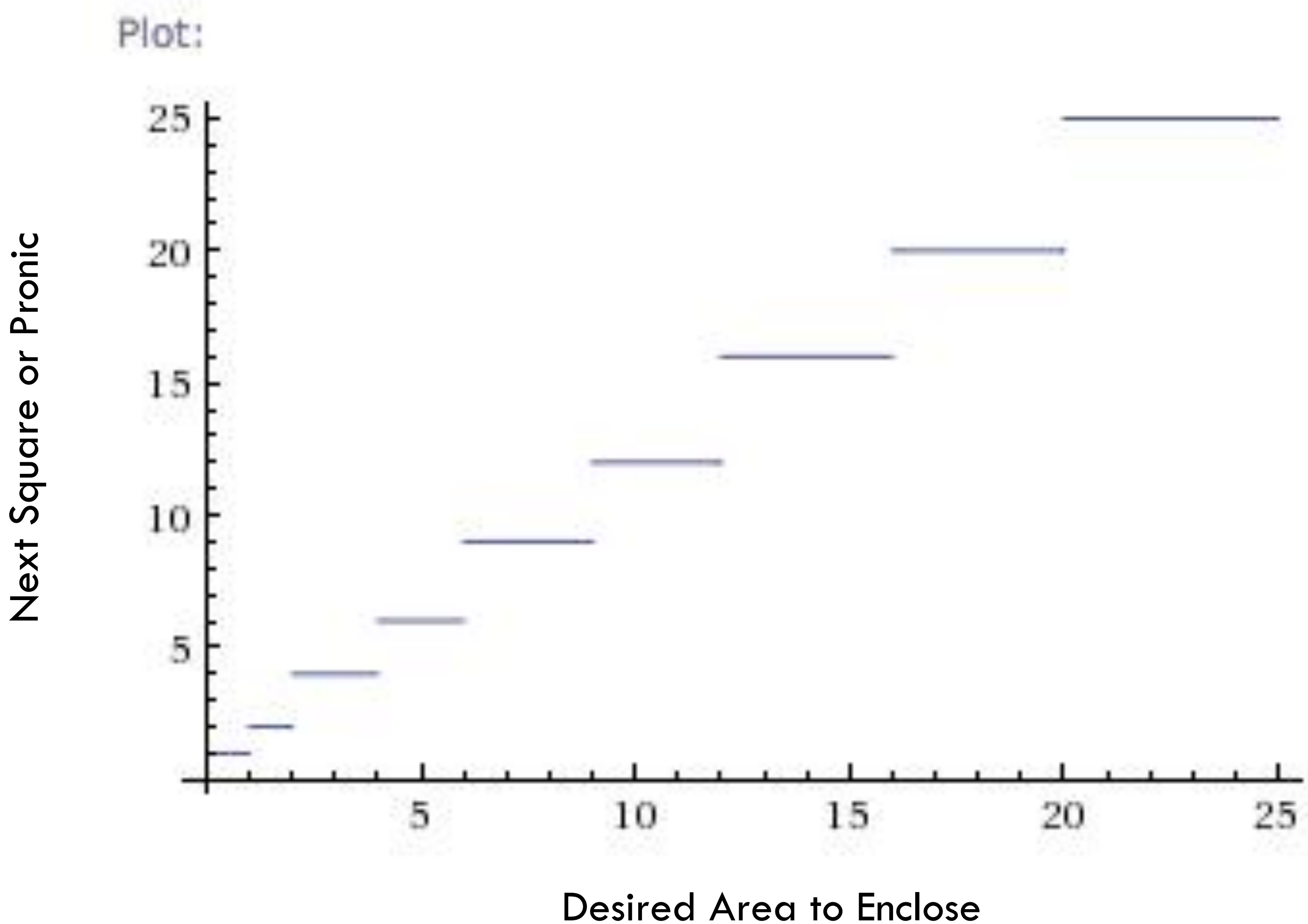
- A = 8

$$A^* = \lceil \frac{8}{\lceil \sqrt{8} \rceil} \rceil \lceil \sqrt{8} \rceil = 9$$

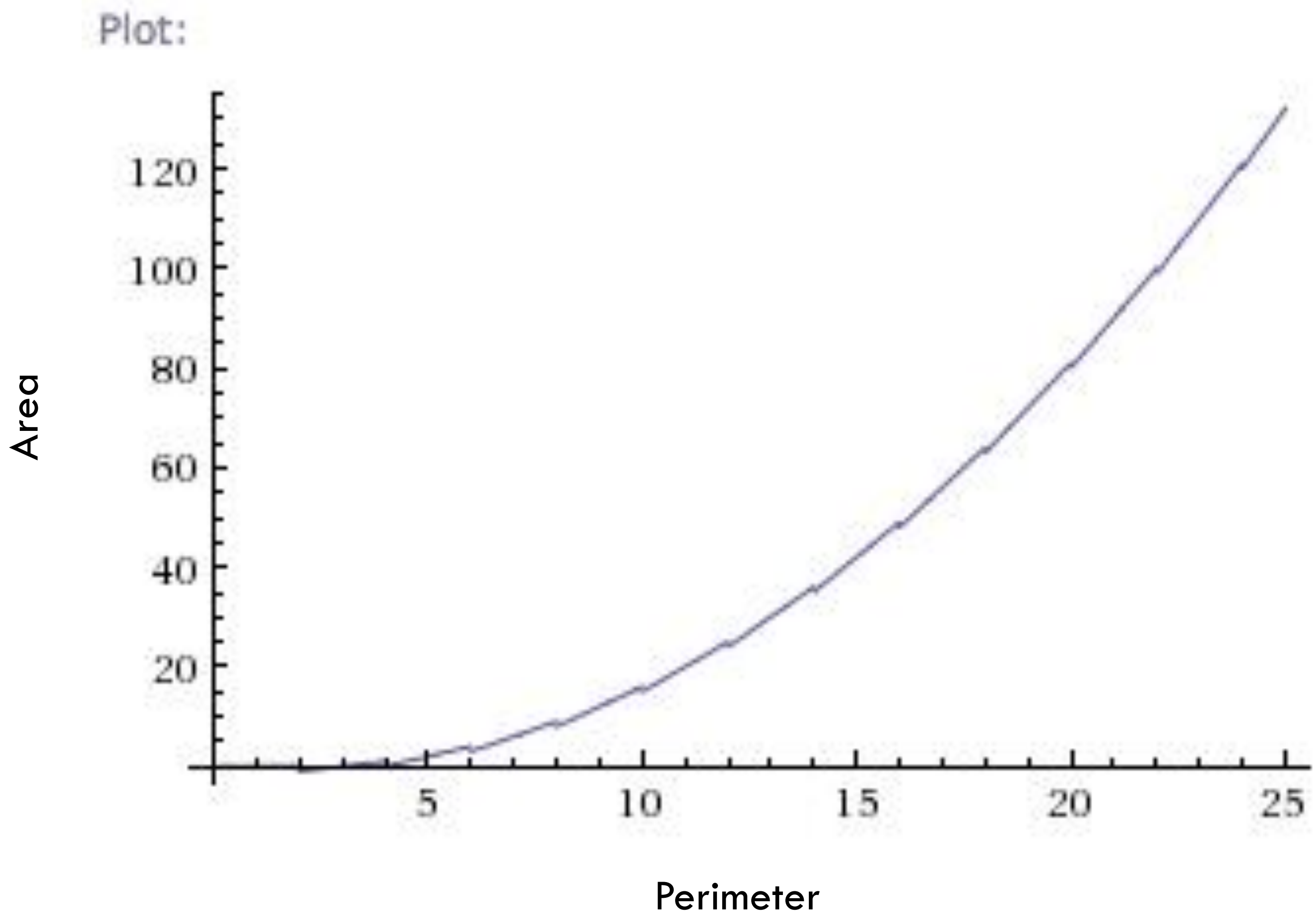
- A = 2012

$$A^* = \lceil \frac{2012}{\lceil \sqrt{2012} \rceil} \rceil \lceil \sqrt{2012} \rceil = 2025$$

Graph of: $A^* = \lceil \frac{A}{\lceil \sqrt{A} \rceil} \rceil \lceil \sqrt{A} \rceil$



Graph of: $MA(P) = (\frac{P}{2} - (\lceil \frac{P}{4} \rceil + 1))(\lceil \frac{P}{4} \rceil - 1)$



~~Our Research Problem~~

~~Background Information~~

~~Whale Theorem Proof~~

Hydropronic Numbers

Future Work

HYDROPRONIC NUMBERS

Hydropronic Numbers

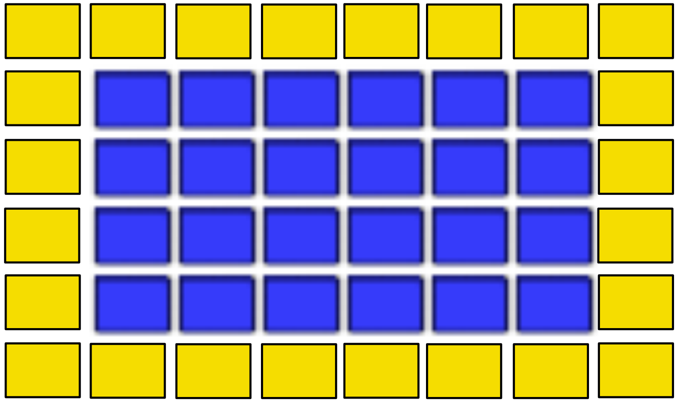
- A number that is not square or pronic and can be arranged as a rectangle using its two closest factors to maintain minimum perimeter.
- 3, 8, 10, 15, 18, 21, 24, 28, 32, 35, 40, 45…etc

Hydropronic Numbers

- Example: A = 24
  - Factors: 1, 2, 3, 4, 6, 8, 12, 24
  - Arrange a rectangle of 6 x 4

Hydropronic

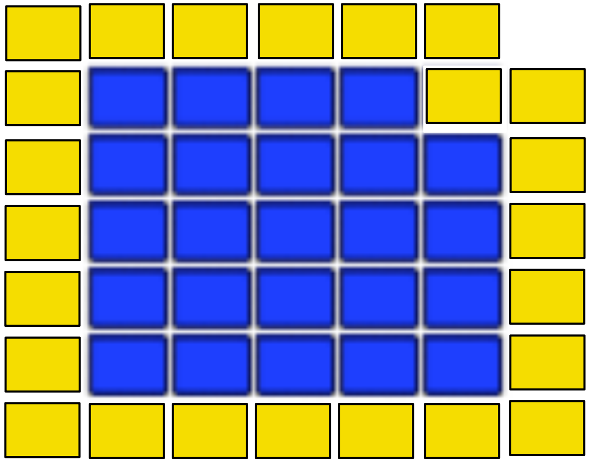
Area: 24
Hydropronic: 6 x 4 (24)



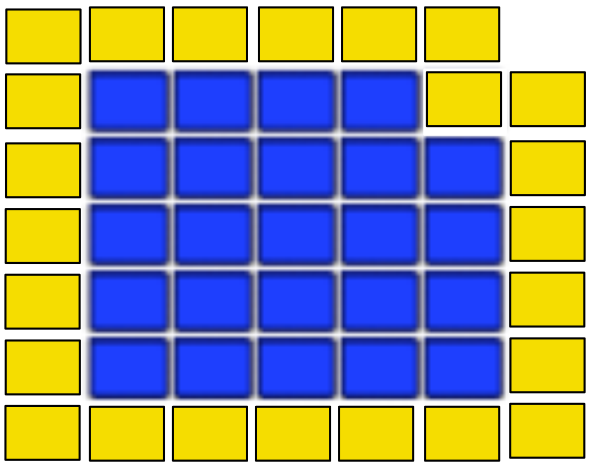
Whale Theorem

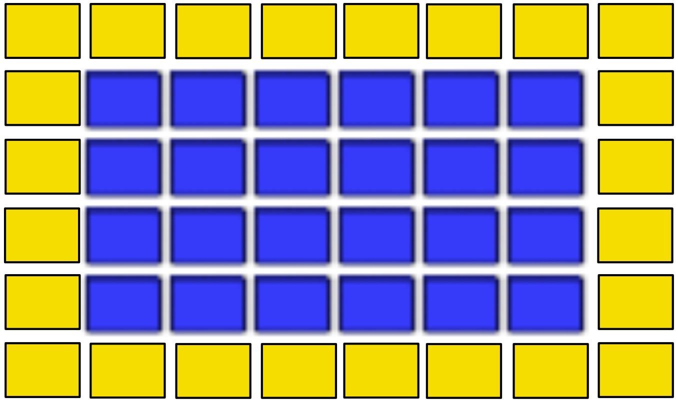
Area: 24

Next Square or Pronic: 5 x 5 (25)



Optimal Perimeters for Area = 24





Finding Hydropronic Numbers

$$A_k = \left\lceil \frac{A}{\lceil \sqrt{A} \rceil} \right\rceil \left(\lceil \sqrt{A} \rceil - 1 \right)$$

$$A^* = \left\lceil \frac{A}{\lceil \sqrt{A} \rceil} \right\rceil \lceil \sqrt{A} \rceil$$

WhaLego Sequence

- Numbers that can be arranged as rectangles and maintain the minimum perimeter are in the WhaLego sequence. All of these numbers can be classified as square, pronic, or hydropronic.
- The first few numbers in this sequence are:

1, 2, 3, 4, 6, 8, 9, 10, 12, 15, 16, 18, 20, 21…etc

~~Our Research Problem~~

~~Background Information~~

~~Whale Theorem Proof~~

~~Hydropronic Numbers~~

Future Work

FUTURE WORK & REFERENCES

Future Work

- Furthering Hydropronic Sequence
  - Finding a generating formula
- Lego Double Bubble Problem
  - Enclosing two areas
  - WhaLego sequence shows up
- 3-Dimensional Spaces

References

- Brower, T.; Espinoza, J.; Green, A.; Roca, A.; Townsend, B. **LEGO™ Isoperimetric Problem,** preprint (2010).

- Capogna, L.; Donatella, D.; Pauls, S.; Tyson, J. **An Introduction to the Heisenberg Group and the Sub-Riemannian Isoperimetric Problem,** 1st ed., *Birkhauser Verlag AG, Basel, Boston,* 2007.

Questions?

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