Modeling Underwater Acoustic Interface Waves

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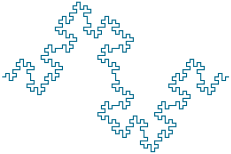

Hudson River Undergraduate Mathematics Conference

Introduction

- Sound = pressure, pressure causes displacement
- Elastic Potential Theory
- Elastic Parabolic equation (PE)

Goal: Compare Elastic Potential Theory displacement to PE displacement

Applications

- Nuclear Test Ban Treaty monitoring

APPLICATIONS

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- Tsunami warning systems

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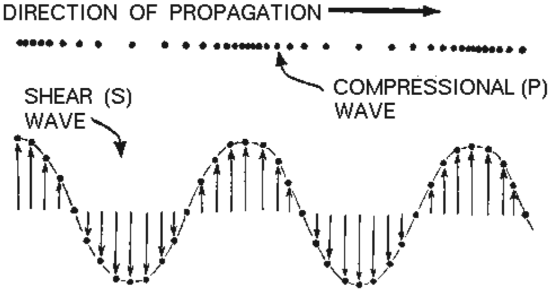
- Nuclear Test Ban Treaty monitoring
- Tsunami warning systems
- Explains anomalies in submarine transmissions

Compressional and Shear Waves

Compressional: (P) Particles travel parallel to the wave
Shear: (SV) Particles travel perpendicular to the wave



- Both types propagate in elastic media

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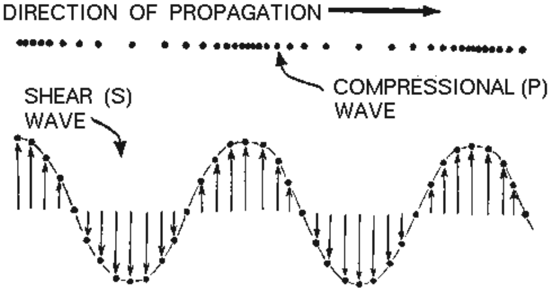

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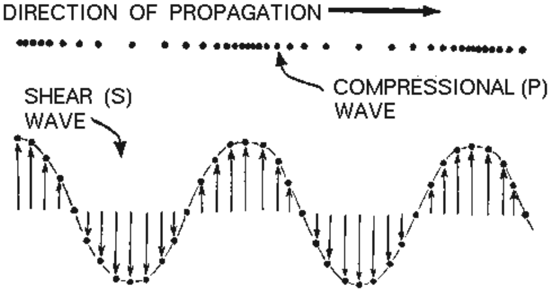

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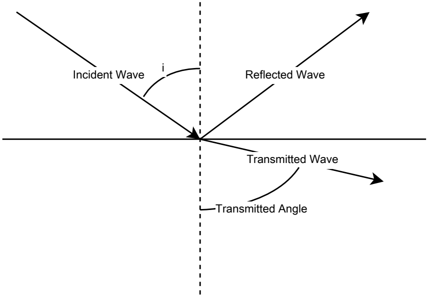
- Only compressional waves propagate in fluid media
- Interface waves require simultaneous incidence of P and SV waves on an interface

Snell's Law



$$\frac{sin(i)}{c_1} = \frac{\sin(\tau)}{c_2}$$

- τ is the transmitted angle, both c_1 and c_2 are propagation speeds
- $c_1 < c_2 \rightarrow \frac{c_1}{c_2} \sin(i) = \sin(\tau)$

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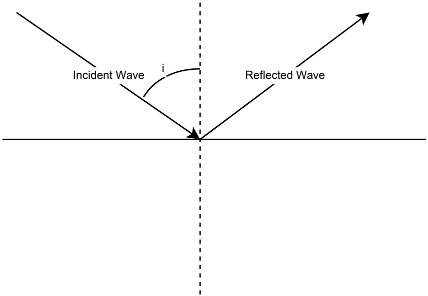

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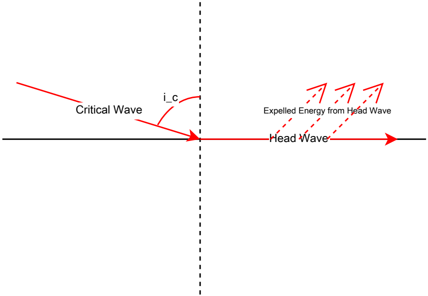
- $i < i_c \rightarrow$ no transmitted wave

Snell's Law



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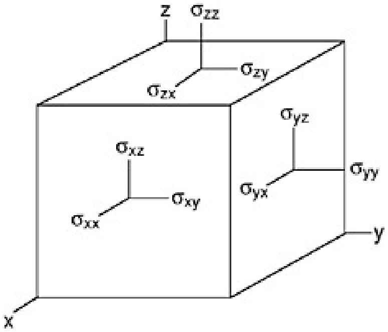
- $i = i_c$ (Head wave)

RAYLEIGH WAVES



Stress components acting on an infinitesimal rectangular parallelapiped

- Stress in elastic media
  - $\sigma_{xy} = \sigma_{yx}$
  - $\sigma_{xz} = \sigma_{zx}$
  - $\sigma_{zy} = \sigma_{yz}$
  - Six independent components remain

Rayleigh Waves

- Potential equations for compressional and shear waves in elastic media

$$\Phi = Ae^{[-\omega\hat{\eta_\alpha}z]}e^{[i\omega(\frac{x-t}{c})]}, \quad (1)$$

$$\Psi = Be^{[-\omega\hat{\eta_\beta}z]}e^{[i\omega(\frac{x-t}{c})]} \quad (2)$$

where $\hat{\eta_\alpha} = \sqrt{\frac{1}{\alpha^2} - \frac{1}{c^2}}$, $\hat{\eta_\beta} = \sqrt{\frac{1}{\beta^2} - \frac{1}{c^2}}$, and c is wave speed

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- Elastic displacement equation

$$\vec{U} = (\Phi_x - \Psi_z)\hat{x} + (\Phi_z - \Psi_x)\hat{y} + (\Phi_z + \Psi_x)\hat{z} \tag{3}$$

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- Apply the free surface boundary conditions $z = 0$, $\sigma_{zz} = 0$:

$$A[(\lambda + 2\mu)\eta_\alpha^2 + \lambda\left(\frac{1}{c}\right)^2] + B(\frac{2\mu\eta_\beta}{c}) = 0, \tag{4}$$

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- and $\sigma_{xz} = 0$:

$$A(\frac{2\eta_\alpha}{c}) + B(\left(\frac{1}{c}\right)^2 - \eta_\beta^2) = 0 \tag{5}$$

RAYLEIGH WAVES

Rewrite equations (3) and (4) as the matrix equation:

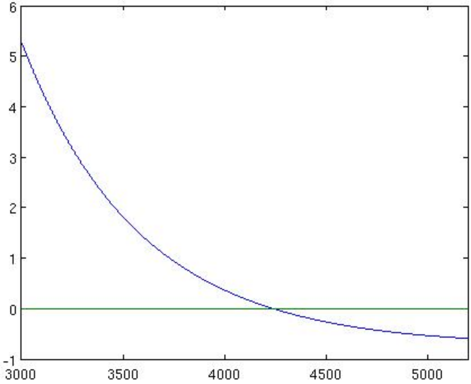
$$\begin{bmatrix} (\lambda+2\mu)\eta_\alpha^2 + \lambda\left(\frac{1}{c}\right)^2 & \left(\frac{2\mu\eta_\beta}{c}\right) \\ \frac{2\eta_\alpha}{c} & \left(\frac{1}{c}\right)^2 - \eta_\beta^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{6}$$

Nontrivial solutions exist where determinant equal to zero:

$$\left[(\lambda+2\mu)\,\eta_\alpha^2 + \lambda\left(\frac{1}{c}\right)^2\right]\left(\left(\frac{1}{c}\right)^2 - \eta_\beta^2\right) - 4\mu\left(\frac{1}{c}\right)^2 \eta_\alpha\eta_\beta = 0 \tag{7}$$

Rayleigh Waves

$$\left[(\lambda+2\mu)\,\eta_\alpha^2+\lambda\left(\frac{1}{c}\right)^2\right]\left(\left(\frac{1}{c}\right)^2-\eta_\beta^2\right)-4\mu\left(\frac{1}{c}\right)^2\eta_\alpha\eta_\beta=0$$



Find c that allows a solution for A and B

Future Work: Use roots in eq. 3 to find Rayleigh Wave displacements for different media

Parabolic Equation Derivation

- Helmholtz equation

$$\left(L\frac{\partial^2}{\partial x^2} + M\right)\begin{pmatrix} u_x \\ w \end{pmatrix} = 0. \tag{8}$$

- L and M are matrices containing depth derivatives

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- L and M are matrices containing depth derivatives
- Multiply by L^{-1} and factor

$$\left(\frac{\partial}{\partial x}+i(L^{-1}M)^{1/2}\right)\left(\frac{\partial}{\partial x}-i(L^{-1}M)^{1/2}\right)\begin{pmatrix}u_x\\ w\end{pmatrix}=0. \tag{9}$$

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- Assume outgoing energy dominates incoming energy

$$\frac{\partial}{\partial x}\begin{pmatrix}u_x\\ w\end{pmatrix}=i(L^{-1}M)^{1/2}\begin{pmatrix}u_x\\ w\end{pmatrix}. \tag{10}$$

This is the (u_x, w) parabolic equation for propagation is elastic and fluid media.

Parabolic Equation Model

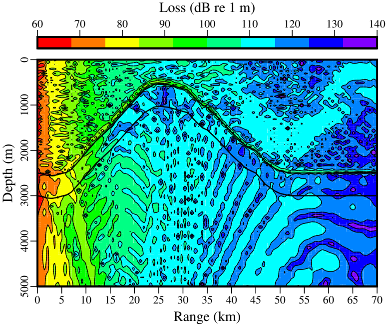
- Marching method
- Range dependent media
- Solves for pressure at every point in a range depth grid

Transmission Loss: Measure of signal weakening as it propagates outward from the source

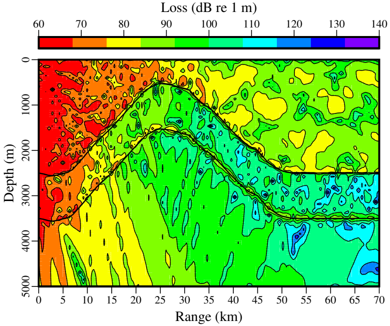
$$TL = -20 \log_{10} \left| \frac{p_r}{p_0} \right|$$

where p_0 is pressure near the source and p is pressure at the receiver

Parabolic Equation Results



Compressional Source

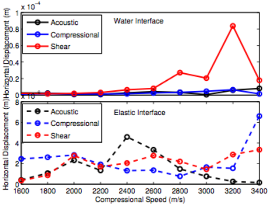


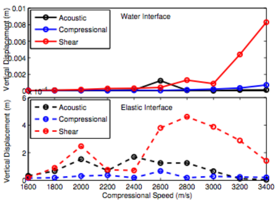
Shear Source

Parabolic Equation Results

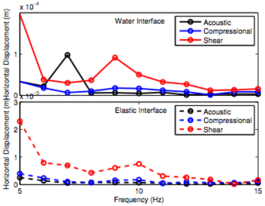
Pressure output form PE at both interfaces

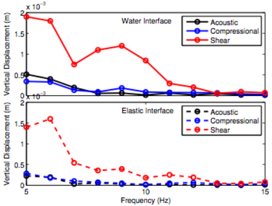
Soundspeed Analysis:





Frequency Analysis:





Future Work: Compare these curves to theoretical displacement curves

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