Catalan Counting

In[292]:= catalancount[n_, numdo_] := Module[{},
(* we have an n x n grid, have n right moves, n up moves *)
(* we will do this numdo times, 
record how many stay on main diag or below *)
(* Right is +1, Up is -1,
success is all running sums non-negative *)
steplist = {};
For[j = 1, j \leq n, j++, steplist = AppendTo[steplist, 1]];
For[j = 1, j \leq n, j++, steplist = AppendTo[steplist, -1]];
success = 0;
For[i = 1, i \leq numdo, i++,
{
 orderedstep = RandomSample[steplist, 2 n];
(* this randomly permutes the 2n steps *)
(* initialize works to 1, if ever have a negative running sum make works 0,
increment success if end with works = 1 *)
works = 1;
runningsum = 0;
For[j = 1, j \leq 2 n, j++,
{
 runningsum = runningsum + orderedstep[[j]];
If[runningsum < 0,
{
 works = 0;
 j = 2 n + 5;
}];
}]; (* end of j loop *)
If[works > 0, success = success + 1];
}]; (* end of i loop *)
Print["Number of successes is ", 1.0 (success / numdo) Binomial[2 n, n]];]

In[293]:= For[q = 1, q \leq 10, q++, catalancount[q, 1000000]]
Number of successes is 0.999901
Number of successes is 1.99851
Number of successes is 4.99845
Number of successes is 13.994
Number of successes is 41.9832
Number of successes is 131.964
Number of successes is 429.97
Number of successes is 1431.3
Number of successes is 4859.66

Out[293]= $Aborted

In[294]:= For[q = 7, q ≤ 10, q++, catalancount[q, 10000000]]
     Number of successes is 428.863

Out[294]= $Aborted

In[281]:= Binomial[40, 20]
Out[281]= 137846528820