Panel on Models for Engaging Undergraduate Students in Research

AMS Committee on Education

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I. Purpose of the Panel

For a long time, many research mathematicians believed that typical undergraduates could not do good, original research, and their usefulness was confined to numerically investigating special cases of conjectures. Though there are numerous challenges that need to be overcome for an undergraduate to have a successful research experience, the growth of REUs, undergraduate dissertations and research projects in the past few years show that these challenges are manageable.

Unlike graduate student research, where the purpose is to lead to a thesis and a productive, professional career, the goals for undergraduate research are different. In many cases, it is most important for students to learn what research is like, to master basic skills, to get a sense of what types of problems they might want to study, and to get a better sense of whether or not graduate school is right for them. These are of great use to all students, regardless of what they elect to do after college.

Undergraduates are always enthusiastic about research opportunities; there are far more students who want to do research than there are professors willing to guide them. We feel it is critically important to expand these opportunities. In the laboratory sciences students are routinely introduced to problems in the forefront of their discipline early on. In mathematics it might not be till late in their junior or senior year (if ever) that students are introduced to open questions and recent results.

We believe that, when given appropriate problems and supervision, students are capable of rising to the task and doing good work, and that the resulting experience will be beneficial to both the student and the advisor. The purpose of this panel is to encourage professors to participate in and run REU-like summer programs and to participate in and develop opportunities for undergraduate research at their home institutions.

There are numerous demands for a professor's time; it frequently takes longer to obtain results with undergraduates than without, and in a publish or perish world this can be disadvantageous to junior faculty. We would like to share our experiences in mentoring undergraduate research with colleagues from other institutions. In particular, how to find good problems, how to get students working independently, how to manage one's time with them, the various roles of mentoring, and how to deal with funding issues.

II. Panelists

- David Damiano (moderator and contact person)
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  Bio: Dave is an Associate Professor of Mathematics at the College of the Holy Cross. He is currently on sabbatical engaged in ongoing research with colleagues affiliated with the HIV/AIDS Clinic at the University of Massachusetts Medical School. His research interests include knot theory and mathematical biology. He is a co-author of *A Course in Linear Algebra* with John B. Little and *Multivariable Calculus* with Margaret N. Freije. He regularly supervises student research, independent study projects and senior theses both in topology and in applications of mathematics to the life sciences. In addition, he has been active in mentoring minority students and has led summer workshops for middle and high school teachers. He was recently been named Teacher of the Year at Holy Cross.

- Dean M. Evasius
  Program Director
  Division of Mathematical Sciences
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  Bio: Dean currently heads the Workforce and Infrastructure programs. He was previously the Program Director for the Probability program. Prior to arriving at NSF in 2004, he was a research mathematician at the National Security Agency. He received a B.S. degree from UCLA, and a Ph.D. from Caltech. His research interests include cryptography, harmonic analysis, applied probability, and signals processing.

- Joe Gallian
  Professor
  Department of Mathematics and Statistics
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  Bio: Joe Gallian came to the University of Minnesota Duluth in 1972 after receiving a Ph. D. in mathematics from Notre Dame. He has directed 34 summer research programs for undergraduate students and given more than 250 invited lectures at conferences, colleges and universities. More than 170 research papers written under his supervision by undergraduates have been published in mainstream journals.

- Jake Levinson
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  Bio: Bio: Jake is a first-year PhD student at the University of Michigan. He received a BA in math at Williams College, where he wrote an undergraduate thesis in L-functions and random matrix theory with Steven Miller, and participated in the SMALL REU at Williams in summer 2009, working in commutative algebra with Susan Loepp. His current research interests include algebraic geometry and analytic number theory, as well as the study of extremely long equations.

- Steven J. Miller (contact person)
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  Bio: Steven is an Associate Professor of Mathematics at Williams College, currently on sabbatical at Smith College and Mount Holyoke College. He has written over 50 papers (many with undergraduates) in accounting, computer science, economics, geology, marketing, number theory, probability/statistics, and sabermetrics, and is writing books on Benford's law, cryptography, linear programming and probability. He is active in high school mathematics, lecturing and mentoring at programs for talented students, participating in education conferences and writing problems for the American Mathematics Competitions. His math riddles webpage is in the top 10 in google searches, and his book *An Invitation to Modern Number Theory* (with Ramin Takloo-Bighash) is based on undergraduate research classes he has team taught over the years. He has supervised almost 200 high school, undergraduate and graduate students in research projects in the past 10 years; most recently, in the past three years at Williams he has had 21 summer REU students and 4 math thesis students.

- Gina-Maria Pomann
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  Bio: Gina-Maria Pomann is a third year Ph.D. student in Statistics at North Carolina State University (NCSU). While at NCSU she was awarded the National Science Foundation Graduate Research Fellowship, the AT&T Graduate Research Fellowship, the North Carolina State Graduate Research Fellowship, and the National Heart Lung and Blood Institute Traineeship. She received her Associates of Science degree in Mathematics from Middlesex Community College in 2006 and her BA in Mathematics from the College of New Jersey in 2009. During her time at TCNJ she found her passion for research by conducting undergraduate research at the Mathematical Sciences Research Institute of Berkeley, California, Princeton University, and Stanford University. Her current research interests focus on, the problem of change detection, functional data analysis, and statistical image analysis. Her ultimate career goal is to conduct research related to environmental and biological statistics while working to promote diversity in the mathematical sciences. She is happy to mentor any students interested in pursuing a graduate program in Statistics as well as students transferring from community college to a four-year institution. More information about Gina-Maria can be found at http://www4.ncsu.edu/~gpomann/.

- Ivelisse Rubio
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  Bio: Ive's research interests are in the area of finite fields and applications to coding theory. She has directed undergraduate research projects in computational mathematics of numerous minority students and has been involved in many activities to promote undergraduate research in mathematics. She co-founded and co-directed the REU Summer Institute in Mathematics for Undergraduates (SIMU) (1998-2002) and the undergraduate research program MSRI-UP (2007-present). Ive is also a member of the US National Committee for Mathematics and Associate Editor of the American Mathematical Monthly.

III. Panel Discussion

The following notes are from the panel discussion. We are grateful to Nadine Amersi, Olivia Beckwith, David Moore, Ralph Morrison, Ryan Ronan, and Liyang Zhang for taking the notes that served as the starting point for this write-up.

(a) Introductions

David Damiano (moderator): Welcome. The format is 40-45 minutes of panel discussion, unscripted, with the remainder of the hour and a half devoted to questions and answers with the audience. My experience at Holy Cross has been working with individual students during the school year, and we have a summer program funded by the college that is similar to an REU (Research Experience for Undergraduates) but also supports students in the sciences. Here are the panelists:

Steve Miller: I was an undergrad math/physics major and started by doing undergrad research in physics. In grad school I helped design an undergraduate research class. I've worked with over 200 students on research projects in the past 15 years.

Ivelisse Rubio: I have co-directed undergraduate summer research programs since 1998: SIMU (Summer Institute in Mathematics for Undergraduates) from 1998 to 2002 and, since 2007, MSRI-UP in Berkeley.

Jake Levinson: I'm a first-year grad student at Michigan. I did the SMALL REU at Williams in 2009, and an undergrad thesis with Steve.

Gina-Maria Pomann: I'm a third year PhD student in statistics at NC State. My first experience with undergraduate research was when I participated in the MSRI-UP program. I then went on to work on undergraduate research projects at Princeton, Stanford and AT&T.

Joe Gallian: I’ve run an REU at Minnesota-Duluth since 1977.

Dean Evasius: I’ve been an NSF program manager and the head of the DMS Workforce program. This includes managing the DMS REU Sites program, Mentoring Through Critical Transition Points (MCTP), and the Mathematical Sciences Postdoctoral Research Fellowships (MSPRF).

(b) Before the research

***David*: This discussion is broken down into before, during, and after the REU. First question: what is the goal of undergraduate research for students?**

Ive: Our programs target under-represented minorities; the goal is for them to pursue graduate studies in the mathematical sciences. We expect them to raise their mathematical level and to work hard on their research problems so they can be confident of succeeding in graduate school.

Jake: I was pretty sure I wanted to go to grad school before doing SMALL, but I wanted to verify that by seeing what math research was like, as opposed to coursework. My secondary goal was to learn the subject area (commutative algebra).

Steve: My main goal is to give students a taste of what it's like to ask questions and design a research program, and to help them decide what they want to pursue. This leads us to pursuing problems at many different levels in several areas.

***David*: How do you choose students and how do students choose advisors?**

Joe: UMD has funding for our students. They have to write a research proposal and get it signed by an advisor. The research is done during the school year; almost every proposal is funded because we want students to get involved. If I have an idea, I look in the classes I'm teaching and see if anyone's a good fit. If I don't have a student in mind, I ask around the faculty. Sometimes I email all the math majors.

For summer REU students, it's a national pool. I target students who are ready to do professional-level research under the right circumstances. I only have problems in combinatorics and number theory, so I turn down great students who aren't focused in those areas. I want students who are well prepared, self-starters who have spent a lot of time working on math on their own. People who have done math camps, study abroad (like Budapest), or previous REUs are well prepared. I'm looking for people who want to be professional mathematicians, not those trying to choose between math and English or chemistry, or a math career versus working at a hedge fund.

Ive: For summer programs, there's recruitment and selection. We want to serve under-represented minorities and need to recruit them; we send advertisements and emails to departments and contacts that teach under-represented minorities, and we go to the SACNAS (Society for Advancement of Chicanos and Native Americans in Science) conference and recruit there. For the selection process, the five co-directors of MSRI-UP look at all the applications, read the statements, letters of recommendation, and the transcripts, trying to identify talented under-represented minorities who are interested in math. We specially look for those who haven't had the opportunity to work on research before and might not be able to do it during the academic year at their own institutions. We accept 18 students and try to have balance in gender, home university, academic background, etc..

In the academic year, I work in a Computer Science department where students take many math courses, and I have contact with all the freshmen students. I talk to those taking the Discrete Mathematics course during their first year, and try to get them to work on research early to get them excited about math. I have a lab where the more advanced students mentor the younger students.

Gina-Maria: I was recruited to MSRI-UP, the summer after transferring from community college, so I had a weaker background in math and probably wouldn't be able to get into Dr. Gallian's program. But had I not gone I wouldn't have learned what grad school was like, that you could be funded to go to grad school, what research was, and I would never have gone on to do other more individualized projects at Princeton, Stanford, and AT&T. I think it is very important to catch students at the community college level as well as students of diverse backgrounds and include them in these undergraduate research experiences. A big underrepresented group is students with disabilities. These students can be recruited by contacting the disabilities services offices at your university.

Ive: At MSRI-UP we focus on students with no previous opportunities, but we also recruit strong students to have a good mix.

Steve: Another way is that whenever I give a talk, for example at a math club, I always have a couple of open problems and I offer to work on them with students.

Gina-Maria: In my same program, there were students with stronger backgrounds and we were paired up, and did very well. Our work resulted in the publication of a technical report and numerous presentations. The group dynamic is important.

Joe: Group dynamics are a selection criterion. I want to find people who are good socially, work well and cooperate with others and support others. Students write an application essay that is useful for this, and if recommenders don't address these issues I sometimes write back to ask about social skills. My students work individually, but spend a lot of time together. I want them to help each other and bounce ideas around. This makes the program run better.

***David*: how do you choose a good research project?**

Steve: I have some rough ideas of a project I want in number theory or probability, and I look for students with background, but I often don't have an exact problem. Once I know the students interests I work with them to choose a problem, so that they have a sense of ownership. I'm also constantly talking to colleagues and going to conferences, and looking for projects that have a natural next step.

Jake: I had no background in analytic number theory before my senior thesis, but Steve had me look at elliptic curves, l-functions, et cetera. He presented me with an array of problems and I looked for what caught my interest.

Joe: I don't let students choose problems, but I attempt to match students to problems based on their interests (gleaned from essays) and their previous background. Students aren't locked into these problems, but it's a starting place. One really good source of problems is the previous summer, since there are usually a lot of problems left unsolved. When my students finish their final report, they include a section called "future research" with conjectures and open problems. If a new student picks up on that, we have a built-in friend from last year who might come back to visit for a week or two to work with the current student.

There's a conference (CANT) every summer in New York City which starts a week or so before my program, that emphasizes open problems and conjectures in combinatorial and additive number theory. I have two program alumni every year who help me as advisors, and I send them with two or three new undergrad students to the conference to hear the problems and find some they think would be interesting to work on, for themselves or others. One student went and got really excited about an open problem, started working on it at the conference and continued on it for a whole year.

A few years ago Steve gave a talk at this conference, and a graduate student who was a Duluth REU alumnus thought it was great. I handed this idea to a student at Duluth and he got six papers out of it, and that spilled over to more work the next summer. So even if we only get one or two problems, it makes conference attendance worth it.

I also do the standard things, looking at journals and math arXiv for problems. I look for problems that students can get up to speed on by reading one or two papers, and problems that are recently posed The area can be old, but the concept should be new.

Many years ago I had an REU student who had done a Caltech REU the summer before. She told me that by the end of that summer, she finally understood the question she was supposed to work on. I want students to understand the question right away.

***David*: Is the goal of an undergrad research experience to produce a publishable paper?**

Steve: My main goal is to help students decide what they want to do and get a sense of what they're getting into, though in the process of discovering this we almost always get at least one paper. My undergrad advisor was a physicist who was not upset that a lot of his students ended up being mathematicians.

Gina-Maria: I've been in different programs with different goals: teaching students what research is and what the academic life is like, versus having a problem which needed to be solved and they actually needed an answer quickly. The goals depend on the students and on the program.

One way to find problems is to go to colleagues in industry, e.g. MSRI gets problems from Lawrence Berkeley National Lab and the result of our work was a LBNL tech report. AT&T also has problems that are appropriate for undergrads.

Dean: The NSF has a broad view of what REU goals can be, from a “pre-REU” that targets people after freshmen or sophomore year to give them first research experiences, up to programs like Dr. Gallian's that attract the brightest students who are ready for professional-level research. Programs can also have non-academic goals, like targeting an underserved population, or involving students who are not entering the pipeline as quickly as you'd like. We just want a clear statement of goals and a project plan that is consistent with those goals, and we let our peer research community make a decision about whether those goals are valuable and whether the project is likely to meet those goals.

(c) During the research

***David*: What is the schedule like for students and faculty?**

Ive: The students at MSRI-UP work on the same topic. We start with one to two weeks of introductory material so that everyone has the same background on the area before starting the research phase. During that time they work from 9 AM to 4 PM at MSRI and continue to work at home; the students and the TAs are housed in the same facilities. When the actual research is taking place students get excited about the problems and work even more, often working a lot harder than we expected.

Jake: At Williams, there is a one-month period called the Winter Study. Students take only one course during this month. For thesis students we work on our research full-time, receiving credit for it. It's a great opportunity for us to focus on our work. This was an important period for me as I had a lot of time to devote to it, and was able to really understand the project and started getting my main results.

***David*: How to you manage the group dynamics? What issues are there?**

Steven: Trying to predict the group dynamics is extremely important. I try to find students who have the same background so they can work together and share projects, and have an equal sense of ownership. While I'll often have 6 or 9 students working with me over the summer, I try to split them into (sometimes multiple) groups of 1 to 3 with similar skill sets and interests. I find this works well for me and for them, though others do well with different models.

Ive: Students work in groups of three. Because of the goal of the program, we try to group students with different backgrounds. The goal is to have a diverse group where the students learn from each other. We also work with them prior to the start of the research so we know their strengths. We form the groups after the first weeks of introductory material, which allows us to get to know them before forming the groups.

Gina-Maria: I had a weaker background during my first REU than most people who were working on my project. But working with people who have done research or had a stronger background helped me get up to speed. At the same time, I also motivated the other students because of my eagerness to do research.

(d) After the research

***David*: What credit do students and professors get for such research?**

Steve: As a junior faculty member, it's important to be clear with your institution what credit they give for advising students. I enjoy working with students and find it refreshing, and now it helps with my own research. However, it took me several years to get to the point where I was good at finding problems and seeing the project through to publication at a good journal. For tenure-track faculty or postdocs, you should think long and hard about what is best for not just your students, but yourself. I'm a much better and broader mathematician because of all the mentoring I've done, but I had a few slow years when starting out.

Jake: For a senior thesis, I received 2 semesters of course credit. At the end of the year I still had more ideas and techniques I wanted to try; fortunately my advisor was able to find funding so that I could continue working over the summer.

Joe: There is a UROP (Undergraduate Research Opportunities Program) during the year for which students get paid to do research. One of the goals is to offer an alternative to working in the library or bookstore for those who need campus jobs. At Duluth, doing a research project allows students to graduate with departmental honors. Also there is the opportunity to earn credits for independent study. I believe students should get some reward for their work, be it money, honors, or credits towards graduation

Ive: For the students that conduct research during the academic year in our institution, we like students to get course credit. An actual course has been created entitled Undergraduate Research in Computer Science. Also there is an Undergraduate Seminar, which all freshman students take, where they learn skills such as LaTeX, how to look up references, give presentations, et cetera.

***David*: How do you help students after the program? What conferences do you want them to attend?**

Steve: I want my students to meet professionals from other institutions working on similar problems. It's important to get students involved in the research community. It helps them not only in their research, but also in multiple ways when applying to graduate school. First, it lets students see who does what and where. Also, it gives them contacts that help when applying to graduate school; they're no longer just a name on a piece of paper, but often someone met at a conference.

Jake: After SMALL I presented a poster and gave talks at the joint meetings. This year I presented my thesis at the Young Mathematicians Conference at Ohio State and the Joint Meetings, as well as at Michigan.

Gina-Maria: After MSRI all students attended the SACNAS conference where they present their research, attend workshops and seminars, and network with other scientists. This was instrumental for me as it was my first conference and during this time I met Dr. Carlos Brody, a professor in the department of Molecular Biology at Princeton. After we discussed my research and my interests, he invited me to work with him on a statistical problem related to his work. The year after I participated in the MSRI-UP program I also attended the Infinite Possibilities Conference (IPC, a "national conference designed to promote, educate, encourage and support minority women interested in mathematics and statistics"). This conference played a key role in my decision to pursue a PhD in Statistics. I think sending undergraduate students to conferences to present their work is absolutely vital.

***Dave*: How does the NSF assess the success of a program?**

Dean: We expect the leadership of each REU site to utilize an assessment mechanism that provides insight into the effectiveness of their program. The outcomes from that assessment are part of the annual report that is submitted to NSF. When an REU site submits a renewal proposal, the reviewers will look at the outcomes and successes of previous students of that REU. Occasionally the NSF conducts an external study of an entire program, such as the assessment of the VIGRE program that was carried out by the National Academy of Sciences. It's likely that we'll do the same for REU sites in the future, although there are no specific plans in that direction.

(e) Questions from the audience

***Question #1*: Do you ever have any issues with industrial funded programs or collaborating with other universities and companies?**

Ive: At MSRI-UP, in 2007, Juan Meza directed the research of the students. The research problems were draw from current problems being worked on at Lawrence Berkeley National Laboratory.

Steve: I was contacted by the IRS about one of my paper years ago, and developed a nice working relationship with them. A few summers ago I had some students working with the IRS on a project. The students were told that they would not get a publication for their work and couldn't discuss their projects, but we made valuable contacts and I could use this (in general terms) in letters of recommendation.

Dean: NSF has no specific concerns about this as long it's acceptable to the students and their faculty mentors.

***Question #2*: I find it challenging to find problems that are approachable but not too easy for the students. (Dave asked for a clarification in what area, and was told differential equations and various areas.)**

Steve: I love the CANT conference (which Joe mentioned earlier) because the problems are fresh, and the people there love collaborating. The conference has a problem session each day, where I take notes and distribute the write-up to all participants. Also constantly look at the literature, and when you find something promising make a note of it. Good places to look are new fields or topics, as often there is a lot of additional work that can be done. Finally, it's good to constantly keep in touch with your colleagues. A few years ago I was advising some work in random matrix theory. I checked to make sure it was still open, and all the experts I asked told me it was. We were making good progress, but halfway through the summer a friend responded back that he remembered hearing a talk that was similar. We got in touch with the other group, and found we had lots of common results, but also some different ones, and we thus made sure we went in different directions for the remainder of the papers.

***Question #3*: What do you think about how the NSF has nudged the mathematical community in the last 15 years, and how will it continue to nudge the community over the next 15 years?**

Dean: I agree that the NSF has seen and supported growth in the last 15 years. I'm not able to address whether the NSF has consciously nudged the community in any particular field or direction during that time. DMS acts primarily in response to and in support of the community. In the next 15 years it will continue to be aware of the progress being made and support it.

Steve: It's important to get students to conferences and working with faculty, and the NSF has been generous in these areas.

Dean: NSF supplements to individual grants are an underused resource. These requests are strongly encouraged, and they are a good complement to the REU experiences because they allow students to work on their own campus rather than traveling to other campuses. If you or a member of your department has an active grant, please consider this option. For an REU Supplement, the description of the REU activity should discuss the following: (1) the form and nature of each prospective student's involvement in the research project; (2) the experience of the PI (or other prospective research mentors) in involving undergraduates in research, including any previous REU Supplement support and the outcomes from that support; and (3) the process and criteria for selecting the student(s).

***Question #4*: A follow up question: what would YOU like to see in next 15 years, Joe?**

Joe: When I was President of the MAA, I publically said in 2007, my goal for 2015 (I chose that year because it is the 100th anniversary of the MAA) was to have 1000 undergraduates at JMM and 500 undergraduates at MathFest, and it looks like we will make that. My goal in 15 years? More, more, more! More conferences, more REUs, more honors projects. Why put a cap on it?

***Question #5*: Directing this to Steve and Joe, are your research projects a one person show or do you involve colleagues at all?**

Joe: I am the only Duluth faculty person involved in my REU. I have graduate students who are program alumni come back to assist me. Other faculty in my department work with UMD students during the year.

Steve: Two extremes. At some institutions I've taught at, few faculty are involved in undergraduate research, while at others there is significant involvement. When I started at Princeton, I was a graduate student and helped create and implement our undergraduate research class with several senior faculty in many different areas. It was a wonderful mentoring experience for me as well, and it taught me how to design a research program. At some schools, there aren't too many students working in the summer, and they're somewhat isolated. The Williams College REU is the other extreme. We have a lot of students, they're all living together, going to talks and meals together, discussing math together round the clock. It's not an individual effort. To run an REU at this level is very different from directing 1 or 2 students. We work hard on building a sense of community. All students have a specific assignment for the summer: athletic directors, social directors, tea and cookies after talk and reception. They need to get together and talk. This is the strength of doing work at an REU sight, just the synergy of people (students and professors) working together.

Jake: I agree, this was my first experience making connections with other mathematicians. We were all enthusiastic about what our research. I sometimes discussed the problem we were working on with other students at the REU, even if it was a very different problem from what they were studying. I was in the Commutative Algebra group, others had taken a course in that, and I could talk to them about the problems. I also heard about their research, which meant that I was exposed to several other areas of mathematics.

Steve: And as a supplement to that supplement: The students give practice and final talks at the end of the summer, which serves two purposes. The first is to practice giving a good speech, how to highlight the key ideas in such a way that another mathematician (who doesn't necessarily know the field) can get something out of it. The second is to get a group dynamic going; here the group is all the summer students. Professors speak at least once on their current research, so even if you're not working on Commutative Algebra (say), you hear enough of these talks to get a sense. Just because someone is working in a field one year doesn't mean they're going to be doing that later in graduate school. We want to show students an array of problems, so they can find out what they like to study.

Jake: Learning to give a math talk was a great benefit of doing SMALL. I don't even know how many talks I've given by now. Ten, maybe twelve have come out of the research I've done, and I can definitely see the difference in quality now from when I gave my first talk at SMALL.

***Question #6: From Dave to Ive*: Are there any connections to programs going on at MSRI?**

Ive: Yes that's the unusual thing about our undergraduate summer program. There are many things happening at MSRI: lots of research, many graduate students …. Our undergraduate program has a weekly colloquium, people at MSRI can also attend, and our students can meet the other researchers. The undergraduates can meet the graduate students at MSRI and picture themselves there a few years from now; MSRI provides and ideal environment.

***Question #7: To Dean*: Are there any REU summer programs that will provide support to foreign undergrads? I know NSF will not support them.**

Dean: That's correct. Students supported by NSF have to be US citizens or permanent residents, but if you have outside funding you can bring in students from abroad. Can others comment?

Steve: There are a small number that provide funding, a lot of times students can get funding from their home institution.

Joe: Usually the Minneapolis REU takes some -- I also usually take one per year.

***Question #8*: Sometimes I get a student saying "I'll pay my own way," but I say my time is too spread with NSF paperwork and other activities. Do you take student up on that?**

Frank Morgan (audience member, AMS VP and perennial mentor at SMALL): We've done it, it's extra admin trouble, but sometimes it works out.

Steve: I've taken additional students when I feel like I can take them without impacting the other students. It's never just as easy as "they have the money," have to worry about which funding to use for which things, dealing with the paperwork and visa issues, …. If I feel like I can add them without impacting other students, I'm all for doing it.

***Question #9*: I'm in position of having students to send to an REU and I've gotten a sense of what you're looking for, but are there any comments about what recommenders should say to show a student is a good fit?**

Steve: For me, one of biggest things I look for is drive in students. Background experience is secondary. I have a range of problems to choose from in number theory and probability, some that far more accessible and most students can start working on them on day one, while others require almost graduate level courses. I have enough problems at different levels that I'm not worried about background, though I do take into account when I assign students into different groups on different problems, as I want students to move through problems together. What really matters for me is whether or not they have the attitude: they can't stop thinking about math, love it, want to find if math is right for them.

Jake: Take home message: some programs will care about background, some won't.... It varies from one REU to the next. An REU whose goal is to publish to professional-level research might care more about student backgrounds than an area whose goal is to help students decide whether to pursue mathematics further.

Steve: In terms of looking at motivation, I want some students with extensive background to work on problems related to my main research, but I also want students who want to decide if math is right for them. I had a student who found out grad school is free and was excited; it was a great moment. Before then it wasn't on their radar screen. In terms of the joint research, these students often have a fresh perspective as they haven't been looking at problems the same way as I have, and often find great approaches and avenues to pursue.

Dave: In helping my students, one question is how to choose an REU that's a good fit for them. Besides requirements, that's not an easy question to answer. The AMS website has a very long, daunting list of REUs. There are a slew of different topics, which isn't easy for students to decipher. They need our help going through the list to get a handle of which problems are of which types. It's not enough to say to a student "go pick 6-8 to apply to." It takes some work to help place them. Unfortunately there are problems with the list, as it has programs that aren't funded yet or haven't been funded in years.

Frank Morgan (audience member, AMS VP and perennial mentor at SMALL): It's a huge thing, to tell the truth I don't feel confident going through that whole thing. It would be nice if the community has annotated list.

Dave: That's what I'd like to say, instead of a passive list, student could search the list in a reasonable way. Maybe partition into area with sophomore or junior experience, or will the program be around in the following year, or what is the goal of the program.

Ive: Things we like to see in letters of recommendation: student motivation, hard working student, curiosity, but also some explanation as to why they are a good fit for that particular program.

***Question #10: From Dave to the audience*: What are people in the audience interested in concerning the focus of REUs? Goes back to future of this. More industry? More experienced students in REUs? Where are weakness, where can we expand, bearing in mind funding?**

Response 1: As an advisor, I look at the number of applications. We have 200 applications for only 12 spots, and 120 of them are qualified. Access is what I'd love to see more of.

Response 2: It's heartbreaking to turn somebody away.

Ive: I would like to see more research opportunities for students during the academic year.

***Question #11: From Dave*: What does undergraduate research and mentoring do for you as an advisor? There may be very little for you in terms of your own advancement in terms of supervising undergrad research. As a matter of the culture, what you accept and value, is there a way to make it possible for faculty to be rewarded in some way.**

Steve: There are two areas where people notoriously do not get proper credit (in my opinion) for work they do. One is writing programs to numerically investigate problems, and making the code available. The other is in guiding undergraduate research. Junior faculty especially have to think about how their work will be credited towards promotion and tenure. This will of course be evaluated differently at a research institution or a liberal arts college. It's important to have a discussion with your department (and perhaps higher up). I've found mentoring undergraduates in research very rewarding, both personally and professional. One caveat, though, is that it takes awhile to get up to speed with mentoring, especially finding good problems and working with the student to a completed, submitted paper. Long term I find it a great positive, but short term I would have been more productive in my early postdoc years by not mentoring. In terms of completing a project, one issue I face, unless students are doing a thesis and getting course credits, is that students are busy with other coursework and extracurricular activities in the academic year. It's very hard to continue research into academic year. That's why we finish work over summer, concentrating on making sure we at least have all the research done. If we're lucky we have a good first draft of the paper as we're writing up as we go, but often the writing has to wait till the year for odd free moments.

***Question #12: From Dave*: Last comments?**

Joe: Everyone knows the NSF has been driving funding force. Another important funding source that is often overlooked is the NSA. They support many research opportunities for undergraduates.

***Question #13: For Dean*: How many sites do your support? How many applications?**

Dean: We support 50-60 REU sites each summer. In a given year, DMS receives 40-50 applications for REU sites and makes awards to about 15 sites. Since the duration of each REU site is three to five years, making fifteen awards each year produces a steady state of fifty or so active sites.

IV. Links

The following is a partial set of links of useful sites and resources, and is meant to serve as a starting point.

Elements of a good NSF proposal: http://sites.williams.edu/Morgan/2011/12/19/elements-of-good-nsf-reu-proposal/

NSF REU homepage: http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5517&from=fund

AMS List of REUs: http://www.ams.org/programs/students/undergrad/emp-reu

SMALL (Williams REU) Homepage: http://math.williams.edu/small/

Young Mathematicians Conference (Ohio State): http://www.ymc.osu.edu/

AMS Undergraduate page: http://www.ams.org/programs/students/undergrad/undergrad

AMS Undergraduate Conference page and reports:

- http://www.ams.org/programs/edu-support/undergrad-research/undergrad-research
- http://www.ams.org/programs/edu-support/undergrad-research/PURMproceedings.pdf
- http://www.ams.org/programs/edu-support/undergrad-research/REUproceedings.pdf

MAA Conferences: http://www.maa.org/rumc/upcoming.html

LaTeX templates (paper, talk) and Mathematica primer:
http://web.williams.edu/Mathematics/sjmiller/public_html/math/handouts/latex.htm

V. Summary

The panelists brought a range of experiences and perspectives to the discussion of undergraduate research and REUs.

Joe Gallian, the senior member of the panel, has worked extensively with undergraduates at the University of Minnesota, Duluth, both in his own long-running REU and during the school year with individual students. Ivelisse Rubio is one of the directors of the MSRI-UP REU and mentors research students at her home institution, University of Puerto Rico, Rio Piedras. Steven Miller has worked with a numerous undergraduate students at several institutions and in a variety of formats, including the SMALL REU program at Williams College, his home institution. Jake Levinson is currently in his first year of graduate school at the University of Michigan, and participated in the SMALL program and wrote a senior thesis at Williams. Gina-Maria Pomann is a third year graduate student in statistics at North Carolina State University and as an undergraduate participated in the MSRI-UP program and in research at Princeton in molecular biology, at the Stanford VIGRE program in statistics, and at AT&T in statistics. Dean Evasius moved to the NSF in 2004 from the NSA and is a program director for the Division of Mathematical Sciences, heading the Workforce and Infrastructure programs. David Damiano, the moderator, has worked for many years with individual students both during the academic year and in the summer research program at his home institution, the College of the Holy Cross.

The discussion highlighted a variety of possible approaches to the design and implementation of an REU. Joe Gallian looks for eight or nine students each year who are ready to jump into doing research on their own as a bridge to attending graduate school and becoming a research mathematician. Focusing on combinatorics and number theory he seeks students with the background necessary for work in these areas. He is constantly on the look out for problems and sends students and graduate TAs to the CANT conference each spring in New York for this purpose. The intention is that student projects will lead to publishable research. While Joe runs the program by himself, he uses program TAs to guide the individualized projects that are at the heart of the REU. Although students work independently, he pays attention to the applicants' collegiality when choosing participants.

In contrast, the MSRI-UP targets talented students from underrepresented groups who might not have the background to get into a program like Joe's at UMD. The five directors of the program seek out a mathematically heterogenous group of eighteen participants. The first two weeks of the program, which makes full use of the opportunities available at MSRI, are devoted to course work and homework preparing students for that summer's research focus. This is an intense period for the students and also allows the onsite program leaders to identify effective groupings of students, since all the projects are group projects. The groups purposely match students with different levels of ability and different skill sets. While publishable research is desirable, it is not the primary goal of the program, rather it is to bring students into the pipeline leading to graduate study in mathematics.

The Williams SMALL program also brings together students with different mathematical experiences. Students are grouped by background and interest so that within a group students are frequently at roughly the same level, and problems are targeted at this common level. The program is larger than MSRI-UP and involves several faculty. However, close attention is paid to creating a strong sense of community, done through social activities and numerous math talks that all attend. In contrast to program at UMD, a successful applicant need not be committed to graduate study in mathematics or computer science. Similar to the MSRI-UP program though with a different applicant pool, publishable research is desirable but not the sole measure of success for the program. Steve also commented on always being on the lookout for good problems either in the literature, at conferences, or when communicating with colleagues, as well as giving open problems in talks math clubs.

Each program encourages students to present their work at conferences either in sessions devoted to student research or as posters. Their goals are two fold: to develop the skill and confidence required for communicating mathematics successfully, and to begin early on to network with professional mathematicians and with other students who will someday be their professional peers. Gina-Maria and Jake confirmed the importance of these opportunities in their own professional development and highlighted how participation at conferences led to making connections.

The NSF, for its part, is interested in funding well-crafted proposals with clearly stated goals and a well-identified target audience, and that have the pieces in place to attract this audience and achieve the goals. Consequently, a program might target rising juniors with limited mathematics background or underrepresented groups with the goal of providing a significant mathematical boost and drawing them into a career in the mathematical sciences, or it might look for more mathematically experienced students who have already focused on a career in mathematics. It might be run by one or two or individuals, a team of individuals from one institution, or widespread group of directors. It might focus tightly on theoretical mathematics or more broadly on the mathematical sciences, or it might partner with other disciplines or industry. As Jake and Gina-Maria's experiences indicate, students can benefit greatly from programs with widely different goals and structures. Program self-assessment plays an important part in the renewal of REU programs.

Some attention was paid to research with individual students during the academic year. As these students are "home grown," faculty talked about consciously seeking out and mentoring younger students with the intent of bringing them into a mathematics or mathematical science major and of encouraging them to do undergraduate research. There was some concern that working with undergraduates may not be appreciated by one's colleagues or administration, especially for junior faculty in the process of becoming established professionally.

Audience questions addressed topics covered by the panel as well other items. Finding good project topics was a concern expressed by several audience members. There were also questions about what REUs are looking for in their students and how to help students choose an REU. Since there is great variety among REUs, these questions turn out to be non-trivial. Indeed, an experienced member of the audience thought this was an almost impossible task given the available tools and the suggestion was made that the AMS should redesign its webpages on student research opportunities to make them more user friendly, that is, to make it easier for students to find programs that match their backgrounds and interests.

The consensus of the panel and the audience was that there is a tremendous need for more REUs and other research opportunities for undergraduates. While there has been remarkable growth in student research in the last twenty-five years, it is still outpaced by demand. Many programs report ten or more applicants for each slot. Concerning the direction of undergraduate research, Dean noted that the NSF is in the business of responding to the needs of the profession, not in steering the profession in any one direction. Lastly, Dean pointed out that one underutilized source of funding for undergraduates is supplements to existing NSF grants; the grant may either be the student's advisor or another colleague at the institution. Approval is somewhat routine and supplemental funds can be used to creatively to support undergraduates.

VI. Appendix: Finding Research Problems

As discussed above, one of the most important parts of guiding undergraduate research is finding appropriate problems for students / groups of students. A key component is realizing not just their backgrounds, but also being aware of what your and their goals are (ranging from publishable papers at various levels to a sense of what research is like). Below are two examples of research problems one of the panelists (Steven Miller) chose, what led him to these choices, and when possible what happened.

(a) More Sums Than Difference Sets

When I was a postdoc at Brown, one year Mel Nathanson came and gave a great talk on More Sums Than Difference (MSTD) Sets. What immediately intrigued me about the problem was how easy it was to state and understand; this is a common feature of many number theory and combinatorial problems. The problem is as follows: given a finite set of integers A in {0, 1, …, n}, one can form the set of sums A+A = {x+y: x, y in A} and the set of differences A-A = {x-y: x, y in A}. As addition is commutative and subtraction is not, most pairs of elements of A generate two differences but only one sum, and thus we expect A-A to have more elements than A+A (and be difference dominated); if there are more sums we say it is sum dominant or a More Sums Than Differences (MSTD) set. Because of this, Mel earlier had stated that with the right way of counting, as n tends to infinity almost all subsets of {0, 1, …, n} should be difference dominated. Surprisingly, it turns out that a positive percentage of subsets are MSTD.

I heard more about this and related problems at the CANT (Combinatorial and Additive Number Theory) 2007 conference. At the conference, I started playing with what would happen if we didn't draw the subsets uniformly. There were lots of ways to play with how to choose the subsets; a colleague Peter Hegarty and I quickly settled on a decreasing function f: N → [0,1] and if A is a subset of {0, 1, …, n} then k is in A with probability f(n). We proved many results depending on how fast f decays.

From this, I generated several projects for undergraduates. The first was for a summer REU student of mine at Brown. All constructions proving a positive percentage of sets were MSTD were probabilistic; could an explicit construction be done? While we were unable to get a positive percentage, we did show that the percent of subsets of {0, 1, …, n} that are MSTD is at least C/n^4, significantly better than the previous result of $C/2^{n/2}$.

I then presented these results at another CANT conference, attended by some of Joe Gallian's students looking for problems. They brought this back to his REU, and looked at many generalizations, including new methods to get a larger explicitly constructible family of MSTD sets, as well as generalizing from the integers to other abelian groups.

I've continued working in the field. When teaching a number theory class, some of my students wanted to try their hands at research. I wanted a nice problem building on earlier work with students. Before my student and I looked at explicit constructions of MSTD sets; a reasonable generalization was to look at finding sets A such that A+A+A+A beat A+A-A-A. This was a great project as the students could start with numerical experimentation on day 1, and slowly work into the theory. While we weren't able to get a positive percentage, we were able to elementarily get much better than C/n^4 in this case.

I recently gave additional problems related to MSTD sets to some students in my SMALL REU. We looked at kA – jA versus pA – qA (here rA means A added to itself r times), where k+j = p+q, we looked at trying to have strings of comparisons, and we looked at k-generational sets (A, A+A, …, kA are all MSTD) among other problems. The students did a great job, completely solving the problems. Their research began with a review of the literature, which didn't take too long in this case, and then they quickly began their investigations.

To date, at least 10 papers have come from the talk Mel gave at Brown, most with student involvement. What I loved about the problem is that it was not only easy to state, but it was a `young' problem. It hadn't been exhaustively studied for years, and thus there was an excellent chance of students making a real contribution. The key was to look at the parts of the subject that were solved, and ask good questions on what to do next.

(b) Origami

The next example comes from a talk I heard at an AMS Special Session on Undergraduate Research (Dave Damiano and I were two of the four professors chairing the session; there were also five undergraduates chairing too). There was a nice talk titled `The Origami Box Problem', presented by Brittany P. Baker (Saint Michael's College), which was joint work with Patrick Dukes (Clemson University), Elias Halloran (Bard College), Anne Ho (Colorado State University), Audrey Hubbard (Ave Maria University), Alexander James (University of Nebraska at Omaha), Victoria McCoy (Yale University) and Robert Rumely (University of Georgia).

> Abstract for The Origami Box Problem: *What is the largest volume that can be enclosed by folding a square sheet of paper, one unit on a side, into a closed box? Different origami boxes with increasing volumes in our attempt to find the maximal volume of an origami box. We have not solved this problem; as will be seen, a solution would require a deep understanding of curved paper surfaces. However, we analyze a series of designs with larger and larger volumes, and we identify a class of designs, which we call Inflated Sealed Sacks, to which the optimal design likely belongs. We believe that our best design is within a few percent of the optimum. After doing this research we found significant related work done under similar names, "Paper Bag Problem" or "Tea Bag Problem". We finish with a few related open questions in origami, including our own origami cup problem. The Origami Cup Problem: What is the largest possible volume of a cup folded from a square sheet of paper, one unit on a side.*

I was in the audience, and enjoyed the talk. I asked the speakers about possible generalizations. They hadn't looked at many of these, and I filed them away for the future. (Currently, Kathy Andrist of Utah Valley University, who was in the audience at the panel, is considering studying some of these with students there.)

Here are some possible generalizations. While many of these might already have been studied, the point of these notes is to discuss how to seize on features of a project when looking for new problems.

- The original problem has a unit square. What if it isn't a square but a rectangle? Note without loss of generality one may assume the rectangle is 1 by x for some real number x at most 1.
- The original problem was a square, which we generalized to a rectangle. What can be said about other shapes? In general this is probably very difficult, but looking at these problems might give some insight into what is happening for the square or rectangle. It might be worthwhile to consider an equilateral triangle, then a right triangle. For the right triangle, it might matter whether or not the angles are rational or irrational.
- Returning to the square, another generalization is to allow ourselves the ability to cut the paper. There are lots of restrictions one could place. We could allow ourselves only one cut, or only a given number N of cuts. We might require that the pieces stay connected, or allow ourselves to separate pieces. Are the cuts only allowed to be lines (if yes, must they be parallel to the sides of the square), or are more general curves permissible?
- Instead of having just one square, what if we were given two squares? If the two squares are identical, does anything nice happen? Can we get a trivial upper bound on what we can do with one square by considering 6 squares? What if the two squares are not identical? (Note without loss of generality one may be taken to be a unit square, and the other x by x for a real number x less than 1.)

There are many other questions one could ask related to both of these talks; the above is meant to give a flavor of how to look at the conditions in a project and ask what happens if we change them.

(c) Number theory and random matrix theory

This section describes another great source of projects: recent graduate theses in your area. My main area of research is in number theory and random matrix theory. In the 1970s a shocking behavior was noticed: the same mathematics that successfully models the spacing between energy levels of heavy nuclei also beautifully models the spacings between zeros of the Riemann zeta function. Since this connection was observed, numerous researchers in mathematics and physics have explored these connections. The common theme in these studies is the behavior of eigenvalues of random matrices, which seem to describe these and many other systems. My thesis was in exploring some of these connections for families of elliptic curve L-functions.

A few years before I graduated a mathematical brother of mine, Michael Rubinstein, looked at these connections for families of quadratic Dirichlet L-functions. A useful statistic (which we don't need to describe for our purposes here) is the n-level density, which measures the behavior of zeros near the central point for L-functions, or eigenvalues near 1 for the random matrices. This statistic depends on test functions whose Fourier transform is compactly supported; the greater the support, the more we can deduce about the behavior. Rubinstein (in his Princeton PhD thesis) was able to show the scaling limit of the behavior of the n-level density of the zeros of these L-functions agrees with that of eigenvalues of random matrix ensembles for all n for suitably restricted test functions. A few years later Peng Gao (in his Michigan PhD thesis) was able to double the support for the calculations on both the number theory and the random matrix side; unfortunately, due to the horrendous combinatorial expressions, he was only able to show agreement in this larger window between the two calculations for n up to 3.

All the conjectures in the field say that number theory and random matrix theory should agree; it was quite annoying to have both calculated and yet not know if the results were equal. I saved this problem for a few years, as I believed it would be a great thesis project. I figured that with *a lot* of work, a student could brute force it and at least prove equality in the expanded region for n equals 4. This would mean they would have a result, and not spend a year getting nothing. I thought that maybe, if we're lucky, we could get a sense of how to do the general case.

Last year, Jake Levinson at Williams decided to do a thesis with me. We began by reading several papers and notes on number theory in general, and L-functions and random matrix theory in particular. Eventually this problem interested him. As I hoped, Jake was able to solve the $n = 4$ case. In doing so, he found a significantly cleaner approach. He created canonical formulations for the terms in both expressions to facilitate comparisons. In the end, this new vantage point (which relied heavily on symbolic programs such as Mathematica to implement the comparisons) was able to fully solve the problem up to $n = 6$, and led to a possible approach for higher n. He reduced the general case to a specific identity, which itself is now an excellent future project!

A big part of a successful project is knowing the student. Jake had a good background, and was willing to spend a month learning what he needed. This project had a component where the plan of attack was pretty clear, so we could start working fairly quickly. Further, it had tempting generalizations and extensions where the path was not known, which allowed Jake to have a chance to shine and control the research. Finally, it was building on a recent result in mainstream number theory research, and thus is of interest to many people. I've created many problems along these lines, and these lead to some of my most successful research experiences with undergraduates.

(d) Final thoughts

In conclusion, while I've found these recipes work very well for me for talks and papers, obviously different methods will work better for others. These are just meant as a suggestion as to one way to find appropriate problems. David Damiano pointed out a very nice feature about building on other undergraduate projects, be they ones you have supervised or ones you've heard at a conference. Since your students are continuing work of other undergraduates, the new group immediately knows that the research here is doable and of interest. This also applies to building on recent PhD theses or papers you've heard at a conference, though in this case the problem may require significantly more time spent mastering the background material.