THE FRIENDSHIP PARADOX

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A SIMPLE QUESTION:

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❖ Do you have more Facebook friends than your Facebook friends do on average?

A FUN FACT (2011)

❖ The average Facebook user has 245 friends.

❖ Yet, the average friend of a Facebook user has 359 friends.

➔ The "friendship paradox" (Scott Feld, 1991)

GOAL

Compare:

The average # of friends of individuals

vs

The average # of friends of individuals' friends

GOAL

Define "score"= the number of friends one has
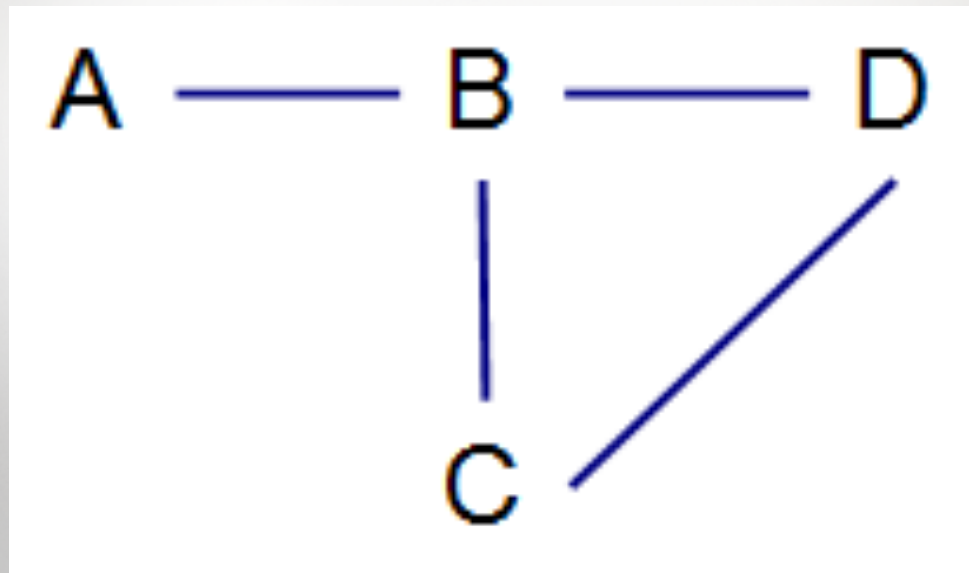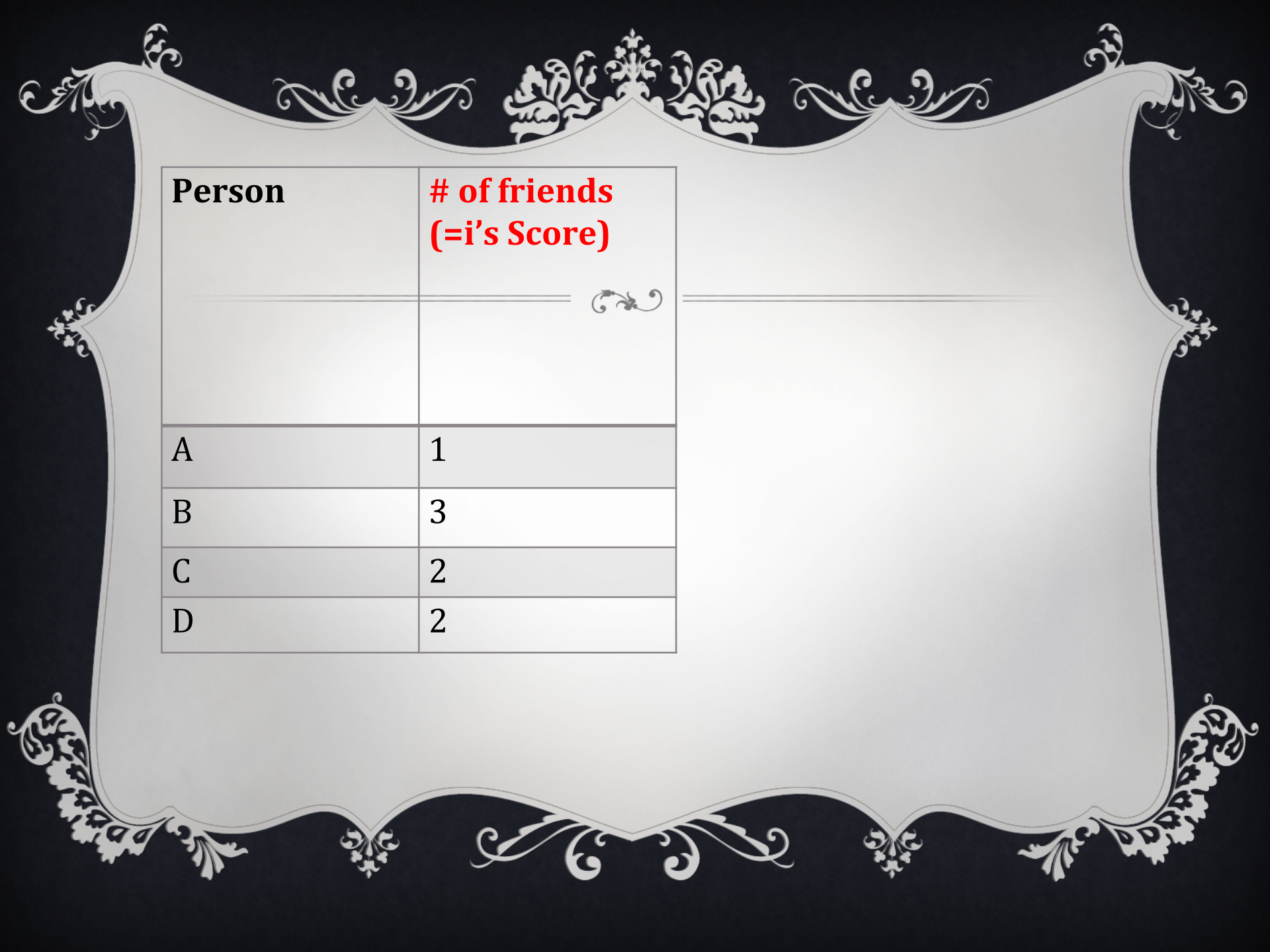
Thus, compare:

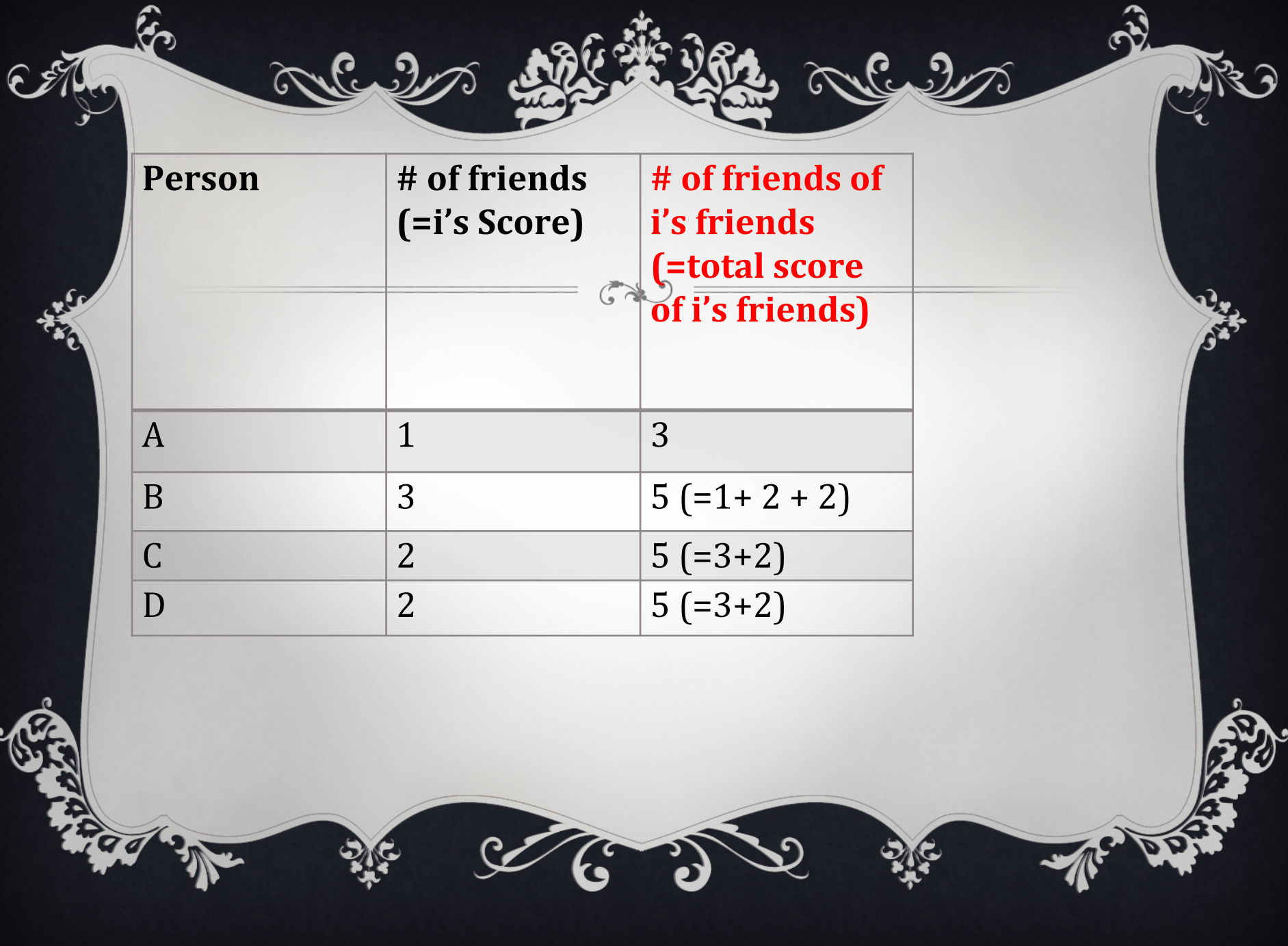The average score of individuals

vs

The average score of individuals' friends

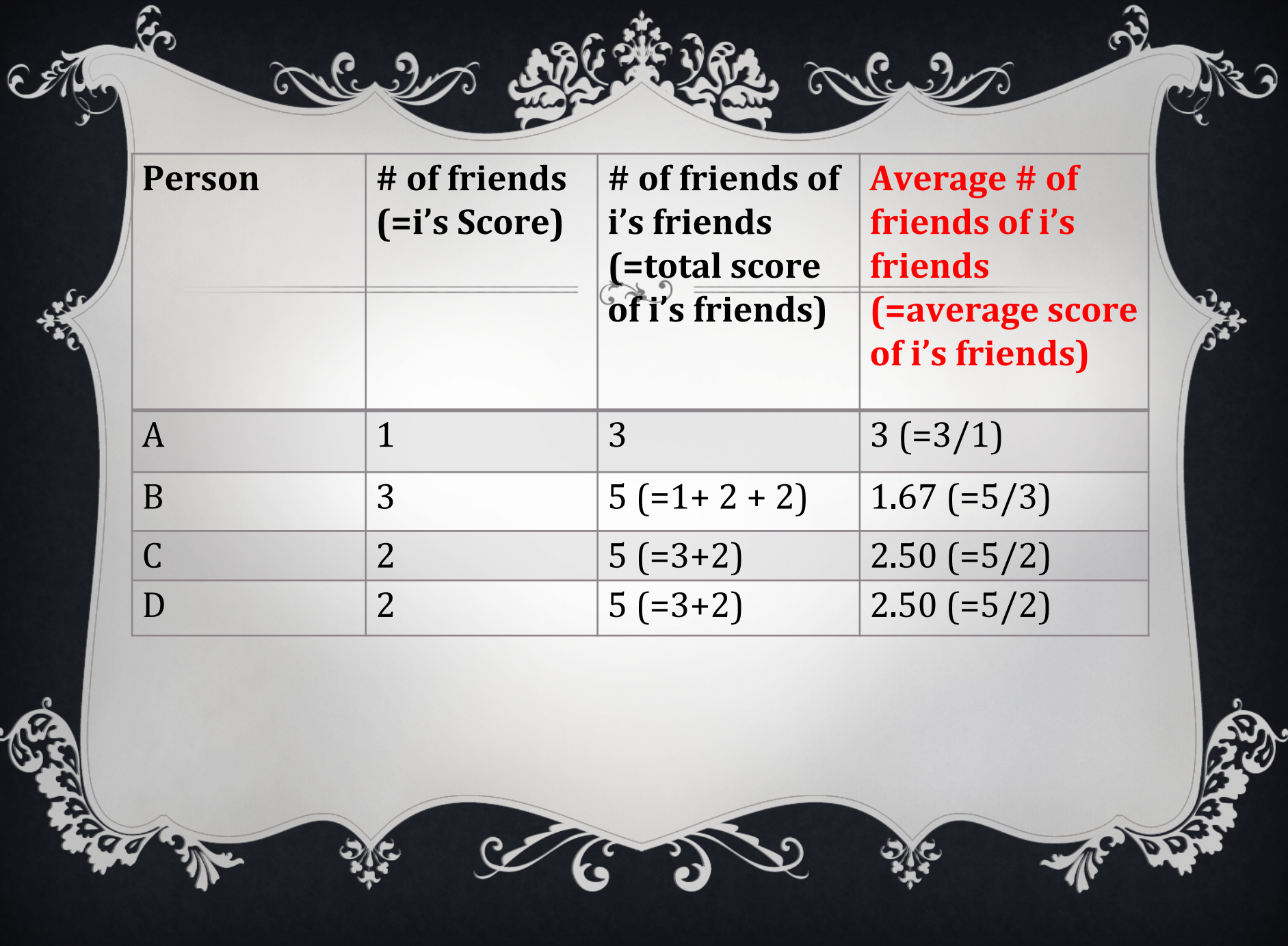# A CONCRETE EXAMPLE

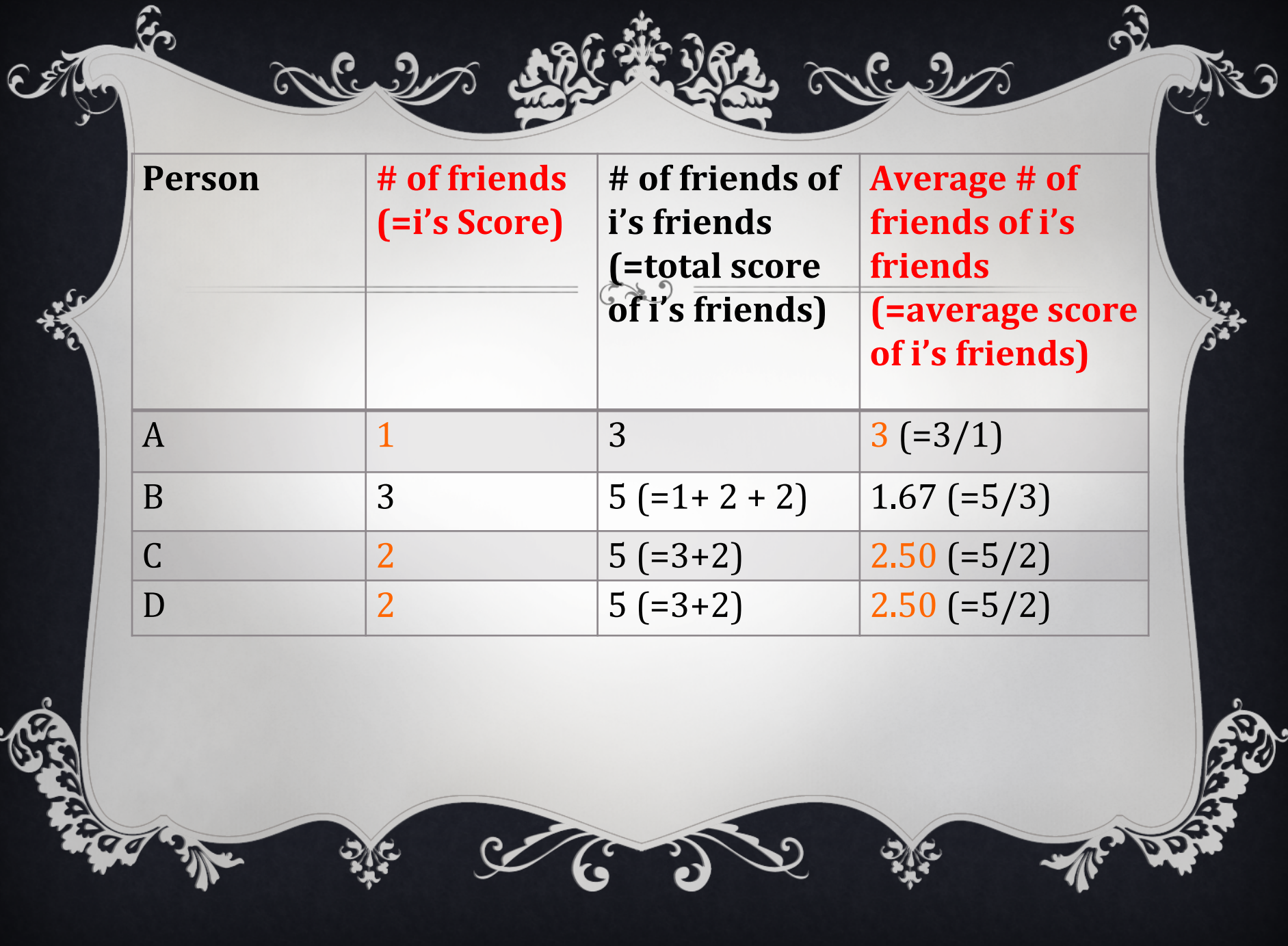| Person	# of friends (=i's Score)
A	1
B	3
C	2
D	2

Person	# of friends (=i's Score)	# of friends of i's friends (=total score of i's friends)
A	1	3
B	3	5 (=1+ 2 + 2)
C	2	5 (=3+2)
D	2	5 (=3+2)

Person	# of friends (=i's Score)	# of friends of i's friends (=total score of i's friends)	Average # of friends of i's friends (=average score of i's friends)
A	1	3	3 (=3/1)
B	3	5 (=1+ 2 + 2)	1.67 (=5/3)
C	2	5 (=3+2)	2.50 (=5/2)
D	2	5 (=3+2)	2.50 (=5/2)

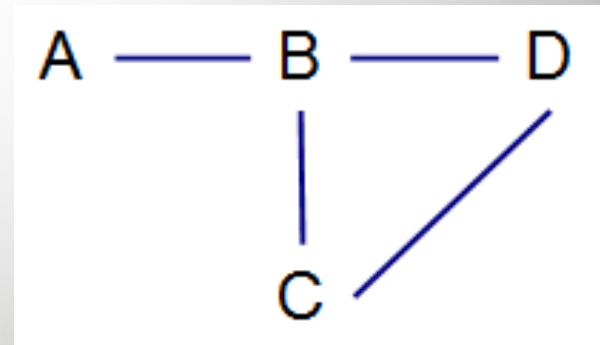| Person	# of friends (=i's Score)	# of friends of i's friends (=total score of i's friends)	Average # of friends of i's friends (=average score of i's friends)
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Total	8	18	
Mean	2		2.25 (=18/8)

A CONCRETE EXAMPLE

❖ On average, each individual has 2 friends

❖ Whereas on average, each friend has 2.25 friends!

THE MATHEMATICAL PROOF

❖ Suppose that there are n individuals in the system

❖ Suppose that person i (i = A, B, C, D, etc.) has score x_i

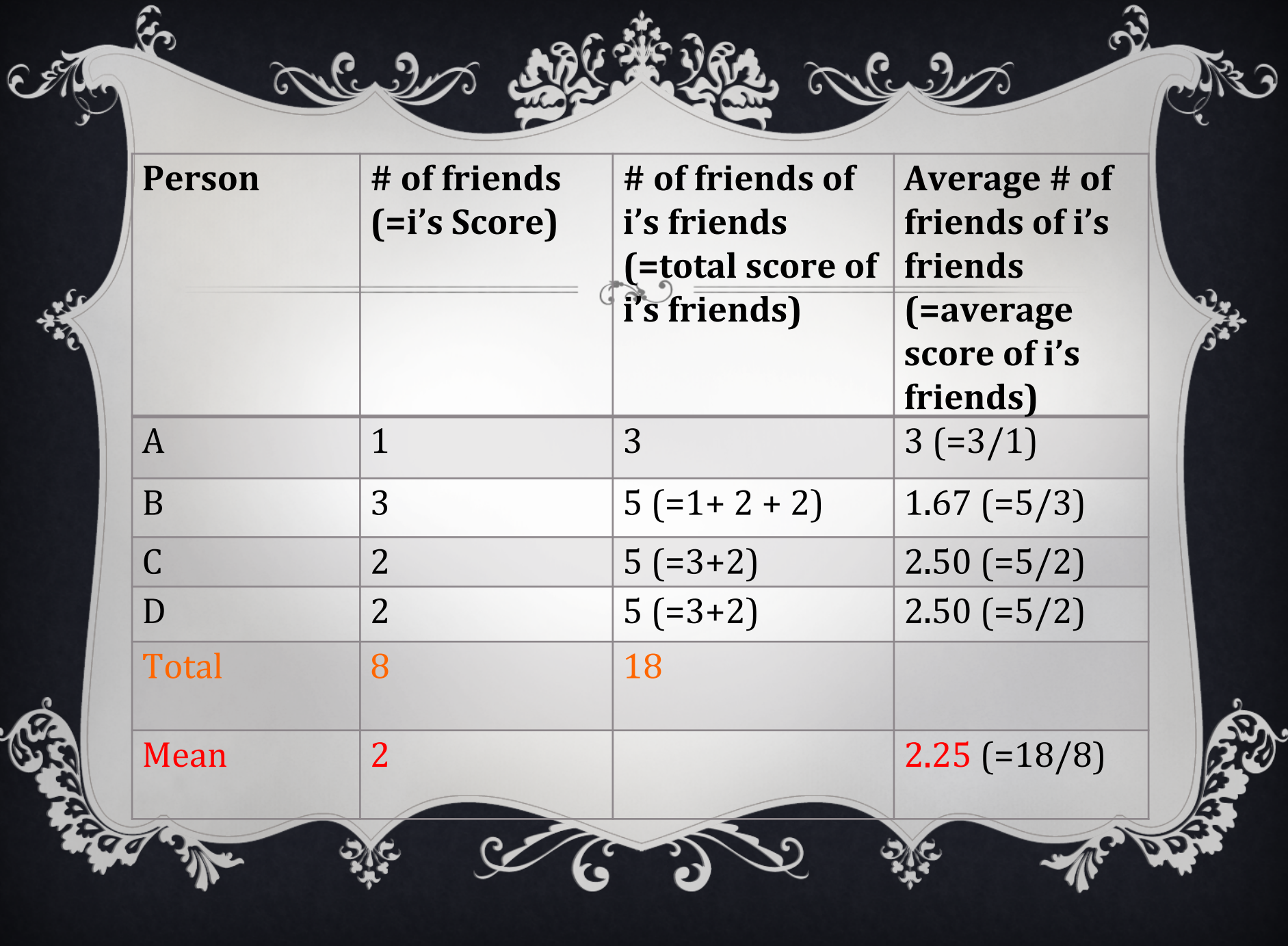❖ The average score of individuals: $\mu = \sum x_i / n$

THE TOTAL SCORE OF INDIVIDUALS' FRIENDS

❖ To calculate the total score of an individual's friends, we simply need to add up his friends' scores. (e.g., if B is friends with A, C, D, we will need to add up the scores x_A, x_C, x_D.)

THE TOTAL SCORE OF INDIVIDUALS' FRIENDS

Notice: The only times we need to include the score of a person i (i.e. x_i) to the total scores of friends in the system is <u>for his friends</u>. (e.g., the only times the term x_B appears in the total sum is for B's friends A, C, and D)

❖ For each person i, the sum has the term $(x_i)(x_i) = x_i^2$

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THE AVERAGE SCORE OF INDIVIDUALS' FRIENDS

❖ The total score of individuals' friends $= \Sigma (x_i)^2$

❖ The average score of individuals' friends=

$\Sigma (x_i)^2 / \Sigma x_i$

BY STATISTICS…

❖ Variance $\sigma^2 = (\sum (x_i)^2/n) - \mu^2$

To re-arrange it:

➔ $\sum (x_i)^2 = (\mu^2 + \sigma^2)n$

➔ $\sum (x_i)^2 / \sum x_i = (\mu^2 + \sigma^2)n/(\mu n) = \mu + (\sigma^2/\mu)$

HENCE…

❖ The average score of individuals= μ

❖ The average score of individuals' friends= $\mu + (\sigma^2/\mu)$