Mentoring through Predator-Prey: Realizing Hopes through Polymath Junior

Wael El Khateeb
Department of Mathematics
University of Toledo
TBD@TBD
Steven J. Miller
Director, Polymath Jr. Summer Research Program
sjm1@williams.edu

1 What is the Polymath Jr. Program?

Fortuna est quae fit cum praeparatio in occasionem incidit.
Luck is what happens when preparation meets opportunity. –Seneca

The Polymath Jr program provides summer opportunities for group undergraduate research in mathematics, including applied mathematics and modeling, via remote mentoring online. The experiences of the program have provided many opportunities for the students and mentors involved. The purpose of this note is to describe and advertise the program, and encourage people to apply to be mentors and student researchers.

There are many people who could write this article (see [1] for another article by many of the professors involved in the program from the start); while the broad brushstrokes would be similar, the tune would be slightly different. The two of us are doing this as our experiences are a great realization of the quote above. We gave a talk at the Joint Mathematics Meetings. At the time of the conference, the first named author was a graduate student and on the job market. Brian Hollenbeck, chair of mathematics at Emporia State University, heard our presentation (and Wael's remark that he was seeking employment); fast forward a few months and the two of them are now colleagues, as he has begun a tenure-track position there!

This story illustrates the driving force behind the Polymath Jr. program: *we create opportunities.* We realize that not everyone will seize them, but those who do can have life-changing outcomes, ranging from acceptance to strong graduate programs to having their prayers answered and getting a good job.

Below, we briefly describe the program's history and how it works. We then turn to some general issues (both good and bad), and end with a brief summary of the project the two of us did, a predator-prey problem. For those interested in more details, from participating as a student or helping to run a project as a graduate mentor or professor, contact the second named author or see the webpage (linked later in the article).

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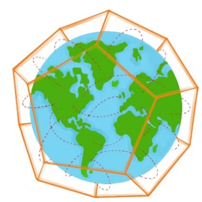
1.1 Purpose

Η *αναγκη μητηρ εστιν τῆς ηυρεσεως*
Necessity is the mother of invention. –Plato

In late 2019 or early 2020, the second named author was serving on a regional school committee, and at a meeting wondered if the district should develop contingency plans in case this COVID infection turned out to be much worse than preliminary fears suggested. He also expressed these concerns on a listserv of directors of math REUs (Research Experiences for Undergraduates); these are wonderful summer programs, mostly in-person, where students spend several weeks doing research guided by professors and often postdocs and graduate students.

The math community quickly agreed that there should be a Plan B, and one of the outcomes of these conversations was the Polymath Jr. Program; see

https://geometrynyc.wixsite.com/polymathreu



for more details. The following description is a slightly modified version of the website's summary of our goal. Namely, we were (correctly!) afraid many in-person programs would be cancelled, and we wanted to provide opportunities for research both to those who lost their chance and to those who never had one.

> *Our goal is to provide research opportunities to a large number of undergraduates who wish to explore advanced mathematics. This online program consists of research projects in a variety of mathematical topics, and runs in the spirit of the Polymath Project (see https://polymathprojects.org/ and https://en.wikipedia.org/wiki/Polymath_Project; researchers collaborate by posting ideas and partial results on difficult problems, and by building on each other's work several open, difficult problems have been solved). For us, each project is run by an active researcher with experience in undergraduate mentoring.*
>
> *A typical project consists of 15-25 undergraduates, at least one professor (sometimes more), and graduate students and postdocs as additional mentors; however, sometimes we have much larger and sometimes much smaller groups – we create a flexible environment that allows mentors to run their groups in a way that works best for them. The group works towards solving a research problem, frequently writing a paper and presenting at conferences. Each participant decides what they wish to obtain from the program, and participates accordingly (this can range from being a passive participant just interested in seeing some mathematics to a part-time contributor balancing this with a 9-to-5 job to someone whose sole summer focus is this work). The program has been generously supported by the National Science Foundation, which provides stipends for graduate student mentors and support for participants' travel to conferences.*

While we created the program to provide new opportunities replacing those lost during the pandemic response, we quickly realized that the program made sense even in normal times. We found we had created a scalable system with a phenomenal force multiplier component. We have hundreds of students from all over the world gaining research opportunities they never would have had otherwise. Additionally, many junior faculty and graduate students gain experience in mentoring. The program is old enough that many of these aides are now running their own groups!

Note the program is easily scalable; the limiting factor in the number of students we can take in a summer is the number of graduate assistants and faculty mentors. If several institutions pooled resources, they could easily create a version of the program serving many students for a minimal cost. Of course, any such interested groups are encouraged to contact the Polymath Jr. program about joining forces, as we have a lot of infrastructure to assist in everything from creating reasonable problems to having resources for first time mentors.

1.2 Logistics

Le mieux est l'ennemi du bien.
The perfect is the enemy of the good. –Voltaire

It's amusing when professors advise students and junior colleagues about taking on too much, because, as a group, we are often reluctant to say "no". When creating the Polymath Jr. program, we were very cognizant of the above quote. We knew that we couldn't create a program as good as the standard in-person ones that so many of us run. We would not have funds to provide students with stipends. We would not have people working and living together, which facilitates conversation and community. We wouldn't have the time to be as involved with them and the projects.

Interestingly, these disadvantages are also strengths of the program. As students are not receiving a stipend and living together, they can work a standard job and live with family and friends anywhere. We accept far more students (in the hundreds as opposed to tens or less) as we no longer have financial and housing constraints. Finally, as the professors are often busy with running an in-person program as well, this necessitates delegating a lot of the responsibilities to the junior faculty/graduate student mentors, which is terrific for their professional development, and gives them a distinguishing item on their resume.

There's also not the same pressure in the selection process; unlike the in-person funded programs where space is limited and thus competition for a slot is fierce, our structure allows us to try to accept everyone who has taken at least one proof-based course. Thus our application process is a lot simpler; rather than a long personal statement and a detailed letter of recommendation, all we require is a note attesting to their ability to write proofs. Unfortunately, we're now victims of our own success; while in the first few years we could take everyone, for 2026 the number of applications exploded to basically 600 applicants for 300 spots, and we had to turn significant numbers away for the first time.

While no two groups run exactly the same, they have many common features. A senior mentor, usually a professor with extensive experience in guiding undergraduate research, has an area to investigate. There might be a specific question in mind, or springboard problems to introduce the subject and give students some control in what is pursued.

Usually a salaried graduate student (sometimes a post-doc, sometimes an advanced undergraduate) assists in a variety of ways, from coordinating meetings and write-ups to giving background lectures and commenting on drafts to brainstorming with the professor on the project. The advantage of online mentoring makes it possible for students to benefit from the breadth and depth of the knowledge of advisors and graduate mentors, whose work spans applied analysis, fluid dynamics, data assimilation, mathematical biology, optimization, statistics, and computational algebraic geometry [11, 12, 13, 19, 9, 27, 16, 2]. Thus, by summer's end, students have not only gained research experience, but we have also trained new people who can run their own groups in the future, thus greatly expanding the number of opportunities for student research.

In addition to the research activities within a group, there are common events. These range from talks by mathematicians for all participants to special mentoring sessions for the graduate mentors.

Polymath Jr. spans several weeks, beginning with an introductory period designed to immerse students in the research environment and expose them to a diverse range of mathematical topics. During the first week, the program features sessions that highlight previous participants' accomplishments, provide guidance on interpreting academic job advertisements, and offer strategies for efficiently finding and reading research papers. These sessions also create opportunities for peer interaction. To foster a supportive online community, the program emphasizes healthy research habits, such as stress management and techniques to avoid burnout.

As the program progresses, the focus shifts toward specialized mathematical fields through lectures on topics like differential equations and combinatorics. Following these presentations, students identify three areas of interest; the program typically succeeds in matching most participants with their first-choice topic. Once assigned, students collaborate in small teams with faculty advisors and graduate mentors in private sessions. This structure allows teams to develop the specific background and tools necessary for their projects. Throughout the summer, mentors provide holistic guidance on both mathematical content and professional development. The program culminates in a final presentation to the Polymath Jr. community, though many teams continue their research and collaboration long after the formal conclusion.

Most groups have a publishable product at the end of the summer or sometime the following year. Frequently students and their mentors travel and present at conferences; for the past several years we've had a special session at the Joint Mathematical Meetings.

2 Results

The results you achieve will be in direct proportion to the effort you apply. –Denis Waitley If opportunity doesn't knock, build a door. –Milton Berle

Overall, the program is a tremendous success. If you look at the number of students gaining a research experience, and the number of people mentored in mentoring, the cost per person is a fraction of what a standard in-person program costs. We give a high-level, general summary here, and then discuss in detail for the project the two of us ran in summer '25. The greatest challenge Polymath Jr. faces is finding enough professors to run groups; the more groups we have, the more students we can accept and still provide a good mentoring experience.

2.1 Setbacks

Every setback is a setup for a comeback! –Joel Osteen

Perhaps the two greatest challenges are making sure there are enough opportunities for everyone who applies, and helping those who elect to participate to be meaningfully engaged.

Before the program was well known, we could take everyone. The number of applicants has grown each year as more and more people learn about us, and we sadly have to tell many graduating seniors and high schoolers that we just do not have space. It's important to resist the temptation to just take everyone and make the groups larger. We have a diverse student population in terms of background, ability, and time available to devote. There is thus already a danger of some students feeling marginalized and not engaging; we fight such tendencies by keeping the numbers smaller so each person is more involved.

In the opposite direction, not every student has the same goals. Some just want to see some good math, while others have every intention of being active participants, but life intervenes. If the student is playing a key role in research, this can be a huge challenge for the project. Frequently, however, such disengagement happens early in the summer. We talk to students and try to keep them engaged, but with finite resources and lots of students who are struggling to contribute needing help, each group typically loses at least a few students, leading to more manageable numbers.

2.2 Successes

Doing the best at this moment puts you in the best place for the next moment. –Oprah Winfrey

In terms of impact per dollar, the Polymath Jr. program might be the best investment as hundreds of students gain research experience, and tens of graduate students and junior faculty gain experience in mentoring.

We have students from all over the globe; everyone from a dreamer in a tiny third world village to a student at a top university with a strong research culture. Students get a sense of what research is like, if this is something they wish to pursue, and a better understanding of some fields of mathematics (helping them answer whether or not they wish to continue exploring such questions in graduate school and beyond.)

In addition, the program prepares them for a variety of possible futures. The skills learned are highly transferable (reading technical papers, doing original work, collaborating with others under difficult situations, and of course writing and speaking well). They get a sense of whether or not graduate school is right for them, and of course an opportunity to impress a senior professor who can advise them on where to apply and write letters when appropriate to help them get there.

Most important, however, is the force multiplier. Anyone running an in-person REU knows how painful it is to turn away eager students. By mentoring graduate students and early-career faculty, we increase the pool of people able and willing to advise students.

3 Representative Project: Predator-Prey

3.1 Creating a Project

Small opportunities are often the beginning of great enterprises. –Demosthenes

Wael gave a research talk at an AMS Eastern Sectional Meeting on Difference Equations, where Steve was also speaking. Both immediately noticed shared interests in conversations during the session, and we began talking about a possible joint collaboration. As Steve was always on the lookout at conferences for young people to assist with the Polymath Jr. program, this quickly led to an invitation to create a research project inspired by Wael's dissertation research. The goal was to design a predator-prey problem that was mathematically meaningful, biologically relevant, and still accessible to undergraduate students, while providing Wael with an opportunity to be mentored in mentoring. What first appeared to be a small academic opportunity became the foundation for a long-term collaboration involving student research, ecological modeling, computational analysis, and professional development.

Polymath Jr. provided a mentoring structure that was both flexible and productive. Though still a graduate student, Wael served as the primary research lead, guiding the mathematical development of the project and working closely with the students on modeling, analysis, and computation. Steven played an essential role in shaping the broader direction of the collaboration. He helped organize meetings, sustain momentum, provide references, give background lectures, and offer perspectives when appropriate, and kept the group focused on goals that were ambitious but realistic. He also opened important professional doors by helping the students find opportunities to present their work, connect with researchers, participate in the wider mathematical community, and apply to more advanced in-person research programs and graduate schools in future years.

Steven's mentorship had a lasting influence on the students' professional formation. He helped them learn how to communicate their ideas clearly through talks, abstracts, emails, slides, and papers. His guidance ranged from broad advice about presenting research to practical details such as managing time during a talk, designing effective slides, and writing polished professional messages. Even small pieces of advice became memorable, such as recommending a green pointer instead of a red one because it appears more clearly in recorded presentations. He also helped the group gain access to important software resources, including Mathematica and Maple, and remained consistently generous with his time and support.

Wael's role focused on the mathematical core of the project. Working with four undergraduate students, he aimed to identify a problem that was applied, meaningful, and appropriately challenging. Since most of the students were sophomores and had not yet taken courses in differential equations or related areas, the early part of the program required careful preparation. The first stage of the project was therefore devoted to building the background needed for the students to participate in the research.

3.2 Balancing Discovery and Learning

I went to a bookstore and asked the saleswoman, 'Where's the self-help section?' She said if she told me, it would defeat the purpose. –Steven Wright

There are many decisions needed to make the research possible for students with minimal background. What did they have to learn well, and what results could be black-boxed and just used? If too much is taken on faith, then there is a danger of using results incorrectly; if too much is taught, then the entire summer is spent on lectures, and no original work is done. Further, new material is internalized better if some is discovered by the student through homework rather than through lectures and readings.

We chose to introduce only the concepts most essential to the project, including differential equations, mathematical modeling, systems of ordinary differential equations, Jacobian matrices, eigenvalues, local stability, and phase portraits. Because the program was short, we focused on the ideas that were most directly connected to the research problem rather than attempting to reproduce a full course in differential equations.

Even within the limitations of fully online mentoring, we tried to make the mathematical ideas concrete and engaging. When introducing the Hartman-Grobman theorem (or linearization theorem), which characterizes local behavior near hyperbolic equilibria via lower-order terms, we asked students to compare the behavior of x, x^2, and x^3 for large values of x [14, 15]. They quickly observed that the highest-order term dominates. We then asked what happens when x is very small, such as $x = 0.0001$, allowing them to see why lower-order terms become more important near an equilibrium. We also used `pplane` to help students visualize how phase portraits change as parameters vary [22]. Instead of spending valuable time teaching the internal details of MATLAB's `ode45` solver or Runge-Kutta methods [26, 10, 18, 25], we provided open-source code that students could adapt with minimal editing. This allowed them to focus on the mathematical and biological interpretation of the model.

Once the students had developed enough background, we presented several possible research directions and invited them to choose the one that interested them most. At one point, we placed them in a separate breakout room so they could discuss the options freely with each other without us listening, and decide which path they wanted to pursue. This was intentional. We wanted them to help shape the project,rather than having it be a problem simply assigned to them. The students ultimately chose to study the ecological interaction among Burmese pythons, alligators, and shared prey in the Everglades. The problem was appealing because it combined predator-prey dynamics, competition for resources, and a current ecological issue with clear real-world relevance.

As the project developed, the students helped construct and analyze a three-species model describing how pythons and alligators interact indirectly through shared prey, allowing for the study of how predation and hunting pressure affect long-term system behavior. While the full system quickly became complex, the research produced significant dual insights. From the mathematical side, the model exhibited high codimension bifurcations, revealing a rich and delicate dynamical structure. Biologically, the analysis identified a very thin and non-robust region of coexistence between alligator and python populations. This suggests that the ecological balance is fragile and highly sensitive to small changes in hunting pressure or environmental conditions, making this coexistence regime potentially difficult to maintain in practice.

As the work progressed, the students learned additional computational tools, including Maple and Mathematica, as the initial six-week summer experience grew into an eight-month collaboration. In the early stages, the work focused on the biological meaning of model terms, the interpretation of equilibria, and the role of local stability. Later, the students moved into deeper analytical questions regarding bifurcations and stability switches. During weeks when progress slowed due to exams, meetings were used to introduce related topics such as delay differential equations and reaction-diffusion models, helping the students see how their specific project fit into a broader mathematical landscape.

The project also developed through conference presentations and feedback. Early presentations focused on explaining the ecological motivation, the structure of the three-species model, and the basic mathematical questions behind the work. Over time, the students became more confident, and the talks became more polished, precise, and mathematically substantial. The group presented the project at several venues, including the SIAM TX-LA Sectional Meeting, the Polymath Jr. Online Conference, JMM 2026, SIMIODE 2026, and undergraduate research symposia at the University of Nebraska and the University of Toledo. These presentations helped the students grow as researchers and communicators. They also helped refine the project itself, as audience questions and faculty feedback led to clearer biological motivation, sharper mathematical interpretation, and new directions for analysis.

3.3 Opportunities

If opportunity doesn't knock, build a door. –Milton Berle

The collaboration also created important professional opportunities. Through repeated presentations, the students gained visibility and confidence within the mathematical community. Faculty began to recognize their work and follow the development of the project across different meetings. The experience also helped Wael build new professional connections and contributed to later invitations involving model design, data handling, and fitting techniques in related research settings. These opportunities were not accidental. They grew from persistence, outreach, mentoring, and the willingness to continue building the project after the original summer program had ended.

Like most meaningful collaborations, the experience involved both setbacks and unexpected progress. After the formal six-week program ended, two students were no longer able to continue because of other commitments. Nevertheless, the project continued to move forward. In an effort to obtain data relevant to the model, we reached out to ecologists across the United States. This eventually led to a collaboration with the Croc Docs, a group of ecologists whose biological expertise and data strengthened the project in important ways. Their involvement helped connect the mathematical model more closely to the ecological reality of the Everglades system.

As data became more central to the work, we welcomed two additional students, Anh and Sangam, into the collaboration. Their backgrounds in data science added an important new dimension to the project. They contributed to data cleaning, exploratory analysis, and methods for fitting and evaluating the model. Their work expanded the project beyond its original mathematical framework and helped turn it into a more integrative study combining modeling, computation, data analysis, and biological interpretation.

On a personal level, the experience transformed Wael not only as a researcher but also as a mentor. It strengthened his ability to communicate across disciplines, build trust with students, listen carefully, and turn an initial idea into a shared intellectual project. The collaboration also supported meaningful student growth. Jasmine, for example, is now considering graduate study in applied mathematics and has been admitted to the highly selective Summer Institute for Training in Biostatistics and Data Science at Columbia University. She also received a strong reference letter from Steven that reflected the exceptional quality of her work. Similarly, Anh and Sangam developed stronger data analysis portfolios, giving them useful preparation for future master's programs or competitive professional opportunities.

The project also attracted interest beyond our immediate circle. In particular, Professor Samer Habre of the Lebanese American University in Beirut, Lebanon, was impressed by the work and shared the opportunity with his students. After a short presentation introducing the program and the research, several students expressed strong interest in participating. As a result, two students from Lebanon are expected to join the program in summer 2026. In addition, several graduate students from the University of Toledo have expressed interest in serving as mentors during the upcoming summer. In this way, a project that began with one conference presentation has continued to grow into a broader mentoring and research network.

Figure 1: Anh Quynh Bui and Sangam Dhakal presenting their work at the 2026 University of Toledo Undergraduate Research Exhibition.

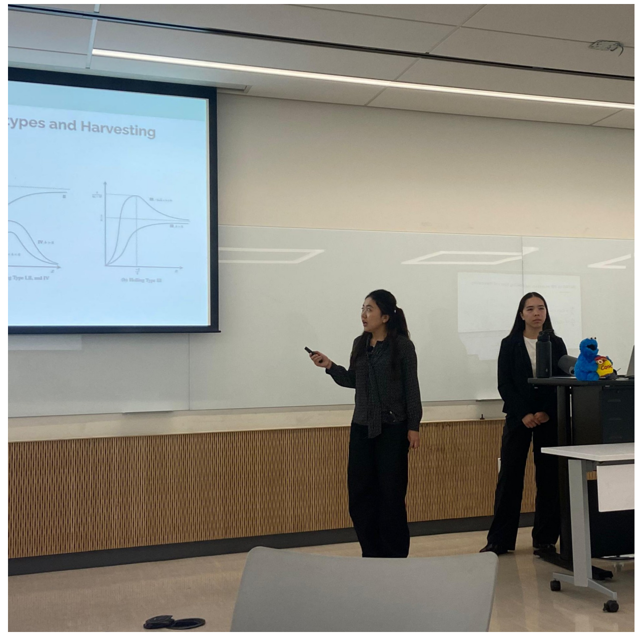


Figure 2: Jasmine Pham and Jessie Wang giving a talk at the 2025 SIAM TX-LA Sectional Meeting.

4 Closing remarks and an invitation

If I have seen further it is by standing on the shoulders of giants. –Isaac Newton

While Polymath Jr. has grown successfully over the years, our goal is to build on these successes and continue to expand its reach. The story we told here is a representative one, highlighting why this program matters and why we remain motivated to keep moving forward.

Polymath Jr. is more than a research program. It builds bridges between people, connects young students with researchers, and creates opportunities for undergraduate students to experience mathematics beyond the classroom. It also offers graduate students and postdocs the chance to assist in designing and running research programs. There is now a thriving community, ranging from participants meeting at conferences (each year we have a special session at the Joint Mathematics Meetings), to staying connected through Discord and other sites. The publications produced by Polymath Jr. students and mentors in both pure and applied mathematics provide strong evidence of the program's success. Students produced applied and applied-adjacent work in machine learning, probability, stochastic games, computational geometry, random walks, Benford's law, and geometric function theory [4, 23, 3, 5, 6, 17, 24, 7, 20, 8, 21].

The more mentors and aides we have, the more groups we can run and the more students we can take. We thus warmly encourage professors, postdocs, and graduate students to reach out about running or helping to run a research group (email sjm1@williams.edu if interested), and encourage undergraduates to apply in future years (applications are through mathprograms.org; the deadline is usually the first week of April, late enough for applicants to know whether they have been accepted into an in-person program.

There's an old joke; professors teach and research for free; you have to pay us to do all the administrative work! Sadly, as the years pass, more and more of your days are taken over by paperwork; we've found the Polymath Jr. program incredibly rewarding and energizing. It's great to see the positive impact one has on lives and careers, and we hope you'll reach out and join us.

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About the Authors

Steven J. Miller graduated with a B.S. in mathematics and physics from Yale and a Ph.D. in mathematics from Princeton. He has supervised more than 600 students and written more than 200 papers in accounting, computer science, economics, geophysics, marketing, mathematics, operations research, physics, sabermetrics, and statistics. He is a founding member of the Polymath Jr. Summer Research Program and the President of the Fibonacci Association. He loves multitasking, preferably with his family, who have published a few papers with him and created some interesting bridge problems. He can also teach people how to poorly solve a Rubik's cube in under an hour.

Wael El Khateeb is an Assistant Professor at Emporia State University. He earned his Ph.D. in Applied Mathematics from the University of Toledo and his B.S. from the Lebanese American University. His research sits at the intersection of bifurcation theory, dynamical systems, and machine learning, with applications to biological and ecological problems. Wael served as president of the AMS Graduate Student Chapter at Toledo, was one of the founders of the SIAM Chapter at Toledo, and worked as a computer science instructor with Johns Hopkins Center for Talented Youth as well as MathCorps. He enjoys cycling, hiking, and working out, and his favorite book is *The Alchemist* by Paulo Coelho.