# Math 12:
# The Mathematics of LEGO Bricks


**Steven J Miller**

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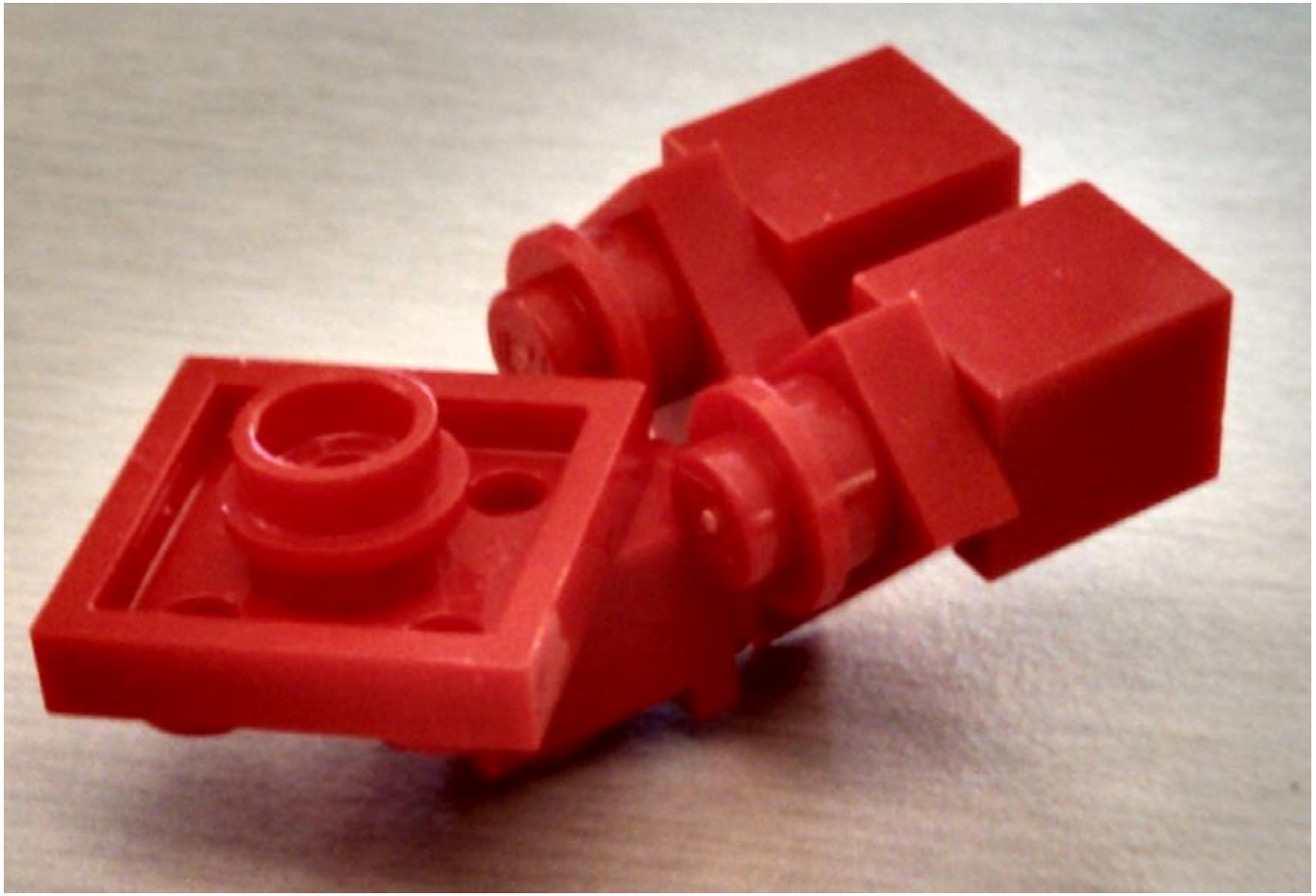
http://web.williams.edu/Mathematics/sjmiller/public_html/legos/

# Quick Overview

- Main Goals for Classes:
  - Lego Idea Challenge
  - MLK Bridge or Speed Build
  - Outreach with Elementary School
- My Qualifications:
  - Know math, know Legos.
  - More forgiving than Darth Vader, less than the Emperor.

# Challenge: Using less than 10 pieces…



# Plan for the Course

- **First Week:**
  - Introductions
  - Legos as a springboard to mathematics
  - Discuss in groups general ideas / purchase blocks
- **Weeks 2-3: meet in groups**
  - Devise build strategy, market research
  - Practice, practice, practice, …, practice.
- **Der Tag: Friday January 25th**

# This is the Flyer you were Looking For

If you're free on the **24th of January** (a Friday), around **3:30 pm** come help us try to set what will hopefully be a **Guinness World Record**: building a Lego Super Star Destroyer in as short a time as possible!

If you're interested, email Professor Miller at Steven.J.Miller@williams.edu!

Hope to see you there!

"Sir, the probability of successfully building a Super Star Destroyer in less than ten minutes is approximately 3,720 to 1..."
"Never tell me the odds!" – Han Solo, replying to C-3PO

# JOBS for The Superstar Build

- 1 second in command (then next year 4)
- 7 Bag Captains
- 7 Assistant Bag Captains
- 7 Sorters
- 7 Strategists
- 1 Publicist
- N-30 or N-32 Builders
- Motivation for jobs / outcome:
  - https://www.youtube.com/watch?v=O1E6To440TA
  - https://www.youtube.com/watch?v=s9t0AHNofgk
  - Instructions: http://web.williams.edu/Mathematics/sjmiller/public_html/legos/6005794.pdf
  - TIME LAPSE: https://www.youtube.com/watch?v=IpSjAYVZFBs&feature=youtu.be

# Possible Activities

- **Outreach Activities**
  - Williamstown Elementary School
  - Millenium Falcon….
- **Movies**
  - Lego Batman Movie, Lego Ninjago (new one in Feb)….
- **Presentations / Events**
  - History of Legos, Games, Challenges

**Always feel free to email me to help coordinate!**

# Grading / Course Mechanics

- Pre-reqs:
  - Basic algebra and a willingness to try suffice.
  - Must come to class, encouraged to participate.

- Grading (Previous Years):
  - Depending on role small paper, presentation, ….
  - High passes available to exceptional work / leaders if we succeed. (No longer possible….)
  - Perfunctory passes may be given for failure….

# UNIT ANALYSIS

Goal is to find good statistics to describe real world.

**Figure:** Harvard Bridge, about 620.1 meters.

# Unit Analysis

Goal is to find good statistics to describe real world.

**Figure:** Harvard Bridge, 364.1 Smoots ($\pm$ one ear).

# From LEGO Bricks to Math: What Cost?

# Unit Analysis

Sieze opportunities: Never know where they will lead.

Oliver Smoot: Chairman of the American National Standards Institute (ANSI) from 2001 to 2002, President of the International Organization for Standardization (ISO) from 2003 to 2004.

# From LEGO Bricks to Math: What Cost?


**560 pieces for $120, or 21 cents per piece (cpp).**

# From LEGO Bricks to Math


**292 pieces for $65, or 22 cents per piece (cpp).**

# Cost per piece


2015 class: Superstar Destroyer: $600 to $800 for 3152 pieces (now about $1000).
Cost per piece of 19 to 25 cents.


London Bridge: $240 for 4295 pieces
Cost per piece of **5.6** cents!

# Investments….


Click to open expanded view

## LEGO Star Wars Ultimate Collector's Millennium Falcon

by LEGO

29 customer reviews | 7 answered questions

Price: $5,999.99 + $41.79 shipping

**Note:** Not eligible for Amazon Prime. Available with free Prime shipping from other sellers on Amazon.

Only 1 left in stock.

**Estimated Delivery Date:** Jan. 6 - 11 when you choose Expedited at checkout.
Ships from and sold by West End Toys in easy-to-open packaging.

- This is the ultimate Millennium Falcon. The most important and most iconic model within the Star Wars universe.
- This BIG stable model measures (in inches): 33.1 (L) x 22.2 ( W) x 8.3 (H)
- Special features include a retractable boarding ramp. The cockpit top can be removed to access mini-figures
- The model can also be lifted in the black technic beams when gun turret is off
- This is it the biggest LEGO Star Wars model ever made - 5195 pieces
- For the first time in mini-figure scale. Noted as one of Lucas Licensing's favorite Star Wars models for 2007
- Mini-figure gallery: Han Solo, Chewbacca, Obi-Wan Kenobi, Luke Skywalker, Leia Organa
- Guns in top and bottom turrets can rotate. The top gun turret can be removed to sit mini-figure

LEGO IDEAS


## Buck Rogers Thu...

LuisPG
Jan 21, 2016

| 1164 | 167 |
|---|---|
| SUPPORTERS | DAYS LEFT |

16k 168 265

1480 recent supports

bramant1
15 days ago

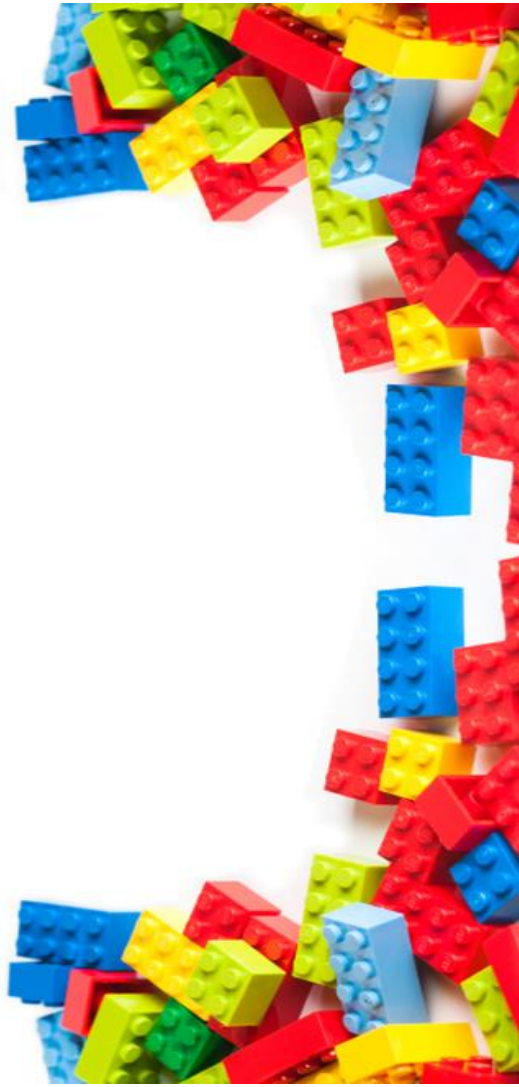
## House from Up

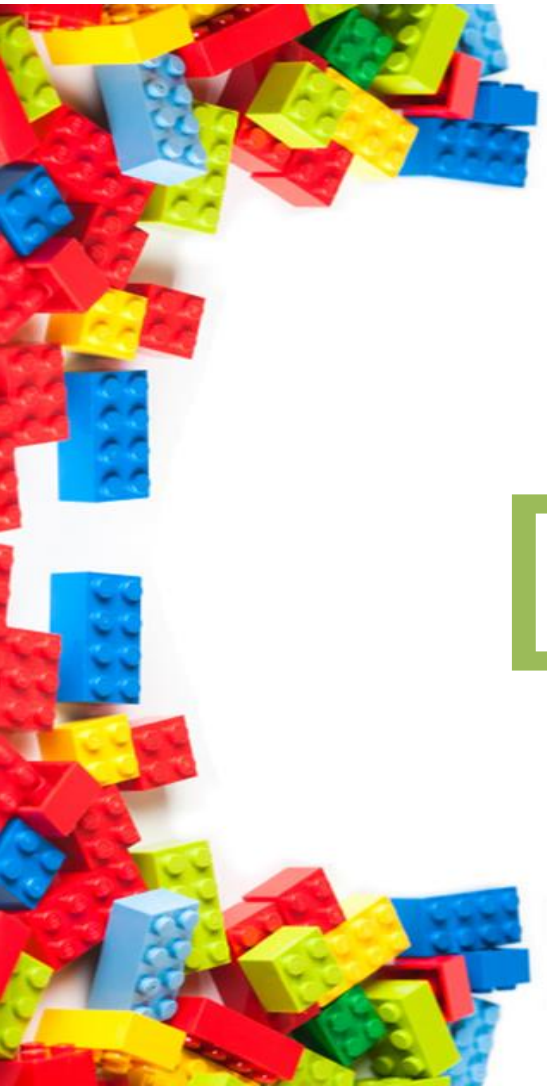
184 ...

Supporters 1550

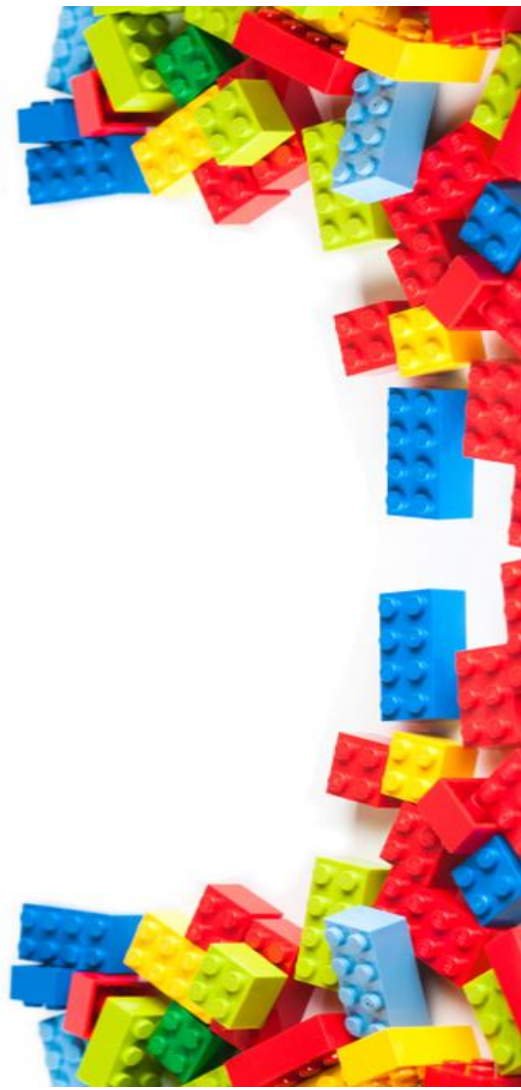
# 2018 LEGO Ideas Team:

Connor Barnes, Creative Supervisor
Hanna Beattie, Ice Wall Specialist
Bridget Bousa, Location Scout
Devon Caveney, Ice Wall Researcher
George Clark, Chief Engineer
Johnny Hinks, Human Resource Manager
Brendan Hoffman, Energy Consultant
Riley Van Der Brook, Red Leader
Neel Jain, Gold Leader
Tori Jasuta, Publicist
Antony Kim, Head of Research & Development
James McFarland, Design Specialist
George Peele, Winter Intern
Will Ruggiero, Assistant Piece Locator
Billy Sperry, Chief Piece Locator
Grant Wagman, Chief Transportation Specialist





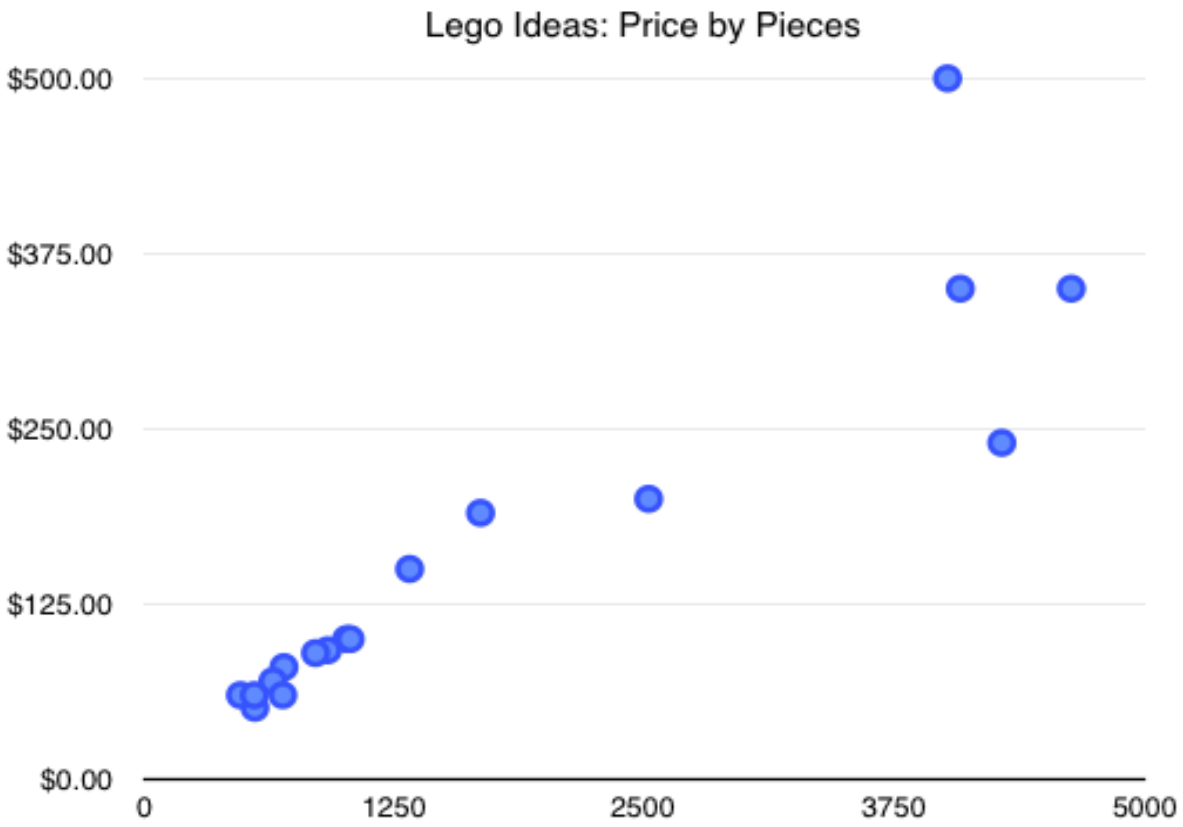
# Design

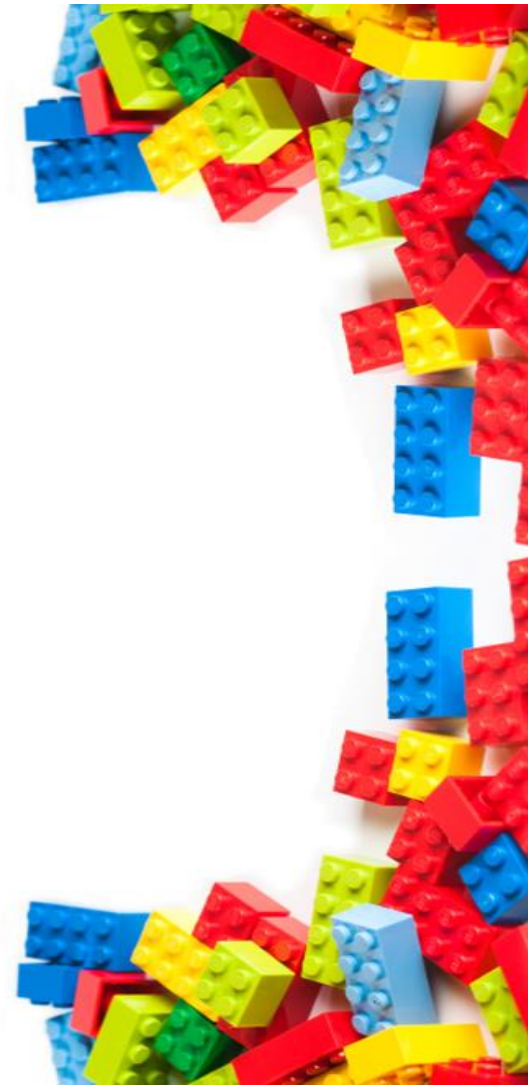


# Determining Price

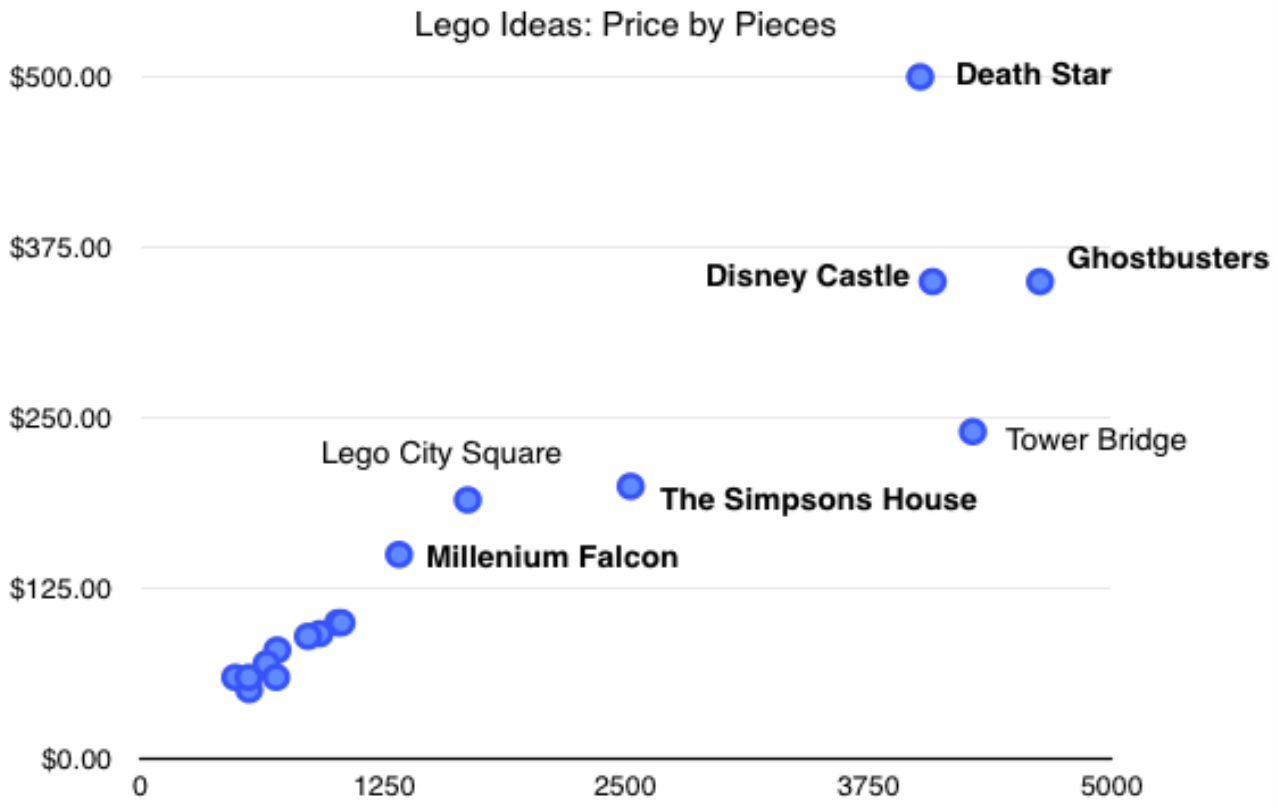

- Copyright costs ↑
- Special pieces ↑
- Number of pieces ↑↓
- Uniform pieces ↓
- Audience ↑↓

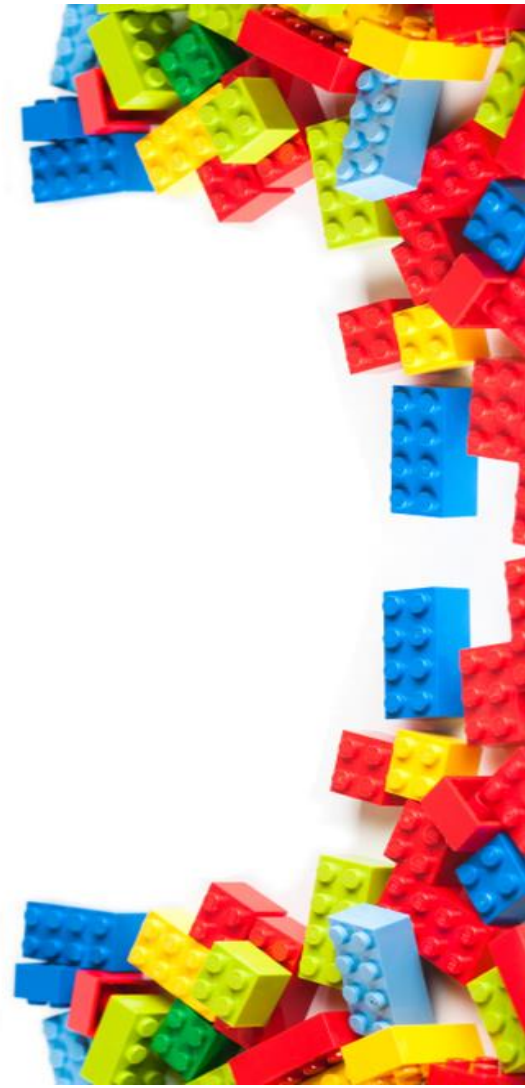
# Price by Piece P



# Price by Piece P



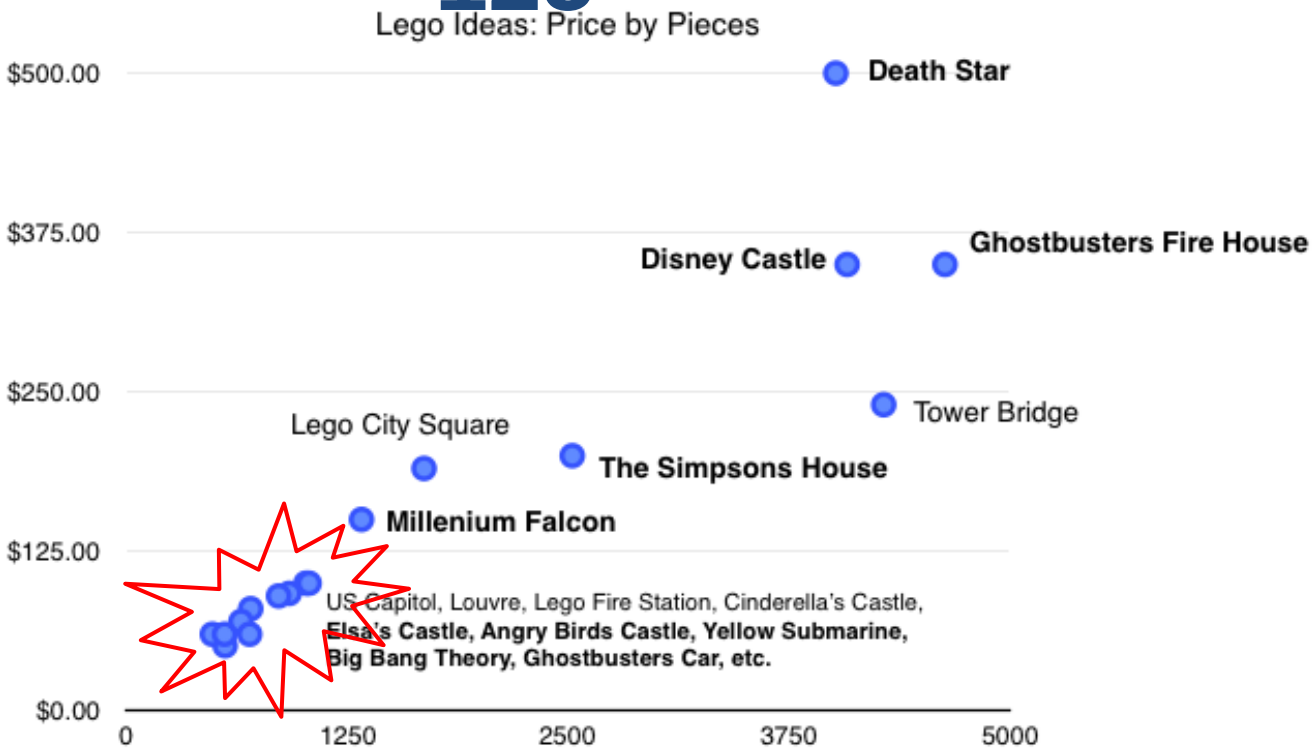



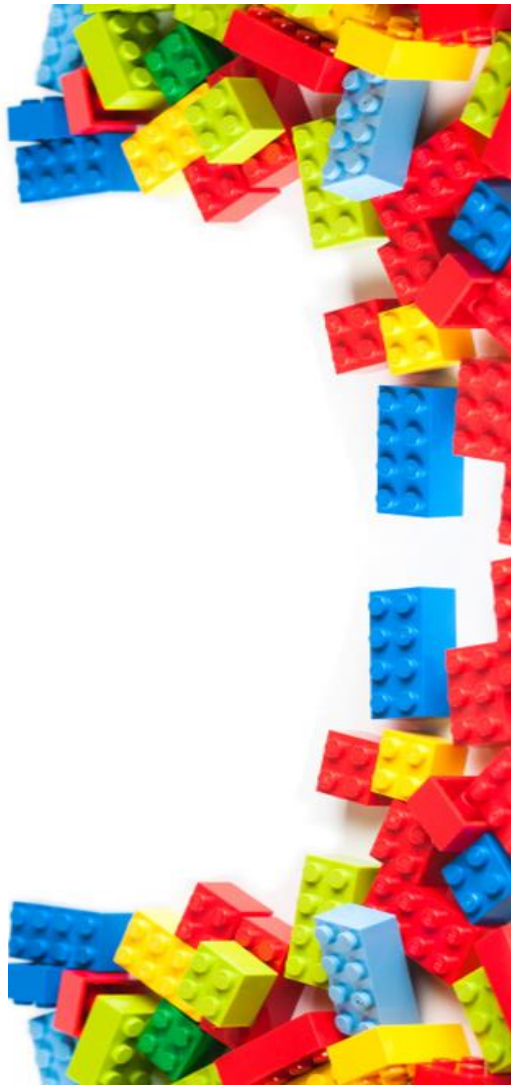
# Our Model

- 700-800 pieces
- Few special pieces
  - Option for LEGO to add on more (ie: mini-figures)
- Lots of uniform pieces (white wall, black fortress)

**Cost Estimate:**
$75-125

# Cost Estimate: $75-125



# LEGO and SCIENCE

https://www.peak-adventure.com/new-products/summer-break-survivor-challenge-week-of-723-727-kdpm2-jsl8t

# Lego Papers
# (opportunity to write book with me!)

MICHIGAN
ROSS SCHOOL OF BUSINESS

WDI PUBLISHING

case 1-429-326
September 3, 2013

**LEGO® Products: Building Customer Communities Through Technology**

# Lego and Science: Bio Complexity

**Reverse Engineering of Biological Complexity**
Marie E. Csete and John C. Doyle
*Science* **295**, 1664 (2002);
DOI: 10.1126/science.1069981

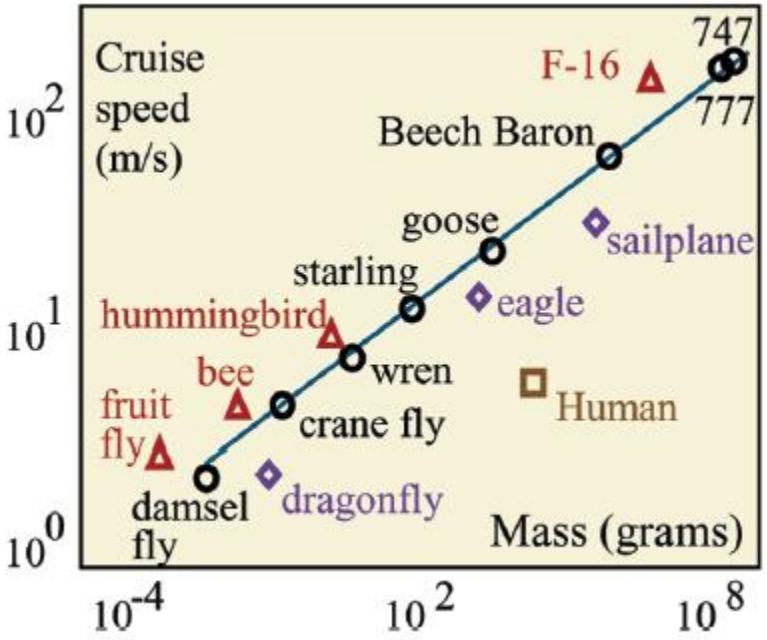
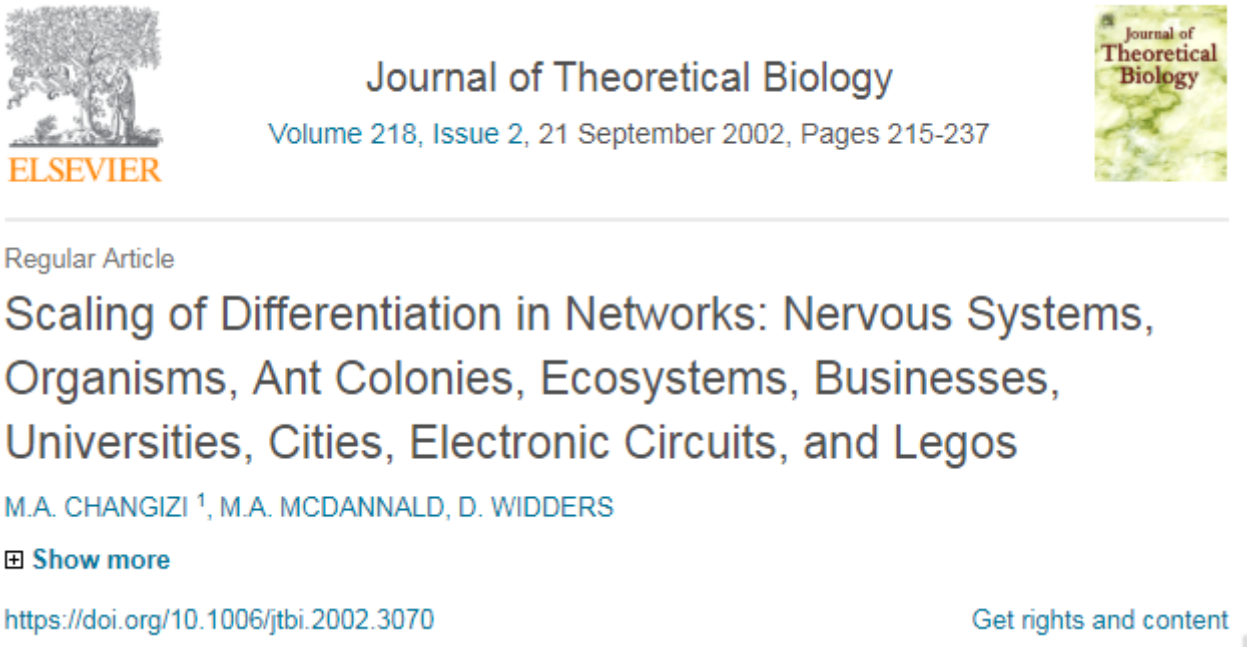



**Fig. 1.** Optimal cruise speed at sea level versus mass (log-log) for organisms and airplanes. Line is theoretical prediction (*12*) with $V = cM^{\alpha}$ and $\alpha = 1/6$ (*29*). Shorter wings for speed and maneuverability (triangles) yield higher cruise speeds than those optimized for soaring (diamonds). Most systems (circles) are compromises. Humans are not selected for powering flight and are far from optimal (square). Data and theory are from (*26*).

# Lego and Science: Piece Complexity



ELSEVIER

Journal of Theoretical Biology

Volume 218, Issue 2, 21 September 2002, Pages 215-237

Regular Article

## Scaling of Differentiation in Networks: Nervous Systems, Organisms, Ant Colonies, Ecosystems, Businesses, Universities, Cities, Electronic Circuits, and Legos

M.A. CHANGIZI [1], M.A. MCDANNALD, D. WIDDERS

⊞ Show more

https://doi.org/10.1006/jtbi.2002.3070



https://ac.els-cdn.com/S0022519302930705/1-s2.0-S0022519302930705-main.pdf?_tid=dd5f971b-f400-4388-a68d-7c0ddb5290dc&acdnat=1548166838_f7e9d0fadc96665912a31ba6d7303dc4

# Lego and Science: Piece Complexity

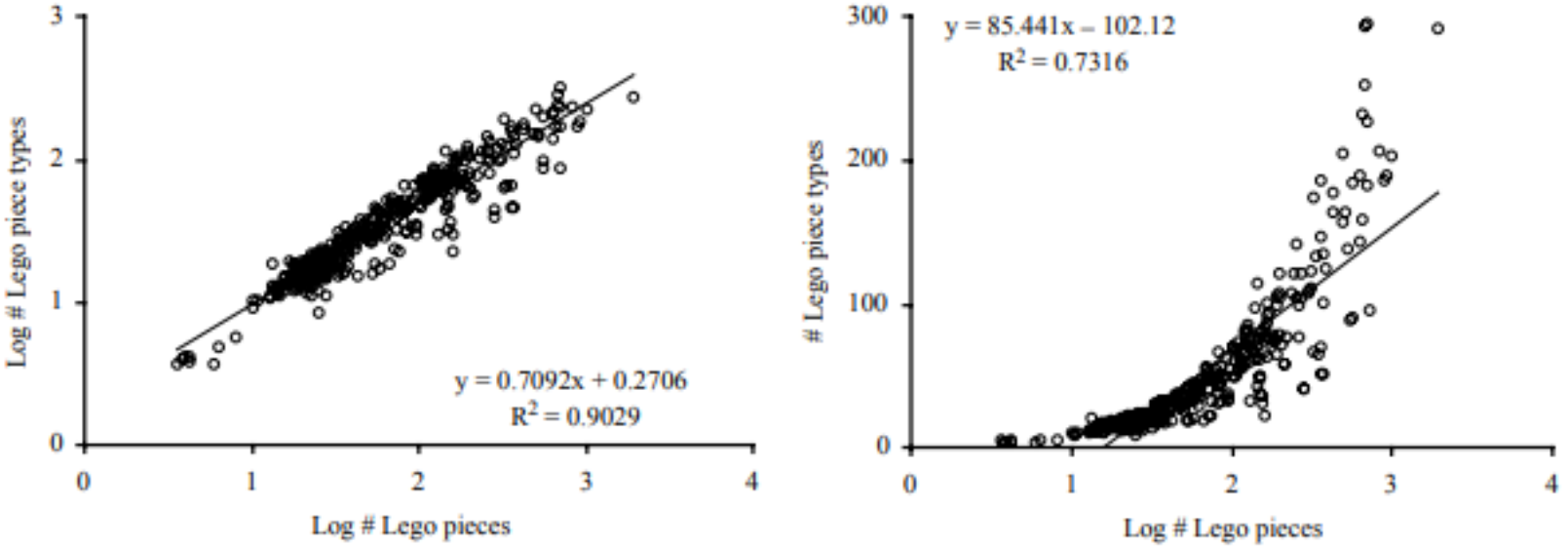
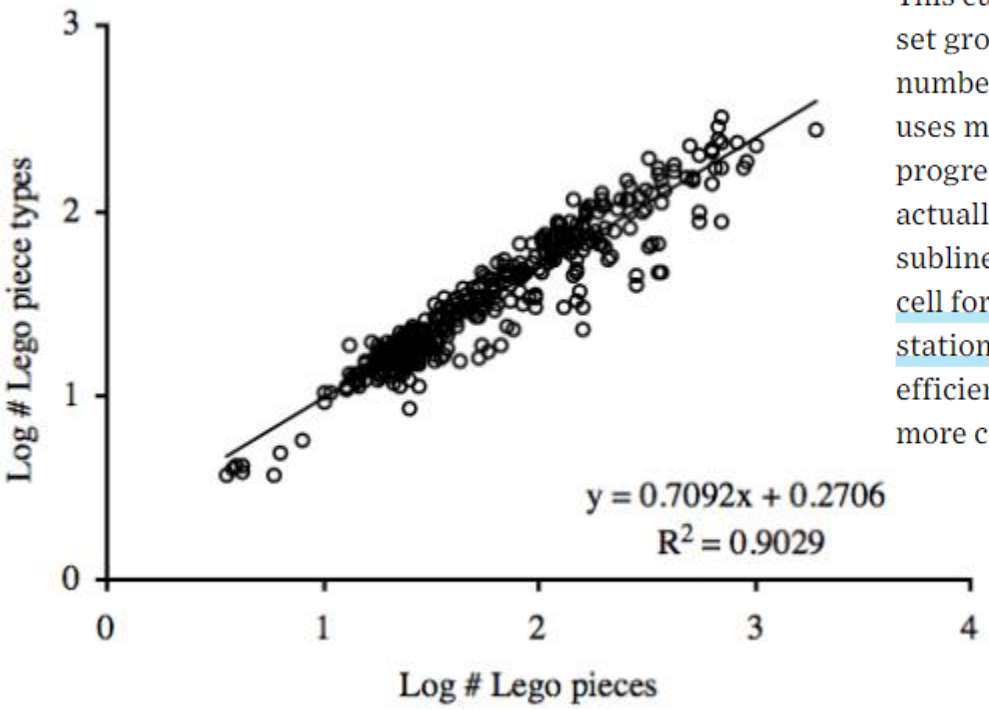



FIG. 3. Log–log (base 10) (left) and semi-log (right) plots of the number of Lego piece types vs. the total number of parts in Lego structures ($n = 391$). To help to distinguish the data points, logarithmic values were perturbed by adding a random number in the interval [−0.05, 0.05], and non-logarithmic values were perturbed by adding a random number in the interval [−1, 1].

# Lego and Science: Piece Complexity

https://www.wired.com/2012/01/the-mathematics-of-lego/




This curve demonstrates that as the number of pieces in a set grows, so do the number of piece types. However, the number of piece types grows *sublinearly*: while a larger set uses more piece types, as sets becomes larger, they use progressively fewer additional piece types (so larger sets actually use fewer types per piece). This is similar to other sublinear curves, where larger animals use less energy per cell for metabolism or larger cities actually need fewer gas stations per capita. Essentially, larger sets become more efficient, using the same pieces that smaller sets do, but in a more complex and diverse way.

# Lego and Science: Piece Complexity

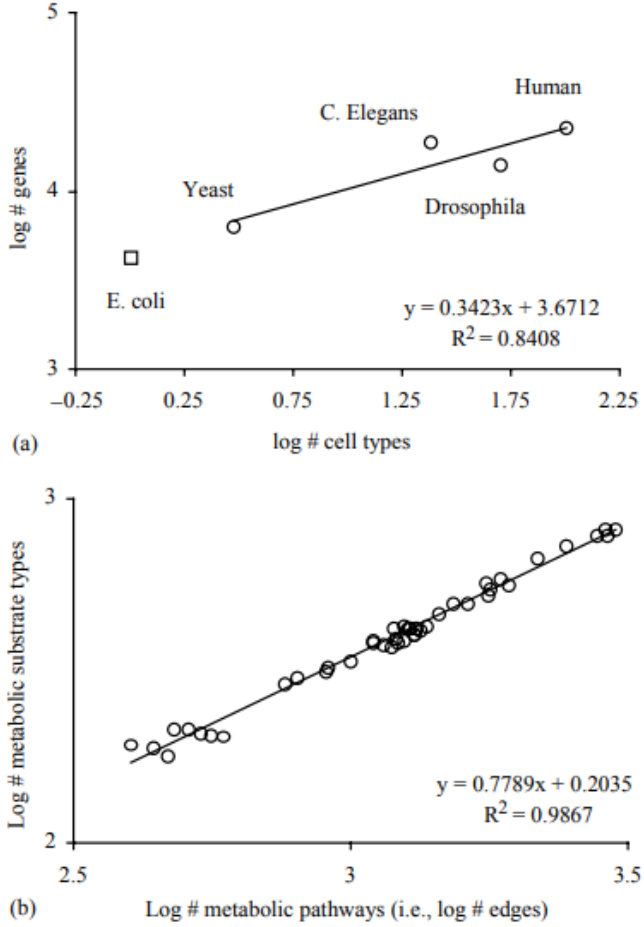
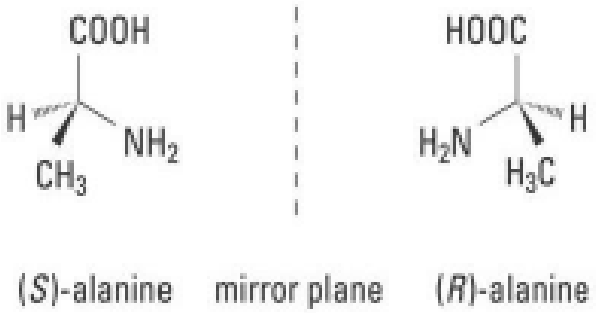



FIG. 8. (a) Logarithm (base 10) of the number of genes vs. the logarithm of the number of cell types. (b) Log–log (base 10) plot of the number of substrate types involved in metabolic pathways vs. the number of metabolic pathways in which they are involved, for 43 organisms (data from Jeong *et al.* (2000); only ingoing links are included here, and plot looks nearly identical for outgoing links).

# Counting and Lego

http://www.rowland.harvard.edu/rjf/fischer/background.php

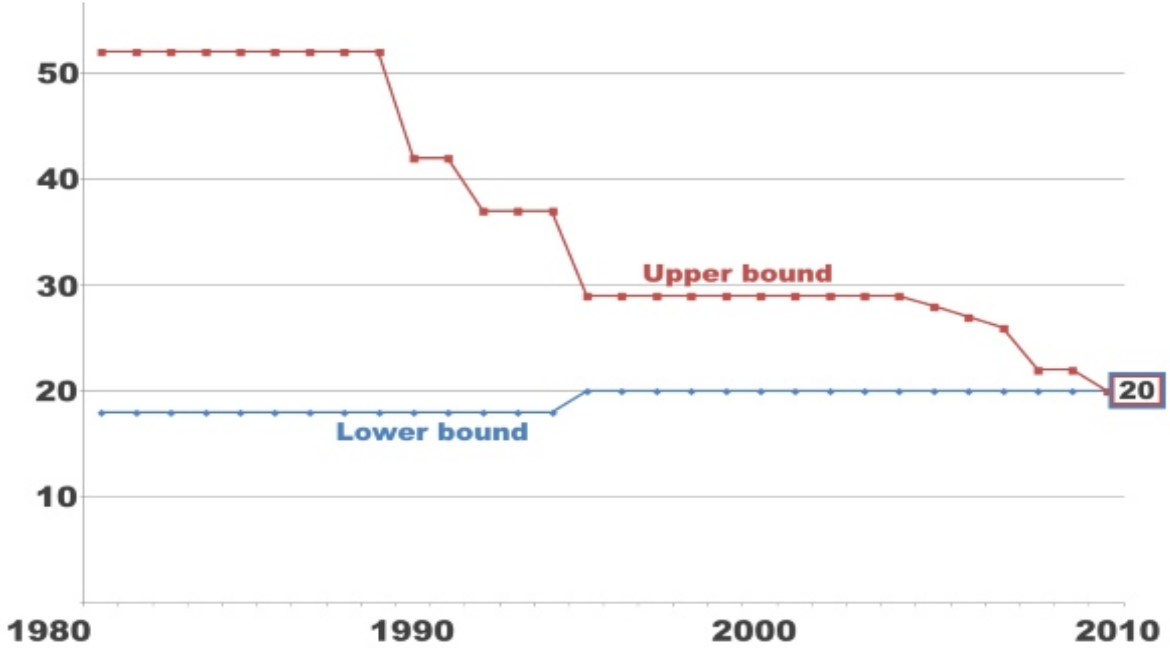
Figure: Chirality in the amino-acid alanine. Structures of the two mirror-image forms. The image is part of the information on the 2001 Nobel Prize in Chemistry.



# Counting toy/puzzle combinations happens in many places….

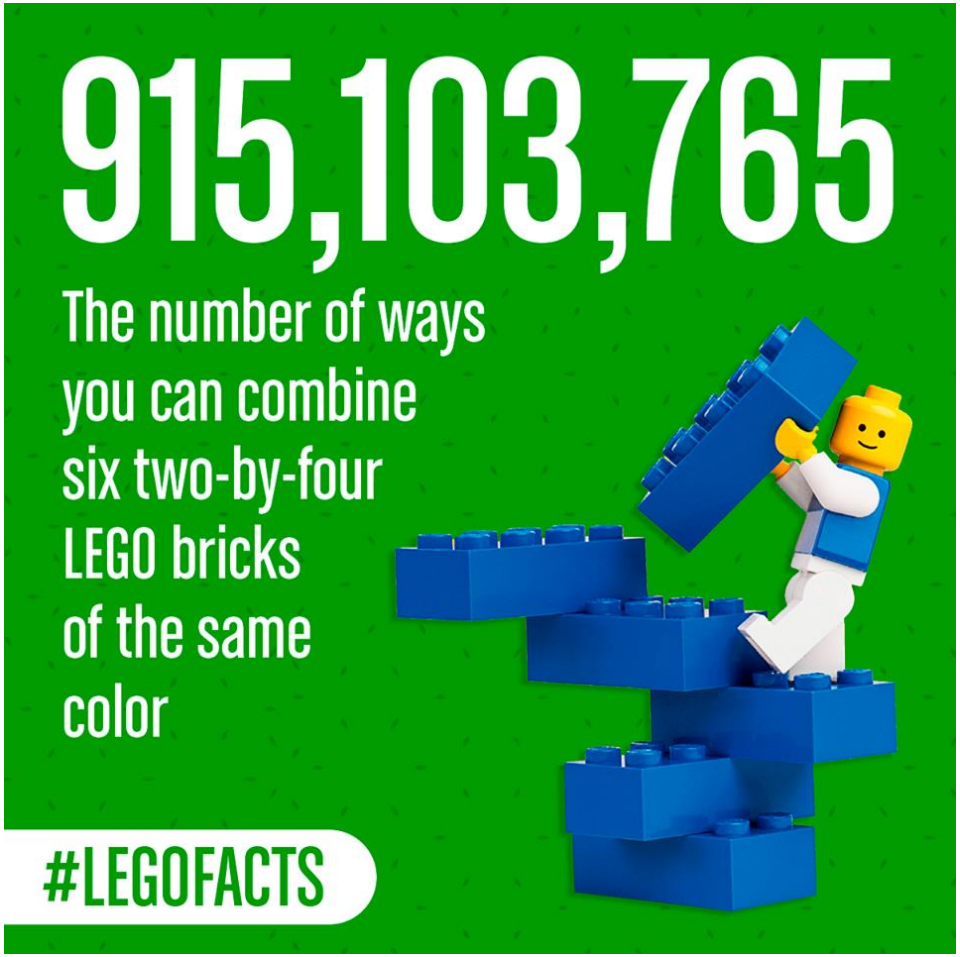
## History of God's Number

Work began on the search for God's Number back in 1981, when a man named Morwen Thistlewaite proved us a complex algorithm he devised himself that 52 moves was enough to solve any of the 43 quintillion different scrambles. This number began to fall slowly as better, more efficient methods were devised for solving the huge number of possible combinations in the fewest moves possible.




God's Number for the 2x2 puzzle (having only 3,674,160 different positions) has been proven to be 11 moves using the half turn metric, or 14 using the quarter turn metric (half turns count as 2 rotations). Unfortunately God's Number has yet to be calculated for the 4x4 cube, or higher. https://ruwix.com/the-rubiks-cube/gods-number/ **For the 3x3x3 there are 43,252,003,274,489,856,000 possibilities!**

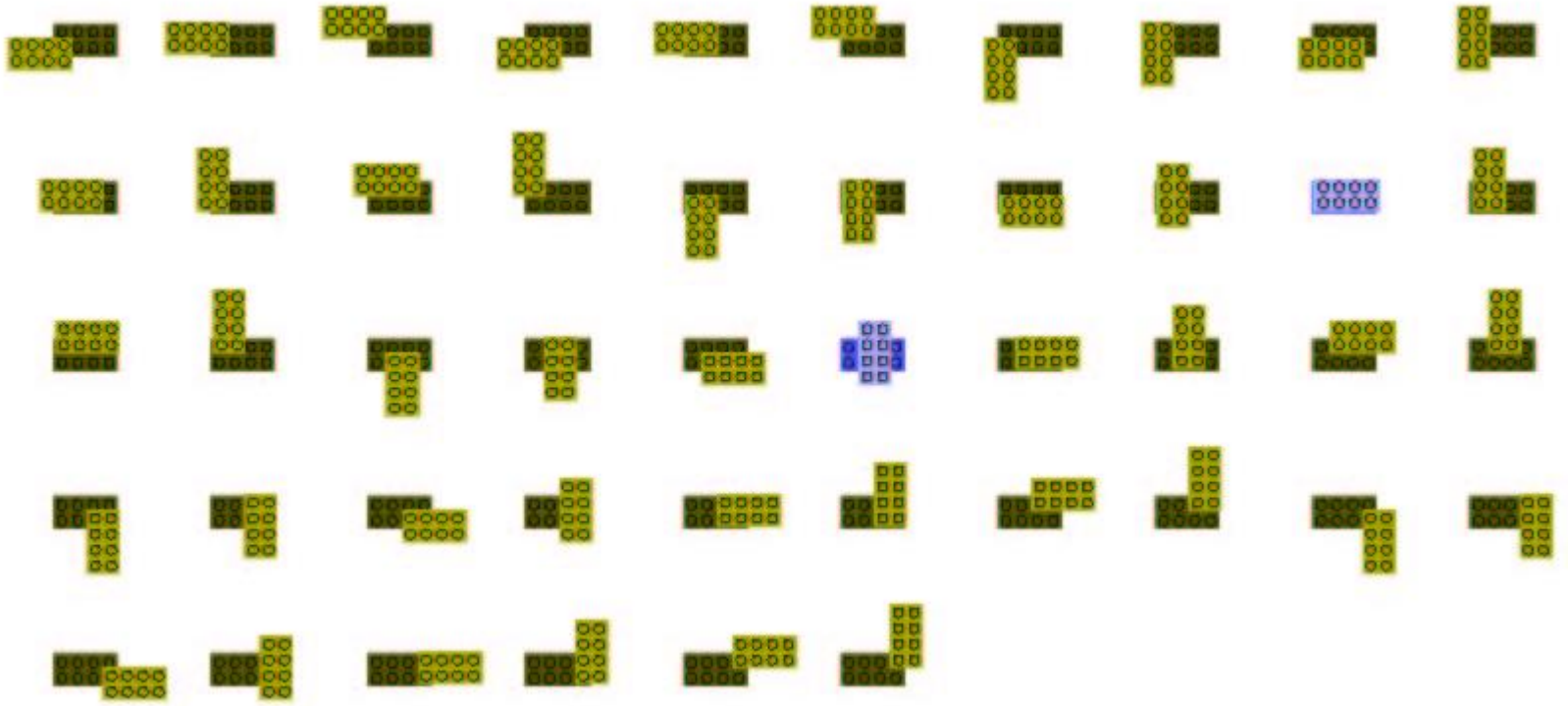
# Counting and Lego




http://www.facebook.com/LEGO/photos/a.101501756747934 03/10156283084713403/?type=3&theater

# Lego Counting

http://web.math.ku.dk/~eilers/lego.html



*Experimental Mathematics*, 20(2):145–152, 2011

ISSN: 1058-6458 print / 1944-950X online
DOI: 10.1080/10586458.2011.564539

Taylor & Francis
Taylor & Francis Group

# On the Asymptotic Enumeration of LEGO Structures

Mikkel Abrahamsen and Søren Eilers

**CONTENTS**



We investigate experimentally the growth regimes of the number of LEGO structures that can be constructed contiguously from $n$ blocks of equal shape and color.

## 1. INTRODUCTION

### 1.1. Background and Overview

https://projecteuclid.org/download/pdf_1/euclid.em/1317924406

# Counting Lego

How many ways to combine 6 pieces (each 2x4)

| Height | Number |
|---|---|
| 2 | 7,946,227 |
| 3 | 162,216,127 |
| 4 | 359,949,655 |
| 5 | 282,010,252 |
| 6 | 102,981,504 |
| **Total** | **915,103,765** |

# OEIS: https://oeis.org/A112389



THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

**A112389** Number of ways, counted up to symmetry, to build a contiguous building with n LEGO blocks of size 2 X 4. 67

1, 24, 1560, 119580, 10166403, 915103765, 85747377755, 8274075616387, 816630819554486, 82052796578652749 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,2

COMMENTS a(6) is often quoted as 102981500, but this is incorrect.

REFERENCES Anthony Lane, The Joy of Bricks, The New Yorker, Apr 27-May 04, 1998, pp. 96-103.

LINKS Table of n, a(n) for n=1..10.

M. Abrahamsen and S. Eilers, On the asymptotic enumeration of LEGO structures, Exper. Math. 20 (2) (2011) 145-152.

B. Durhuus and S. Eilers, On the entropy of LEGO, arXiv:math/0504039 [math.CO], 2005.

B. Durhuus and S. Eilers, On the entropy of LEGO, J. Appl. Math. Comput. 45 (1-2) (2014), 433-448.

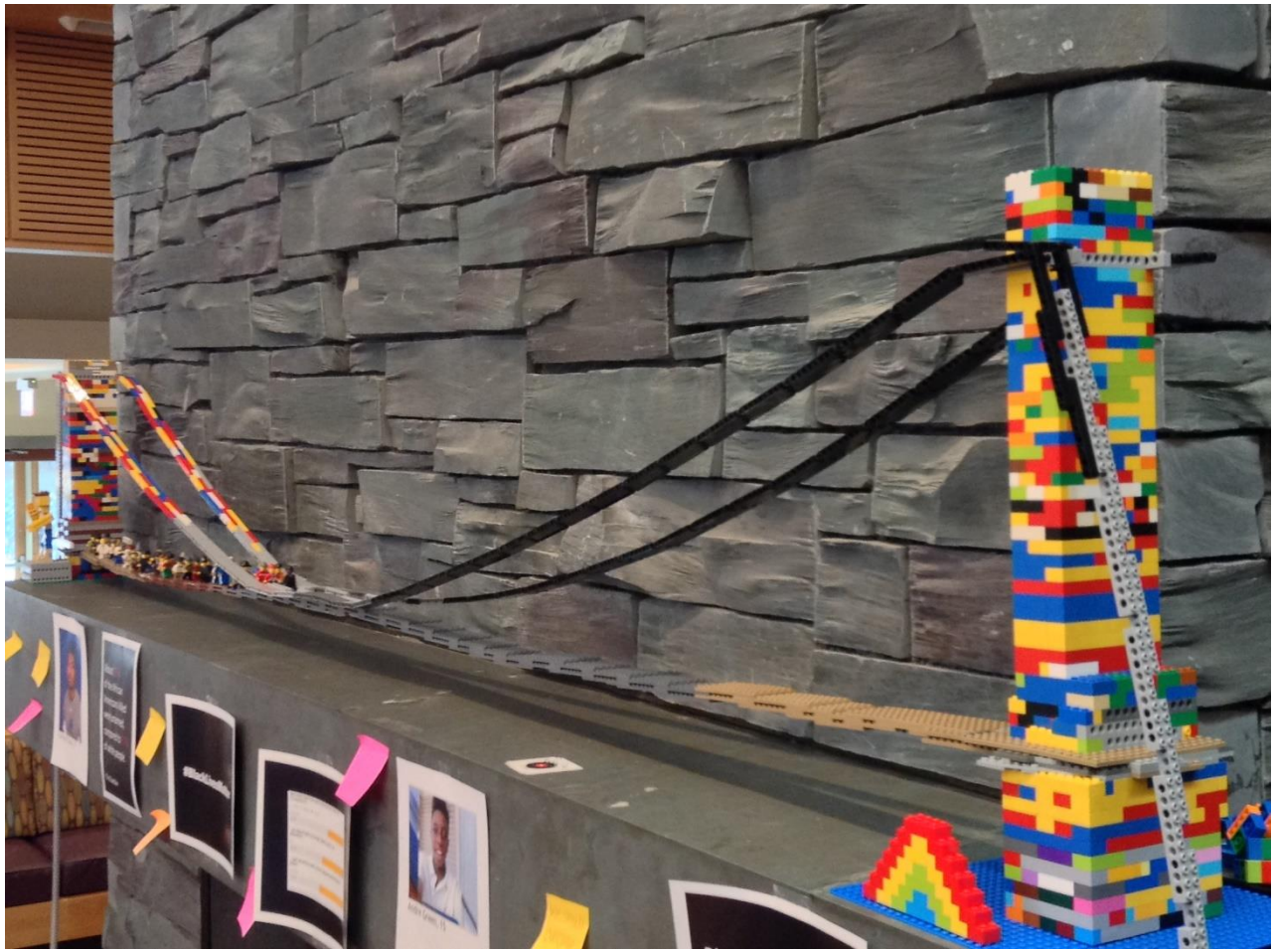
S. Eilers, The LEGO counting problem, Amer. Math. Monthly, 123 (May 2016), 415-426.

S. Eilers and M. Abrahamsen, Efficient counting of LEGO structures, March 30 2007.

Index entry for sequences related to LEGO blocks

# Lego Suspension Bridge

# First Bridge: 2016



# First Bridge: 2016

# Second Bridge: 2017

# Second Bridge: 2017

# Third Bridge: 2018


https://web.williams.edu/Mathematics/sjmiller/public_html/legos/mlkbridge/bridge2018/index.htm

# Mathematics Topics (2015)

- **Day 1: January 5, 2015:** Introductory remarks about the class, basics of efficiency and optimization, game theory and symmetries, how many ways to combine brick....
  - Problems to consider:
    - How many distinct games of tic-tac-toe are there? Do both in the case when we consider mirror images / flips to be the same and when we don't. Remember as soon as there are three in a row the game is over!
    - Redo the problem above, but now do it on a 3×3×3 tic-tac-toe board.
    - Redo the above problem but on a 3×3×⋯×3 board, where we have a total of n dimensions!
    - Write a program to figure out how many ways to combine n bricks, where all bricks are the same. Do this for bricks that are m×n for various m and n.
    - Read about solutions to a Rubik's cube to see more about symmetries.
    - We talked about chirality and mirror images in biology; there are a lot of great articles on this. Look at http://www.rowland.harvard.edu/rjf/fischer/background.php . Related to this is a wonderful story by Isaac Asimov, Mirror Image (click on the pdf file and search for Mirror Image to get to the story).
    - We also discussed scaling issues with LEGO sets, specifically how the number of pieces grows as a function of the set. There are lots of great reads, relating this to biological and other complexities. Go to Wired: http://www.wired.com/wiredscience/2012/01/the-mathematics-of-LEGO/ (BY SAMUEL ARBESMAN 01.06.12), Scaling and LEGO:http://www.changizi.com/org.pdf (Scaling of Differentiation in Networks: Nervous Systems, Organisms, Ant Colonies, Ecosystems, Businesses, Universities, Cities, Electronic Circuits, and LEGO: M. A. Changizinw, M. A. McDannaldwand D. Widdersw, J. Theor. Biology (2002) 218, 215–237). See also Science article where LEGO bricks are mentioned.
    - Related to the above: look at the *price* of different LEGO sets as a function of the number of pieces and what line it is (general city, Star Wars, Harry Potter, Lone Ranger, LEGO Friends, ...). Try to find relationships (if you know regression here's a terrific place to use it!).

# Mathematics Topics (2015)

- **Day 1: January 5, 2015:** Telescoping sums, Babylonian Mathematics, Look-up Tables, Fibonacci Numbers, Recurrence and Difference Equations, Method of Divine Inspiration, Binet's Formula, Binomial Theorem, Derivative of x^r, Evaluating sums efficiently.
  - Problems to consider:
    - Let's say that if you multiply an m digit number and an n digit number that the cost is mn, as this is the number of digit multiplications you need to do (of course, a better approach is to also include a cost of the additions, but that's a little harder as there are possible carries). Try to figure out how to compare the run-time of directly computing a product xy and using the Babylonian formula $xy=((x+y)^2-x^2-y^2)/2$; note that with the Babylonian formula you need to make an assumption about how long it takes to read in a number and then do subtraction and division by 2.
    - Read the notes here on solving difference equations, and try some of the problems. If you know eigenvalues and eigenvectors, use those to attack the matrix formulation of the Fibonacci numbers and reach Binet's formula that way.
    - Read pages 44 to 49 of this talk of mine on generating functions, another way to solve recurrence relations and reach Binet's formula.
    - Notes on analysis review (includes proofs by induction): For us most important part is page 3, where it talks about binomial coefficients and the binomial theorem. Try Exercise 1.1.7 (note it is possible to prove each claim by telling an appropriate story). After proving the binomial theorem find an expansion for $(x+y+z)^n$,
    - Show $x^r=\exp(r\log x$ and use the chain rule to prove its derivative is $rx^{r-1}$. Note the proof of the derivative is very different than the proof of the derivative of xn for n an integer. That just uses the binomial theorem. If we have $x^{a/b}$ for a rational number a/b then the proof is by the power rule: if $f(x)=x^{a/b}$ then set $g(x)=f(x)^b=x^a$, and now we can find the derivative of g(x), from which we can get the derivative of f(x). Fill in the details of these arguments.
    - Create a look-up table for values of sinx and cosx. You need to start with inputs where you know the output; good choices are to take $x=m\pi/2n$ for integers m,n, as we can get these values from the half-angle or double-angle formulas. Continue by using Taylor series (reviewed in the analysis notes, page 6).
    - Come up with a good way to evaluate $\sum_{n=0}^{k}$(n choose k)$x^k y^{n-k}$ by looking at the modification term by term as you go down. In other words, it's expensive to calculate each summand from scratch. If $a_k$=(n choose k)$x^k y^{n-k}$ find a simple formula relating $a_{k+1}$ to $a_k$, and use that to march down the line.

# Mathematics Topics (2015)

- **Day 2: January 6, 2015:** Recurrence relations and roulette (the roulette video is available here: http://www.youtube.com/watch?v=Esa2TYwDmwA), combinatorics (factorials, binomial coefficients, Pascal's triangles, proofs by story, the cookie problem).Problems to consider:
  - What is $\sum_{n=0}^{k}$(n choose k)$^2$? Hint: (n choose k)$^2$=(n choose k)(n choose n−k). Tell a story.
  - More generally, can you figure out what the `right' sum of a product of three binomial coefficients is? One difficulty is you have to figure out what's the right triple!
  - To solve the roulette recurrence from the video involves finding the roots of a polynomial of degree 5; sadly in general there's no analogue of the quadratic formula to give us the solution in terms of the coefficients (there are cubic and quartic formulas for polynomials of degree 3 and 4). Look up methods on how to numerically approximate roots, such as `Divide and Conquer' and `Newton's Method'.
  - Try to impose *upper* bounds in the cookie problem (say 12 people, 100 cookies, no one gets more than 20). Interestingly, I know of no good way to impose upper bound constraints, even though lower bounds aren't too bad. One possibility is to try using Inclusion-Exclusion.
  - Read about the Gamma function. Prove Γ(s+1)=sΓ(s) by integrating by parts. Deduce Γ(n+1)=n! for n a non-negative integer. Look up the proofs that Γ(1/2)=π√; this is a very important result in statistics and probability.
  - The cookie problem can be cast more number-theoretically as Waring's problem where the exponents are 1; look up Waring's problem and think about fragmentation problems where the pieces split so that a sum of squares equals the given number: $x_1^2+\cdots+x_s^2=C$.

# Mathematics Topics (2015)

**Day 2: January 6, 2015:**

- Horner's algorithm, Fast Multiplication, Strassen's algorithm.Problems to consider:
  - The best known algorithm is the Coopersmith-Winograd algorithm, which is of the order of N2.376 multiplications. See also this paper for some comparison analysis, or email me if you want to see some of these papers.
    - Some important facts. The Strassen algorithm has some issues with numerical stability.
  - One can ask similar questions about one dimension matrices, ie, how many bit operations does it take to multiply two N digit numbers. It can be done in less than $N^2$ bit operations (again, very surprising!). One way to do this is with the Karatsuba algorithm (see also the Mathworld entry for the Karatsuba algorithm).
- If instead of evaluating a function at an integer you instead evaluted it at a matrix, could you still use Horner's algorithm? Why or why not?We saw how to do fast multiplication. Show that it takes at most 2log2n multiplications to compute xn.We saw Horner's algorithm does significantly better than brute force, standard polynomial evaluation. What if instead we used fast multiplication to compute the different powers of x; is that enough to beat Horner? Why or why not.Look up RSA and see how fast exponentiation is used to make it useable.Consider the following problem. You're given a large number, for definiteness say 100, and you want to split it into a number of summands such that each summand is a positive integer and the product of the summands is as large as possible. How do you do this, and what is the product?Redo the last problem but now remove the restriction that the summands are integers (they must still be positive). Now what's the answer? How many pieces do you want, and what are their sizes? Th**e** answ**e**r is v**e**ry int**e**r**e**sting.

# Mathematics Topics (2015)

**Day 3: January 6, 2015:**

- Game of Life, Pascal's Triangle Modulo 2, Sorting: Wikipedia page on the game of life: http://en.wikipedia.org/wiki/Conway's_Game_of_Life
  - Gosper's sliding gun: http://www.youtube.com/watch?v=GrIO5RJ76D0
  - Game of life breeder: http://www.youtube.com/watch?v=X3HiczyUDis
  - Conway on the game of life and set theory: http://www.youtube.com/watch?v=cQUAwhhC8cU (2 hours)
  - Sorting algorithms: http://en.wikipedia.org/wiki/Sorting_algorithm (see especially Merge sort: http://en.wikipedia.org/wiki/Merge_sort ).
- Problems to consider:
  - Read about the game of life and cellular automata. Try to come up with your own pattern that causes growth.
  - Read about the various sorting algorithms. Think about how you want to measure run-time: do you care about the worse case or average case?
  - Help me make a good movie out of constructing Pascal's triangle modulo 2. Think about what's the most efficient way to find the levels: do we want to use memory, or do we want to use (n choose k+1)=(n choose k)(n−k)/(k+1)?

# Mathematics Topics (2015)

**Videos:**

- Lecture 1.1 (January 5, 2015): Introduction and class mechanics: https://www.youtube.com/watch?v=0jFLfIhlwdU (unfortunately audio only worked for first 10 minutes)
- Lecture 1.2 (January 5, 2015): From Lego Bricks to Math (tic-tac-toe, metrics, chirality, statistics, ...): http://youtu.be/GZOvuaQykME
- Lecture 2.1 (January 6, 2015): Math Lecture 2: Efficiencies (Horner's algorithm, combinatorics): http://youtu.be/f3AujzMchLc
- Lecture 3.1 (January 7, 2015): Van Halen and Brown M&Ms, Telescoping Sums, Check digits: http://youtu.be/_Q_AKCU0xPk
- Lecture 3.2 (January 7, 2015): Opening up the box: http://youtu.be/ftCWQzZ295E
- Lecture 4.1 (January 8, 2015): Midway, Qwerty, Pascal mod 2: http://youtu.be/lRYN5y_BI6M (unfortunately the Mathematica video doesn't display well, so you need to seehttps://www.youtube.com/watch?v=tt4_4YajqRM).

# Circuit of the Americas

http://www.circuitoftheamericas.com/events

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# Links for Projects….

- Useful links:
  - Individual Bricks: http://shop.lego.com/en-US/Individual-Bricks-ByCategory
  - Lego suspension bridge: https://www.flickr.com/photos/suspensionstayed/sets/72157610808577323/with/3090342269/
  - Lego speed champions: http://www.lego.com/en-us/speedchampions
  - Lego base plates: http://www.ebay.com/bhp/lego-road-plates
  - Lego base plates: http://shop.lego.com/en-US/Straight-Crossroad-Plates-7280?fromListing=listing
  - Lego base plates: http://shop.lego.com/en-US/T-Junction-Curved-Road-Plates-7281?fromListing=listing

# Lego Art


KARATE STUFF, EH? I KNOW A FEW THINGS ABOUT...
...THAT!
...THE JOKER! LET'S GO, WILDCAT!
OOOFF!
SHORTLY, IN THE MOST FAMOUS ARENA IN FICTION...

# Sean Kenney: http://www.seankenney.com/

# Sean Kenney: http://www.seankenney.com/

Sean Kenney: http://www.seankenney.com/

http://seankenney.com/shop/bricks/