# UPPER LEVEL MATHEMATICS AND STATISTICS COURSES SHARED ACROSS CAMPUSES

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ABSTRACT. For undergraduate students of advanced mathematics and statistics, the liberal arts model offers a deep level of engagement in learning with faculty and peers. Due to practical limitations, small colleges cannot usually offer the breadth of courses available at large institutions with graduate programs. To explore collaborative models that may help enrich curricular offerings, faculty and technologists from several leading liberal arts colleges are experimenting with a consortial hybrid/online course sharing model. The goal of this chapter is to report on what we learned from teaching three different courses in this context.

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## 1. Introduction

For students of advanced mathematics and statistics, the liberal arts model offers a deep level of engagement in learning with faculty and peers. Due to practical limitations, small colleges cannot usually offer the breadth of courses available at large institutions with graduate programs. To explore collaborative models that may help enrich curricular offerings, faculty and technologists from several leading liberal arts colleges are experimenting with a consortial hybrid/online course sharing model. These institutions are partner schools in LACOL, the Liberal Arts Collaborative for Digital Innovation (`http://lacol.net/`). The goal of these explorations is to increase the wealth and frequency of the advanced classes our students need, both for future graduate study and to delve deeply in the subject.

In this chapter we report on the challenges and opportunities encountered teaching three different math/stats courses using the hybrid/online course sharing model:

- Graduate Real Analysis, Fall 2017 (Stephan Ramon Garcia, Pomona College),
- Putnam Problem Solving, Spring 2017 (Steven J. Miller, Williams College),
- Bayesian Statistics, Fall 2017 (Jingchen Hu, Vassar College).

The courses were not completely designed from scratch but rather built upon previous classes taught by the authors, some standard and some partially online. For these shared courses, each instructor opened their course to students across LACOL, sharing lectures, assignments, and other class activities through both asynchronous (e.g., recorded lectures and screencasts) and synchronous (e.g., real time video-conference of the lectures, online problem solving sessions and office hours) means. We viewed these classes not just as isolated courses, but as opportunities to try different techniques and technologies and to determine how to structure future classes.

In this chapter, we report on what we learned. In Section 2, Stephan Ramon Garcia from Pomona College reports on teaching Graduate Real Analysis through the LACOL network. In Section 3, Steven J. Miller from Williams College discusses experiences, findings, and recommendations on synchronous versus asynchronous instruction. In Section 4, Jingchen Hu from Vassar College presents her experiences, findings, and recommendations on how to create and foster an online learning community. Section 5 discusses future work.

## 2. Case Study: Graduate Real Analysis (Garcia)

During the Fall 2017 semester, I taught a graduate-level analysis course at Pomona College. As part of a LACOL-sponsored pilot program, the course doubled as a small-scale experiment to examine the feasibility of bringing high-level course offerings to other small liberal arts colleges. I

| | Pomona | Swarthmore |
|---|---|---|
| Classes Begin | Sept. 4 | Aug. 29 |
| Fall Break | Oct. 16-17 | Oct. 13-23 |
| Classes End | Dec. 6 | Dec. 12 |
| Finals | Dec. 11-15 | Dec. 15-23 |

**Table 1.** Fall 2017 schedules for Pomona and Swarthmore.

was motivated by the fact that pure mathematics students at liberal arts colleges often do not have the opportunity to take the graduate-level courses that would make them competitive candidates for elite graduate programs. In addition to a dozen or so local students from Pomona College and Harvey Mudd College, two students from Swarthmore participated in this project, taking the course remotely and asynchronously.

Perhaps the greatest lessons to be learned were on the technical side. Unlike big-budget, professional endeavors, the video recording, processing, and distribution were handled by the instructor. I could not, for example, hire a dedicated camera operator for several dozen lectures spread throughout the semester. Thus, some of my main goals in setting up the course were the following.

(1) Minimizing the reliance on expensive software.

(2) Avoiding the use of auxiliary personnel (e.g., a camera operator)

(3) Minimizing the change to the in-class experience for local students.

(4) Accomplishing this in a repeatable fashion cost-effectively.

I focus here mostly on the logistical and technological aspects rather than the pedagogical aspects of this experience. Indeed, the technical problems that were encountered and some of the solutions that were found may be of interest to a broader audience than the specific pedagogical challenges involved.

2.1. **Scheduling.** The Claremont Colleges, which includes both Pomona and Harvey Mudd Colleges, are located in Los Angeles county. Consequently, classes there are scheduled according to Pacific Standard Time. Having synchronous classes during normal business hours for all participants, while avoiding traditional lunch hours, appeared prohibitive.

Another significant barrier to synchronicity was the incompatibility between the Pomona and Swarthmore academic calendars; see Table 1. The schedules contain several points of disagreement that would have required significant accommodation on the part of the Swarthmore students.

In addition, we had no funding for a technical-support person to be constantly on site, so having real-time streaming video to and from the remote students would be difficult. Consequently, I decided upon an asynchronous approach (recorded video lectures) early in the planning process.

2.2. **Equipment.** I anticipated many technical challenges early on and began researching possible solutions during the Spring 2017 semester. Additional tests and tinkering took place throughout the summer of 2017, especially during the weeks leading up to the Fall 2017 semester. I worked

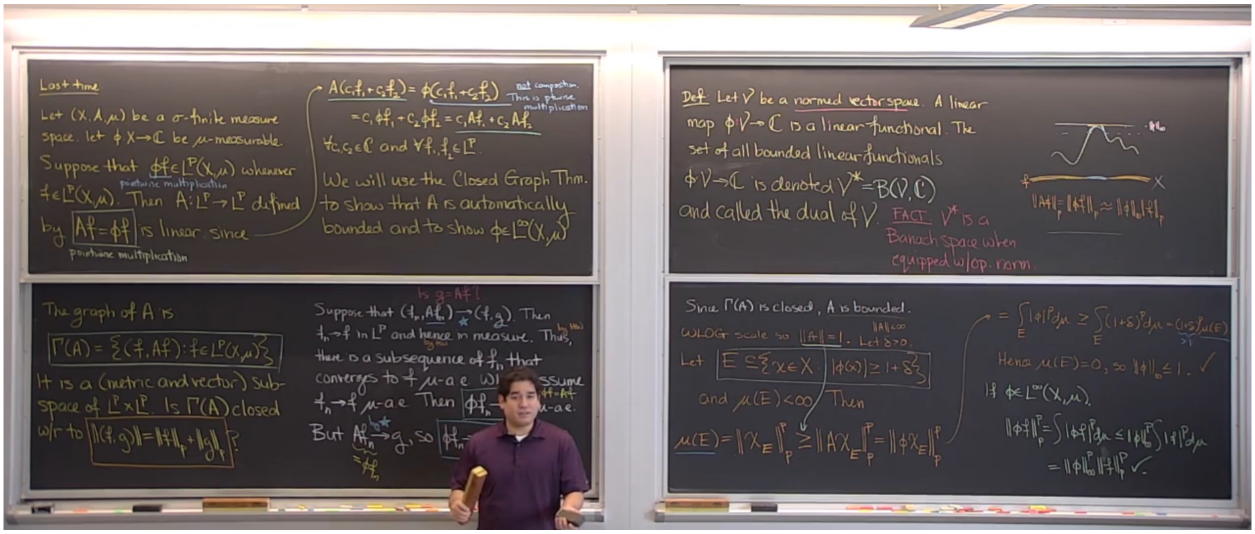


**Figure 1.** The classroom features two sets of raisable boards. This is a typical frame from the M4V video files produced for the graduate real analysis course.

closely with Joseph Brennan, Pomona's Director of Media and Classroom Services, throughout the entire process.

Unlike certain other disciplines, in which the instructor lectures with only occasional boardwork, mathematics often requires technical arguments to be spelled out in detail on the board. Although some professors prefer to use Beamer or PowerPoint slides, I felt that a traditional "chalk-talk" approach was best suited to graduate analysis. Moreover, I did not want to reinvent the wheel because one of my primary motivations was to ensure that the in-class experience for local students (most of whom had already taken at least one course with me) did not vary dramatically from what they were used to. I also did not wish the remote students to simply watch slides accompanied by a disembodied voice. Consequently, it was clear that we needed a way to record high-quality video.

The selected classroom seats approximately 35 students and features two sets of raisable boards. Four boards are typically visible at the same time; see Figure 1. Put together, the boards encompass approximately $8' \times 18'$ (144 square feet), a relatively large area to maintain sharp focus on. In order to utilize all available board space and to keep previous work on the board for the longest time possible, it was necessary to have the camera focused on all four boards simultaneously. This greatly narrowed down the possible cameras that could be employed.

We brought in several models that the Pomona Information Technology Services (ITS) department had in stock. For the initial tests, these were attached to tripods located at the rear of the classroom. However, all of them lacked sufficient resolution to discern fine details on the board. Although most of the writing could eventually be deciphered by the attentive viewer, I could not ask remote students to watch videos of such low quality.

One possible option was to write larger than normal. However, I suspected that consciously writing larger than normal for the entire semester would be difficult and that I would eventually revert to my usual writing style. There was also the risk that focusing on "font size" would be a constant


**Figure 2.** The camera is attached to the rear wall of the classroom and controlled by a joy-stick/keypad device located in a cabinet located inside a podium at the front of the classroom.

mental distraction, something that I could ill afford in such a high-level course. Consequently, the "font size" approach was quickly panned.

Fortunately, these experiments occurred during the Spring 2017 semester and there was plenty of time to find solutions. Since the in-stock cameras were insufficient for our purposes, we had to purchase a new camera. Through a combination of LACOL and Pomona College funds, a Sony SRG-300H camera was purchased and installed in the classroom; see Figure 2. The camera, which cost about two thousand dollars, can record $60$ frames per second at $1080$p. Since the camera would be fixed and the lectures would feature no dramatic movement, we decided to record at $30$ frames per second to reduce file sizes.

Audio recording proved easier to manage. ITS provided me with a Samson Stage PXD1 microphone system. The instructor wears a clip-on lapel microphone and a transmitter, about the size of a cell-phone, that can fit in a pocket. A small USB stick attaches to the recording computer and communicates with the transmitter. There was only one minor issue on the audio equipment front: a cellphone could potentially cause interference with the transmitter, resulting in poor-quality audio. I had to get accustomed to turning my phone off entirely before class.

2.3. **Recording.** The camera was controlled by a joystick / keypad device, located in small cabinet in a podium near the front of the classroom; see Figure 2. Because the classroom was used by several different classes and also served as a student study-room in the evenings, I had to check the alignment of the camera prior to each lecture. Fortunately, only once did I find that the settings had been modified in my absence.

With the camera and microphone up and running, it was necessary to identify the computer that would be used for the actual recording of the lectures. One obvious solution was to bring my laptop to class and have the camera record directly to it. However, this was undesirable for several reasons. First of all, the power usage necessitated by video recording would require the laptop to be plugged in. This would mean dragging to class an AC adapter, in addition to the microphone, transmitter, chalk, lecture notes, and graded homework assignments, not to mention fiddling with audio and video cables, and adapters every day. Surely at some point during the semester I would forget a key ingredient! Because of the additional hassle and added risk, I decided that using my own laptop was not a viable option.

It was clear that the built-in podium computer (a dual-boot PC/Mac OS machine), which was already connected to the audio-video system, was the best option. However, this introduced an additional wrinkle: how to transfer the large video files from the podium computer while packing up and vacating the class for the next instructor. That important issue is discussed in the following subsection.

Now that the recording computer was chosen, we (Joseph and I) needed to find the appropriate software. Although there were several alternatives, we settled on Quicktime Player. This had the advantage of being simple, easy-to-use, and essentially free, since it comes preinstalled on Mac OS. Moreover, I felt that there was no need to employ fancy video-editing tools or special effects. Thus, Quicktime was perfectly adequate.

The generic podium guest account was used by multiple people, meaning that files could be moved or deleted without my knowledge. There was also the risk of having audio and video settings overridden by other users. Consequently, a special "recording" account was created on the podium computer. This had the added benefit of eliminating the risk of accidentally leaving my personal account logged in during the rush to pack up.

2.4. **File transfer.** Now that we were able to record lectures at an adequately high resolution, another serious problem emerged. Each lecture video needed to be transferred from the in-class podium computer to either the hosting computer or, at least, my office computer for further tinkering. This process needed to be rapid since a computer science course was scheduled in the same room immediately after my class. After each lecture I had at most a couple minutes to accomplish this task.

For a typical 1:15-hour lecture, the resulting Quicktime video file was approximately 6 gigabytes (gb). This was far too large to post on the internet directly and large enough to cause difficulty transferring it from the podium computer in a timely manner.

One of Joe's initial suggestions was to use Box, a file-sharing service similar to Dropbox. Perhaps each lecture video could be posted directly from the podium computer to a Box account? Then the students would be able to view the video files through a suitable link and the high-quality video could be streamed immediately after class. Unfortunately, this proved untenable for two reasons.

First, the podium computer, despite being hard-wired to the local network via an ethernet cable, was unable to transfer the enormous video files to the Box account quickly enough for me to log out of the podium computer in time for the next class to start.

Second, video files stored on Box and viewed through a web browser are automatically streamed. Consequently, the high-resolution video that we worked so hard to capture was degraded and pixellated when viewed through Box. This would probably be fine for most recreational videos, but for a mathematics lecture in which the fine details on the board needed to be viewed, this proved unacceptable. Consequently, Box had to be abandoned as a viable option.


**Figure 3.** The only additional objects that I needed to take to class were contained in a small microphone bag: the lapel microphone, transmitter, receiver, and USB 3.0-capable memory stick.

Joe's second suggestion proved the simplest and most effective. He purchased a SanDisk Extreme Pro USB 3.0 128gb memory stick. These were significantly more expensive than standard memory sticks and perhaps they still are. If memory serves me, they ran about $75 at the time. The USB 3.0 interface permitted a 6gb video file to be copied from the podium computer in less than a minute. As part of my "tear-down routine", I would plug the memory stick into the podium computer and copy over the day's lecture video. This permitted me to exit the class room in time for the following computer science class. To ensure that I did not lose or misplace this expensive memory stick, I kept it on a "leash" and in the same bag as the microphone and transmitter; see Figure 3.

2.5. **Video delivery.** Once the raw video files were transferred to the memory stick, they could be copied onto my office machine. This did not resolve the large file-size issue, of course. It merely changed the location of the problem and got things out of the way in time for the next class to use the classroom.

It was clear that the high-resolution video files needed to be compressed without significant loss of fidelity. Moreover, I wanted to accomplish this without purchasing new software and fiddling with advanced tools on a daily basis.

After several experiments, I settled upon VLC. This freely available, multi-format video player is standard fare for Apple users. Although many users might be unaware of this feature, VLC is able to convert between various popular video formats. A little experimentation revealed that the raw Quicktime files, each about 6gb in size, could be compressed without much loss of clarity to

| Class Schedule<br>(Subject to change) | | | |
|---|---|---|---|
| **Date** | **Reading** | **Topics** | **Homework** |
| Tu 8/29 | Course Description (PDF)<br>Notes on Cardinality (PDF)<br>Bass, Sections 1.1, 1.2 | **Foundations**<br>Vitali sets, Banach-Tarski Paradox<br>Lecture #1 (PDF) Lecture #1 (m4v) | Homework Template (TeX)<br>**Due Tr 8/31:**<br>Survey #1 (PDF)<br>**Due Tu 9/5:**<br>Homework #1 (PDF)<br>Homework #1 (TeX)<br>Solutions #1 (PDF) |
| Tr 8/31 | Logicomix<br>(for those interested) | **Foundations**<br>Russell's Paradox, Axiom of Choice, partial orders, ZFC, Godel's Second Incompleteness Theorem<br>Lecture #2 (PDF) Lecture #2 (m4v) | |
| Tu 9/5 | 2.1, 2.2 | **Foundations**<br>Sigma-algebras, ordinal numbers, transfinite induction<br>Lecture #3 (PDF) Lecture #3 (m4v) | **Due Tu 9/12:**<br>Homework #2 (PDF)<br>Homework #2 (TeX)<br>Solutions #2 (PDF) |
| Tr 9/7 | | **Foundations**<br>Borel hierarchy, Goodstein's Theorem, Godel's First Incompleteness Theorem<br>Lecture #4 (PDF) Lecture #4 (m4v) | |
| Tu 9/12 | 3.1 | **Measure Theory**<br>Measures, null sets, completions<br>Lecture #5 (PDF) Lecture #5 (m4v) | **Due Tu 9/19:**<br>Homework #3 (PDF)<br>Homework #3 (TeX)<br>Solutions #3 (PDF) |
| Tr 9/14 | 4.1 | **Measure Theory**<br>Outer measures, constructing measures, the Cantor set<br>Lecture #6 (PDF) Lecture #6 (m4v) | |
| Tu 9/29 | | **Measure Theory**<br>Cantor set trivia, Borel sets vs. Lebesgue sets<br>Lecture #7 (PDF) Lecture #7 (m4v) | **Due Tu 9/26:**<br>Homework #4 (PDF)<br>Homework #4 (TeX)<br>Solutions #4 (PDF) |
| Tr 9/21 | 4.2, 4.3 | **Measure Theory**<br>Monotone functions, Lebesgue-Stieltjes measures<br>Lecture #8 (PDF) Lecture #8 (m4v) | |
| Tu 9/26 | 4.2, 4.3 | **Measure Theory**<br>Lebesgue-Stieltjes measures, regularity properties<br>Lecture #9 (PDF) Lecture #9 (m4v) | **Due Tu 10/3:**<br>Homework #5 (PDF)<br>Homework #5 (TeX)<br>Solutions #5 (PDF) |

**Figure 4.** The course website contained videos of the lectures in M4V format.

M4V files around 350mb in size. The reduction in file size was about 94% and rendered the videos small enough to be posted to a course website.

Each 1:15-long lecture took perhaps forty-five minutes to compress on my office desktop (an iMac that is several years old). Class ended at 10:50am and the compressed M4V version of the lecture would be ready just before lunch. The M4V file, along with the course lecture notes, would be posted on the course website about two hours after the actual lecture. The M4V file could then be downloaded from the course website or watched in the browser; see Figure 4.

All of this was done with local computers and with software that was either free (VLC) or included with the Mac operating system (Quicktime). Just as importantly, this required minimal deviation from my usual teaching routine and it did not significantly affect the in-class experience for the Claremont Colleges students.

2.6. **Homework.** The Claremont Colleges use the Sakai course management system. Similar course management software is available at most American institutions. Each course has a Sakai site dedicated to it and instructors can post announcements, assignments, and files there. I personally find the system cumbersome and limiting, so I chose to post items on my personal course website. However, I used the Sakai system to keep track of grades and assignments.

The remote students, who were fluent in LaTeX, submitted assignments in PDF form via the Sakai "dropbox" feature (this has nothing to do with the company of the same name). In principle, a remote student could scan an hand-written assignment and then upload it in the same manner. The course graders (there were two of them and they alternated assignments) were given teaching assistant access to the course Sakai site. They downloaded the homework assignments and marked up the PDFs using Preview, Acrobat Reader, or other similar software capable of annotating a PDF. The graded assignments were uploaded by the graders to the Sakai site and placed in the remote student's personal directory. In this manner, the remote students could turn in their assignments and receive the graded assignments back entirely online.

The process was smooth and we encountered no technical problems. In particular, it seemed to be a reasonable way to handle a small number of remote students. Although there do not appear to be any significant technical issues that prevent scaling things up to a dozen or more remote students, there are the matters of money (who pays?) and personnel (who grades?) that would have to be resolved.

2.7. **Examinations.** The local students took examinations in class. Remote students took identical exams that were proctored by a Swarthmore faculty member. This arrangement needed to be set up before the course began, since it required a low-intensity commitment from a remote professor.

The completed exams were scanned by the proctor and sent to me via e-mail. This procedure, however, seems less amenable to scaling since it requires each participating institution to have an on-campus proctor. The possibility that an exam might be scheduled on an inconvenient day for one campus or proctor could be uncomfortably high if the number of participating institutions were to increase. Perhaps for advanced classes, "take-home" examinations might be more suitable?

2.8. **Summary.** Our small experiment shows that advanced mathematics courses, even those at the graduate level, can be taught asynchronously online. Perhaps the biggest lesson to be drawn from our experience is: plan early! There are many technical challenges that may arise and it is hard to anticipate all of them. Do not assume that your current in-class technology will be sufficient. Pre-existing cameras in classrooms might not be able to capture video at the resolution required for remote students. Rectifying this may take time and money, although it is a one-time expense.

## 3. Synchronous versus asynchronous instruction (Miller)

An important issue in the creation and execution of shared classes is how the material is delivered: synchronous or not. There are advantages to each. If the lectures are streamed live, the off-campus participants are able to fully participate in real-time. They are able to comment in class, ask questions which guide the lecture, and interact with their fellow classmates. Unfortunately, doing so requires more on the technology side, and also prevents students from taking a class that meets at that block. As many colleges are on different calendars and time structures for the day, these are non-trivial scheduling issues. If the class is entirely asynchronous, many of these issues vanish, though unfortunately now the students are not able to participate in classroom discussions. In this section we report on some of the successes and challenges of these two approaches.

3.1. **Introduction.** For about five years now I have recorded my classes for multiple reasons. The initial motivation was a desire to flip the classroom, either fully or partly; specifically, my Multivariable Calculus class at Williams (where most of the students are freshman, making the transition from high school to college). In college such classes typically meet three times a week for 50 minutes for somewhere between 12 and 15 weeks (though some schools do block off a fourth meeting); in high school classes meet daily for more weeks. Thus there are significantly more contact hours in high school, leading to significantly more time to go through the same material. This gives these instructors a lot of freedom not found in college, such as going at a slower pace and spending more time on worked out examples. I wanted to be able to assign students videos to watch in future years to move some of the more standard material to home viewing, and free up more class time for value added lecturing.

Over time, however, my use of recorded lectures changed. Many students found the recorded lectures useful, as they would occasionally have to miss class (either due to illness, family situations, or travel for athletic competitions or job interviews). Further, if there was a topic that was confusing they could watch a subset of the lecture again. They worry less about taking notes during class. Also, this allowed me to assemble a large list of introductory lectures on a variety of topics which could be used to supplement other courses. For example, if a student in complex analysis has not seen Green's Theorem, I can refer them to my lecture where we cover it.

Additionally, Williams is a small liberal arts college; we often only have one section of upper level classes, and many of these are only offered every other year (if even that frequently). I would often have students wish to take both my course and a class in another department meeting at the same time; by recording the lectures the student could do that. They could still come to office hours and TA sessions, they could still work with their fellow students. The only difference is that they could not participate in class, which means that I do not have the benefit of their perspective in discussion, and they cannot immediately ask questions on material that is confusing or that they wish to see in greater detail.

Over the past few years I have been talking about the rewards and challenges of teaching at a small liberal arts college with many of my colleagues, who have had similar experience, which has led to new thoughts on how to use lecture capturing. One of the greatest problems we all face is class enrollment. Frequently we do not have the manpower to teach all the electives we want, or if we do there is often not sufficient interest on our campus to justify the class.[1] Many of us have explored solving these issues by sharing classes across campuses. For example, I might teach Complex Analysis at Williams while one colleague teaches Algebraic Topology at their campus and a third teaches Functional Analysis at their institution. We open our classes up to all students from our institutions.

There are numerous problems with making such class sharing work well. Some are administrative issues. What credit do professors get for teaching students at another institution? How much extra work is it to have remote students, and how should the professor be compensated for the additional time demands? What credit do students get for taking a class at another institution? How do the classes show up on their transcript? While these and related problems are important and often challenging to solve (frequently one needs a faculty vote to allow credit for a class at another institution, but a work-around often exists by having a local faculty member at the student's college enroll them in a guided independent study), that is not the point of this contribution. My goal is to talk about two main formats for classes with remote students: asynchronous and synchronous. I'll briefly touch upon the technology and administrative issues, but concentrate on some class structures I've tried, highlighting what works and what the challenges are.

3.2. **Asynchronous content.** As a professor, an asynchronous class is enormously easier to run than a synchronous one. Many of the administrative issues and challenges disappear or are easily resolved. In particular, if the lectures are recorded and posted online, students can watch at their own pace and at whatever schedule works for them. As colleges rarely have aligned academic calendars (and further are often in different time zones, or some have classes starting on the hour and others on the half-hour), this is tremendously valuable.

There are some slight annoyances, as a school that starts later than the host institution could have students submitting say the first homework assignment while the local students are on the second or third, but these are minor issues for grading.[2] Slightly more severe is that it is harder for students to collaborate across institutions if they are at different places in the course, but this can be mitigated by having some schools do more work in certain weeks to force alignment.

This last point, on collaboration across schools, is worth dwelling on as it goes to the heart of the goal of such course sharing. Is the primary objective to provide content that would not be otherwise available, or to build bonds between students and faculty across multiple institutions? In the asynchronous setting it is easier to work towards the former. While I try to encourage my remote students in such settings to have at least one friend take the course with them, so they have a local study group, this is not always the case. In comments from them over the years the most common remarks are that they are glad to have had an opportunity to see material that otherwise would not be available and overall they are glad they did the course, but that the interactions are

[1]This is for the more advanced courses; for the introductory classes the problem is the opposite, where courses such as Introduction to Statistics often turn away as many people as they accept.

[2]Of course, there is the potential for the other problem: the remote schools have a semester starting first! If the lectures have been recorded from a previous iteration this is not an enormous challenge; if not, one easy solution is to have those students wait till the local school starts.


**Figure 5.** Swivl system for tracking lectures, image from their homepage (https://www.swivl.com/how-to-use/). Note the instructor can see what is being recorded in the iPad screen, and the system follows the tracker in real-time.

not equal to what they have in live classes. The differences range from the challenges of watching lectures on a small screen instead of live in the classroom to feeling more isolated.

On the technical side, it is very easy now to record and post lectures. As Garcia described many of the technologies in depth, I'll just briefly remark here on two options of varying difficulty.

The easiest is to record lectures using an iPad and post them online through YouTube. One can buy a tracking system online for a few hundred dollars, put the iPad on it on a tripod, and just lecture (see Figure 5). There is a tracking device one can wear which will make sure the system follows you, which has the advantage that comments from students won't be detected and recorded. This is important as some students feel uncomfortable asking questions if they know that others will be able to hear them later; for the lecturer, you just have to remember to repeat or rephrase the question. If students are comfortable having their questions recorded and thus being identifiable, you can purchase more advanced systems which have multiple audio input devices, and place those throughout the room.

I prefer to post my lectures online on YouTube rather than through a college coursepage for many reasons. First, it is simple to go to https://www.youtube.com/upload and upload a file. You can place tags to make it easily searchable, you can create playlists for classes, but most importantly anyone on the web can see the material. One of the great advantages of this is that you do not need to give remote students accounts at your school. Further, if you want to use material from one class in another you do not have to give current students access to a former class, or move content. The material has a permanent home and you can use wherever and whenever you

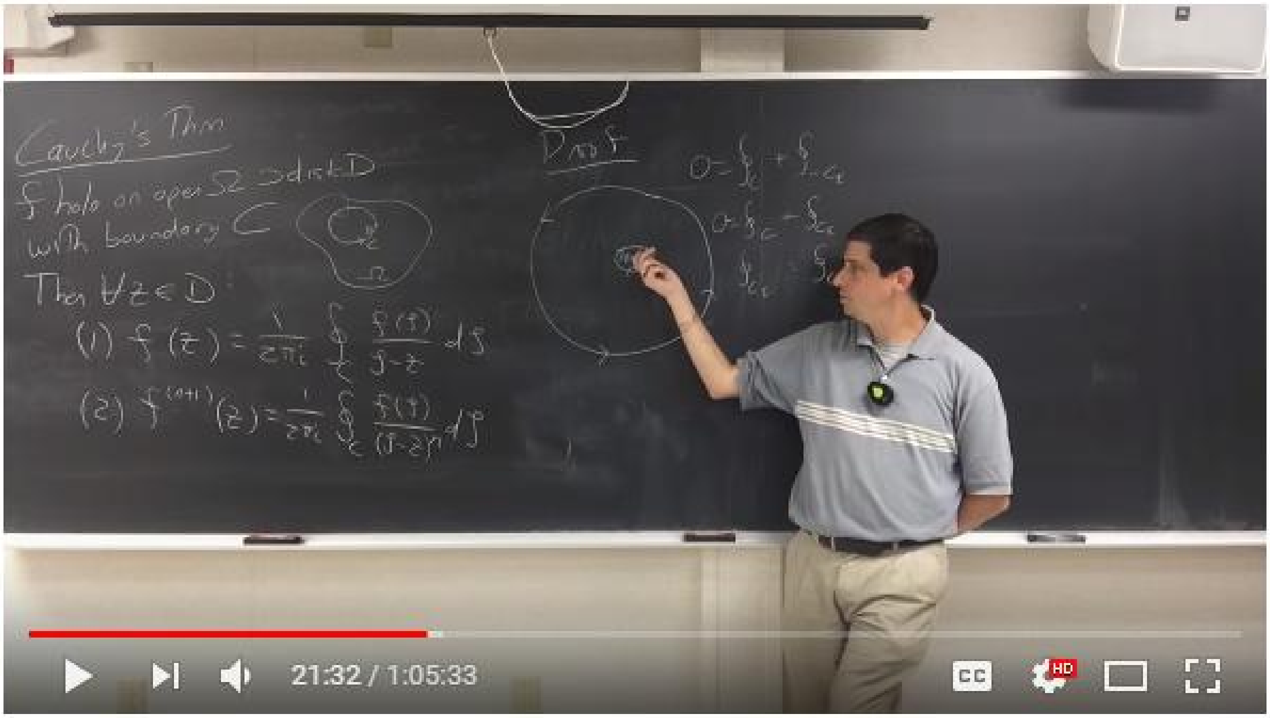


**Figure 6.** Snapshot of a lecture as it would appear on YouTube, recorded using an iPad and the swivl system. In this model the lecturer wears the tracking device on their shirt, which the base follows. Lecture available online at https://www.youtube.com/watch?v=NgHIiZUYI6g&feature=youtu.be.

wish. For me, the only real concern is that anything you say is now public domain. In standard and undergraduate elective math courses this is not a big deal[3], but in a humanities discussion course where very personal discussions are happening this could be a serious concern.

The biggest drawback to this technology is that you cannot zoom in to the material, and thus the writing can be a little small and not as crisp and clear as one would like (see Figure 6); thus it is better for students to watch on a laptop or desktop over a phone or iPad.

The other option I have used is to hire a student to record the lectures (see Figure 7). In addition to excellent tracking, more importantly is the ability of the student to zoom in on the mathematics being discussed. This makes the lectures easier for students to follow, but requires significantly more local resources. Using an iPad and a swivl is relatively easy and can be done without too much work for the professor; hiring a student requires funds and often processing time for the video. That said, if funds are available it is worth having the better recording, as it makes the video significantly more useful for remote students and future classes.

In summary, the above are just two of the many options available for running asynchronous classes. The technology is now cheap enough and reliable enough to make this a real option. The minor administrative issues can be overcome without too much trouble; the greatest difficulty is

[3] That said, there are certain stories that I cannot share with the camera rolling, ranging from certain personal anecdotes to some issues in applying math in practice; these have to be told outside class hours.

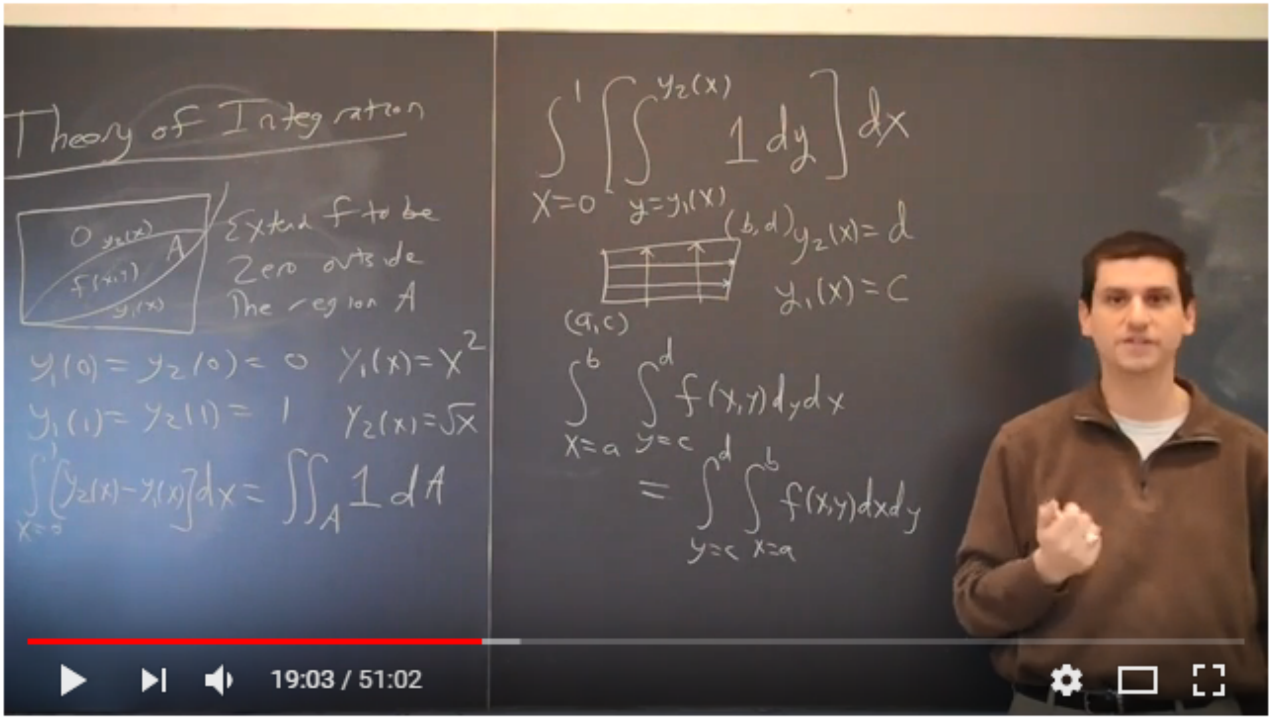


**Figure 7.** Snapshot of a lecture as it would appear on YouTube, taken by a student using a high quality video recorder. Lecture online at https://www.youtube.com/watch?v=G9d9lcYevnM&feature=youtu.be.

the lack of alignment among academic calendars (though this is also a challenge for synchronous content). Liberal arts colleges pride themselves on close contact between students and professors; this is significantly lessened in this set-up, but with work (such as skyping outside of class, or even better visiting the remote campuses) personal connections can still be forged.

3.3. **Synchronous content.** Slightly different technology is required for a synchronous class. To date, I have not done a fully synchronous class. This is due in part to the different academic calendars, but also to students at remote schools (or even the local school!) desiring to take not only my class but another class at the same meeting time. What I have done is taught classes that are mostly asynchronous, but with four synchronous meetings which are scheduled outside the normal time block. Doing so allows real interaction between the remote and local students.

The greatest challenge now is to have the information available to all in real-time. I've used a Microsoft Surface as the input and shared screen technology. At the local school I have my computer connected to a data projector, and thus have the screen viewable by the entire class. For the remote students, there normally are not that many at each institution, and they can just congregate around one connected machine. There are programs now that allow remote users to draw directly on my screen, and thus we can all see the same material at the same time. I've used software available through our class system, Glow, but there are many applications now. The bandwidth requirements are low as most of the time the content has very little change. In

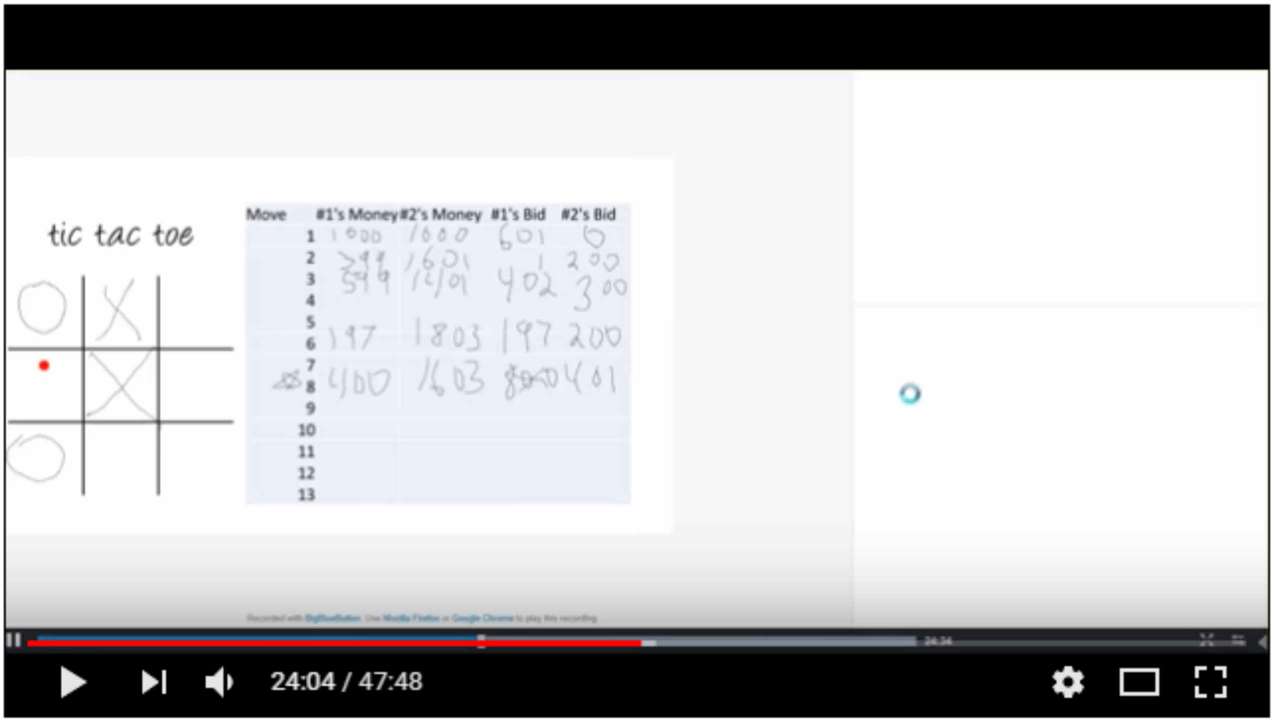


**Figure 8.** Snapshot of a shared lecture. Video online at https://www.youtube.com/watch?v=DRBmLlrztvI&feature=youtu.be.

such situations I'm not lecturing on the board, but instead I've posed a problem that students are pondering and then presenting.

I've done this in a problem solving class. Figure 8 shows what a shared screen looks like; one of the students is about to move where the red dot lies. The problem solving class was ideal for synchronous material. I was able to give a short general lecture first, and then the students remotely played a few games[4] and tried to develop a theory.

These classes were enormously fun, and very different from the standard lecture classes. Students from each institution chatted with one another before class began. To make the atmosphere more festive and special I arranged for food (typically Dunkin Donuts munchkins) at each place; this is also somewhat needed as we met at 8am, well outside normal class hours for today's college students!

Thus, the limiting factor in synchronous courses is not technology (and as the technology is improving significantly each year, any issues on that end will be short lived). The greatest challenges are the different schedules, ranging from unaligned academic calendars to being in different time zones.

[4]The game depicted is bidding tic-tac-toe. Each side starts with $1000, and each turn consists of making a secret bid, whomever bids highest gets the move and gives their bid to the other side; if there is a tie no one moves and you bid again. The game continues until someone wins or all squares are taken.

Finally, one concern I am always aware of is making sure that having a class remotely accessible does not negatively impact the experience for the local students. I will lecture slightly differently with screen capture than I would with chalk on a blackboard; however, that is not necessarily worse, just different. Different media have their advantages and disadvantages, and one must adjust one's lecture style to the method used for presentation.

3.4. **Summary.** The technology exists for a variety of shared class experiences, and what we can do will only expand in the coming years. The need is great, especially at small institutions, where we cannot offer all the classes we want with the frequency our students need. Rather than viewing this as a disadvantage, we can see it as an opportunity to forge connections between students and faculty across institutions. In the course of sharing classes, we also assemble a large database of (hopefully!) excellent lectures, which are a valuable resource.

Right now the challenges surrounding asynchronous classes are easier resolved than those for synchronous ones. While both have administrative hurdles, the asynchronous format avoids the headaches arising from different calendars, as well as allowing students to take two classes that meet at the same time. The value of this latter point should not be forgotten; we often have so few classes that are appropriate for upper level students that we should do all we can to preserve options.

Finally, at liberal arts colleges one of our selling points is the personal relationship between students and faculty. In the synchronous classes I've taught I have made it a point to go to each campus for at least one lecture. There are several advantages to this. First, by visiting for a day or two I get to know the remote students much better than possibly through email and skype. Second, and more importantly, it gives the local students a sense of what the remote students are experiencing!

## 4. Creating and fostering an online learning community (Hu)

In this section we report on student experiences in our model of sharing a course across multiple campuses. This approach is unique, and distinct from most of the existing hybrid/online models. Our class meets in a real classroom at one campus, provides synchronous and asynchronous access to remote students from other campuses. To local students, the class is not so different from a traditional face-to-face class, as they can ask the instructor questions during office hours and/or in person, and interact with other local fellow students as they like. However, the experience can be vastly different for remote students, and it is easy for them to feel isolated and left out. We discuss challenges, and experiments and approaches to those challenges. The goal is not only to make remote students feel part of the class and provide them an enjoyable learning environment, but also to create an online learning community involving both remote and local students. The sharing and exchange should not only happen in the instruction and material delivery, but also students' involvement and engagement and contribution to a shared learning community.

4.1. **Introduction.** Vassar College's MATH 347 Bayesian Statistics was offered through the LA-COL network in Fall 2017 for the first time. Class met in person on Vassar campus, where local students were present in the classroom during lecture. The lecture delivery was through the conferencing software Zoom[5]. The course management system was Moodle[6] at Vassar College. Registered Vassar students had a regular Moodle account, whereas remote students had sponsored Moodle accounts created for them by Vassar's Academic Computing Service. The sponsored accounts gave remote students all Moodle functions.

Typically, I as the instructor started the synchronized lecture via Zoom meeting with the classroom desktop computer, and remote students could join class with the Zoom meeting ID. I used an iPad Pro to join the Zoom meeting as well, where lecture slides and other course material were shared and then projected onto the projector in the classroom. The entire lecture (75-min) was recorded, and then posted on YouTube list of class recordings for anyone to access freely. Therefore, both synchronized and a-synchronized access to the lecture were available to registered students.

The course material and interacting tools were all hosted on the Moodle site. For example, lecture slides and R[7] programming scripts were posted and downloadable from Moodle. Also, the Discussion Board function of Moodle allows the instructor to create a topic within a module, and everyone has the right to post a thread to the topic.

The course started with four remote students. Among them, two were able to join class in real time (synchronized access) and the other two were only able to watch lecture recordings due to schedule conflicts (a-synchronized access). Eventually, only one remote student stayed in the class, who only utilized the a-synchronized access. Nevertheless, the course provided both access throughout the semester.

The challenges of keeping every student involved became obvious early in the semester. With a roughly 4-to-1 ratio of local students versus remote students, there seemed to be no drive for local students to get to know the remote students. To make matter worse, the four remote students came from three different campuses, and their various schedules and availabilities made it difficult to interact even just among them. As the number of remote students gradually dropped to 1, such communication and engagement challenges only grew greater.

Through conversations with experienced colleagues, readings about online teaching and learning, and a series of trial and error experiments, I have identified a few useful tools to create and foster an online learning community. In each of the upcoming subsections, I would like to introduce the tool, illustrate with screenshots when possible, and discuss the advantages, my experiences, and my reflection.

4.2. **Self-introduction posts.** The self-introduction posts idea was introduced to me by a blended learning expert in the LACOL network. To me, it was an after-thought, as it occurred to me soon after the course started that I needed to do something to make everyone get to know everyone else, virtually in some cases. The request of a self-introduction post was given to all students in the third week, instead of before the course started or in the first week, which obviously would be more ideal. Still, better late than never.

[5]For more information about Zoom, visit `https://zoom.us/`

[6]For more information about Moodle, visit `https://moodle.com/`

[7]R is a statistical programming language. For more information about R, visit `https://cran.r-project.org/`

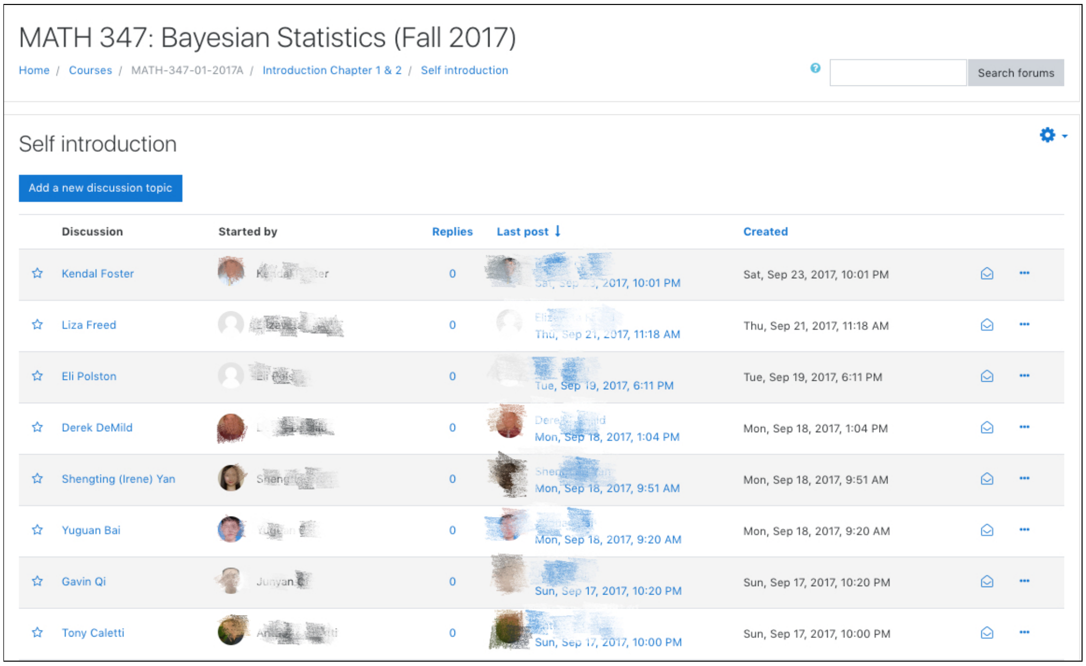


**Figure 9.** Students' self-introduction posts (first page).

I started the first post to introduce myself, and then invited every student to make a post on the following aspects:

(1) Your name, year, school.

(2) Your prior statistics exposure (note that we have 22 students from 4 different schools, so when you talk about your statistics course experience, be more specific than just write down the course number).

(3) Your prior R exposure (be specific).

(4) Your potential project interest, if you already start thinking about if. Even if you haven't, anything that would interest you is good to be put down.

After all posts were made, I went in to read each post carefully, and summarized a list of project interest topics and shared on Moodle. This process helped me to get to know the students better.

Overall, I think requesting a self-introduction post is a good practice. It not only allows students to introduce themselves, but also encourages them to reflect their previous experience, ask themselves what they want to get out from this course, and think ahead about their course project ideas.

There are few things to keep in mind to maximize the effectiveness of these self-introduction posts. First, do it early, ideally before the semester starts. Second, try to make it more interactive. As you can see the "Replies" column in Figure 9, none of the self-introduction posts were replied to at all. This does not mean that no one read it (well, who knows), but it does mean little online interaction activities among the students. If some kind of "reply to one" mechanism can be introduced, that might help improve the actual interaction among students. Third, try to make it more personal. In the current format, the self-introduction posts were made in words. I later received the advice of making a self-introduction video instead of text-based posts, so students can see the face and hear the voice, which sounds very appealing and potentially useful. In fact, I incorporated the video idea into the project introduction posts, which is the second tool that I turn to now.

4.3. **Project introduction video posts.** Like many statistics courses, the Bayesian Statistics course had a project component as a capstone experience. The projects can be done individually, or as a pair of two students. At the end of the semester, a poster session was held for students to present their projects.

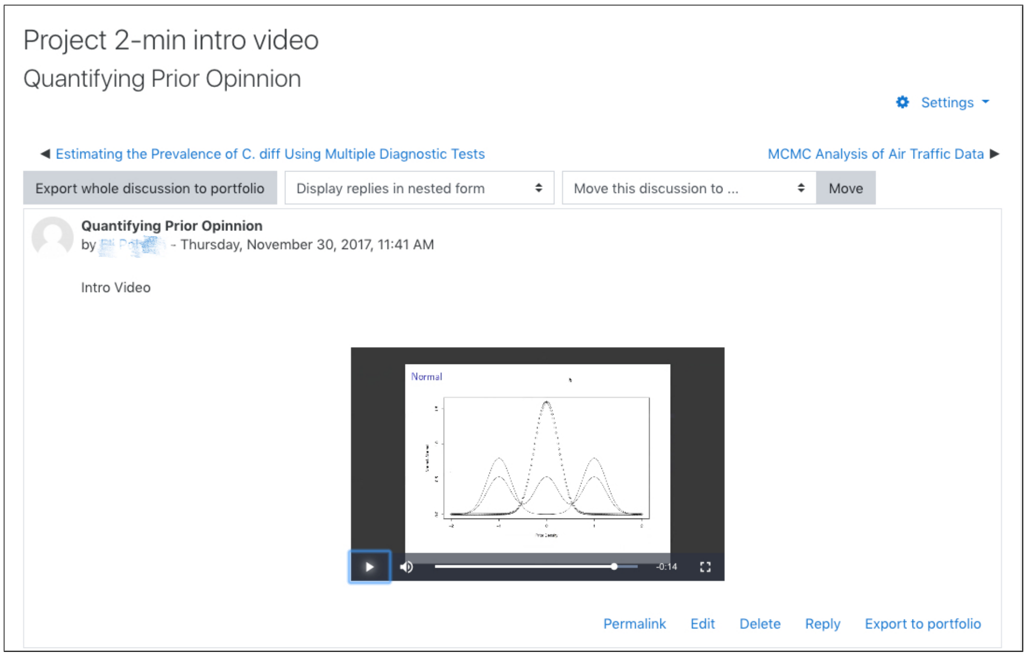


**Figure 10.** Sample project 2-min intro video on “Quantifying Prior Opinion”.

The poster session worked in a similar way as any other poster session at academic conferences. All projects were put into two groups. The first half of the poster session was for the first group to present, so the students from the second group were able to walk around, read poster and interact with the presenters. Then the groups switched for the second half. Between the switch and after the second group’s presentations, there were allotted time for discussion, summary and Q & A.

Almost all students were new to the poster session format. To help them prepare their posters, and think about how to present the material from the posters in person (smaller group or even one-on-one, versus a 10 or 15-min presentation at the front of the room to the whole class), I asked each project presenter to prepare a 2-min project introduction video and post it on Moodle. All students were required to watch all videos before the poster session took place.

These 2-min project introduction video posts served like a pitch talk of students’ projects. Many students used their poster slides to go over the project within the time frame. These videos not only helped the presenters to organize their material, thoughts and ideas, but also gave the other students the chance to get to know the gist of the presenting poster, and help them to navigate the poster session more efficiently and effectively. Figure 10 is a screenshot of a sample project introduction video, "Quantifying Prior Opinion".

In particular, this project introduction posts tool was useful to engage the remote student with the local students. The only remote student in the course could not attend the poster session in person, but other students had the chance to learn about the remote student's project, particularly through the posted 2-min video. In fact, the remote student created a longer video (around 5 minutes, acted as if it was the actual poster in person) and posted it on Moodle too. Several local students became very interested in the topic and they had the chance to learn more about it through these two videos.

Overall, the project introduction video posts tool is a very useful and effective practice. It pairs up well with the poster session format in particular. To improve it, in addition to requiring all students watch all videos before the poster session, requirements such as "comment on at least two videos" could increase out-of-classroom interaction to an even higher level.

Fall 2017 MATH 347 — Jingchen (Monika) Hu

Reading Guide for Explaining the Gibbs Sampler

Casella and George (1992)

1. [Section 2] How does Gelfand and Smith (1990) suggest to obtain an approximate sample from $f(x)$? How is it different from or similar to the approach we talked about in class? What are the advantages and disadvantages of each approach?

2. [Section 2] The authors claim "Gibbs sampling can be used to estimate the density itself by averaging the final conditional densities from each Gibbs sequence." What is the theory behind this claim? How does Figure 3 support this claim?

3. [Section 3] Write down marginal distribution of $y$, and verify the conditional probabilities $A_{y|x}$ and $A_{x|y}$. Also, verify $A_{x|x} = A_{y|x}A_{x|y}$ and $f_x A_{x|x} = f_x A_{y|x} A_{x|y} = f_x$.

4. [Section 4] What is a fixed point integral equation in the bivariate case? How does it help illustrate how sampling from conditionals produces a marginal distribution? Hint: check equations (3.5), (4.1), and (4.2).

5. [Section 4] The authors claimed "a defining characteristic of the Gibbs sampler is that it always uses the full set of univariate conditionals to define the iteration." Explain this claim by illustrating how a Gibbs sampler works with $k$ parameters $(\theta_1, \theta_2, \cdots, \theta_k)$.

6. [Section 5] Summarize different approaches to sampling the Gibbs sequence.

**Figure 11.** A 6-question reading guide for the paper Explaining the Gibbs Sampler.

4.4. **A learning environment for reading a research paper.** Reading and discussing accessible research papers are important components of an advanced statistics course. When covering the Gibbs sampler[8] module in Bayesian Statistics, I chose to have my students read *Explaining the Gibbs Sampler*, a short paper by George Casella and Edward I. George, which appeared in *The American Statistician* in 1992. The paper set up the Gibbs sampler background nicely, and discussed its features and practices through a series of designed simulations. It is a relatively accessible research paper, even to undergraduate students. Some practices are different from current

[8]For more information, visit https://en.wikipedia.org/wiki/Gibbs_sampling

ones now, which make the paper a good reference to learn and discuss the characteristics of the Gibbs Sampler, and Markov chain Monte Carlo (MCMC)[9] in general.

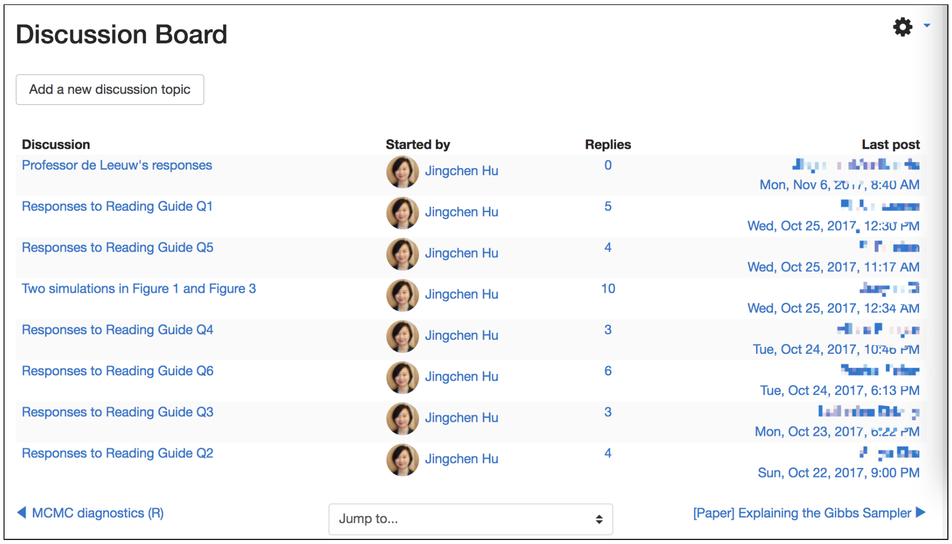


**Figure 12.** Discussion board for the paper Explaining the Gibbs Sampler.

In addition to the original paper, I provided a 6-question reading guide to the students; see Figure 11 for a screenshot of the list of questions. Some questions were related to verification of presented results, while others were more associated with practices of the Gibbs sampler, leading to discussion of similarities and differences between practices after the introduction of the Gibbs sampler (when the paper was published) and practices about 25 years later (what people typically do now).

To engage students in reading and discussion both in-class and outside of class, a request of one pre-class post and another post-class post was given to all students. Separate discussion topics for each question on the reading guide were created on Moodle's Discussion Board (see Figure 12). There were two more topics added after the class discussion, one was supplementary topic raised up during class, and the other from colleague's comments. As we can see, the effect of the two posts requirement can be seen in the counts in the "Replies" column. In those topics with a good number of replies, actual online discussion (e.g. later comments made reference to previous comments) can be identified (see Figure 13 as an example).

I went through the pre-class posts before the class discussion to see what students had been thinking and how their understandings were. I was able to refer to some comments during in-class discussion as well, and students' reaction and interaction in class showed their engagement with the pre-class posts, and overall this practice facilitated in-class discussion well.

For the post-class posts, though I was not able to discuss them in detail in later lectures due to time constraints, I was able to respond to their posts directly on Moodle, and also make a summary of the reading guide questions in general.

[9]For more information, visit https://en.wikipedia.org/wiki/Markov_chain_Monte_Carlo

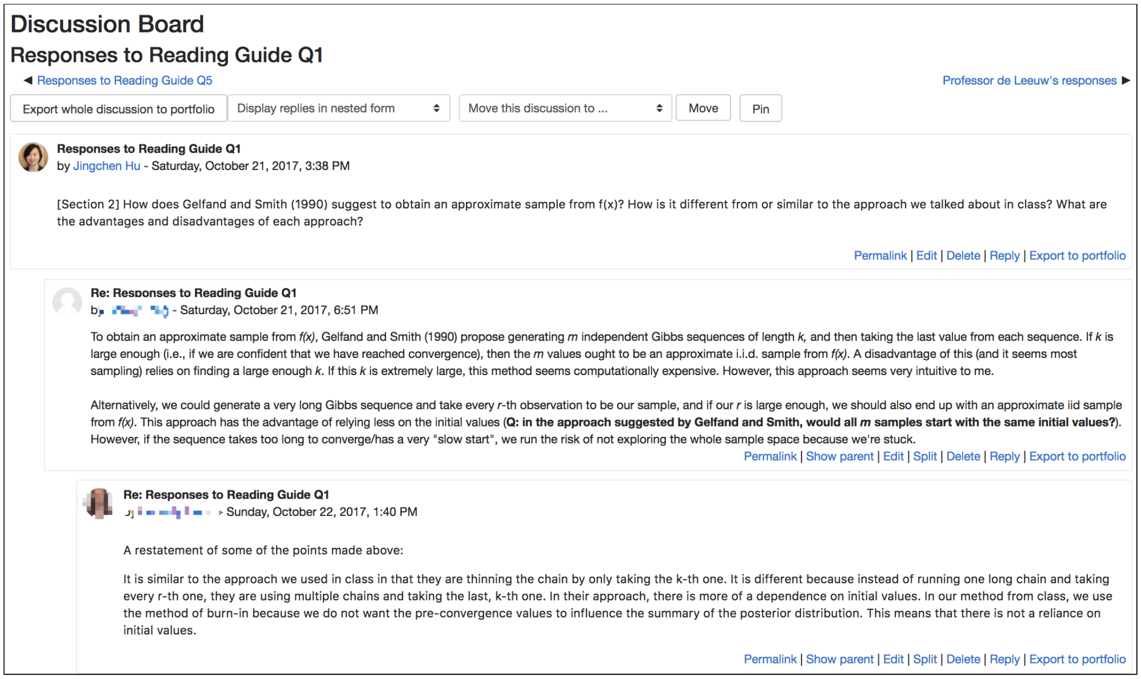


**Figure 13.** Discussion board (partial responses to Reading Guide Q1) for the paper Explaining the Gibbs Sampler.

The online engagement of reading and discussing a research paper complements the in-class discussion, and both practices together greatly enhance students' learning and engagement. I strongly recommend creating a reading guide to help students read and understand the paper. It helps the orientation and flow of discussion too. The pre-class and post-class posts mechanism worked great for the Bayesian Statistics course, and I can imagine variations of this mechanism, depending on the difficulty level and suitability of the paper to the course material, to be effective.

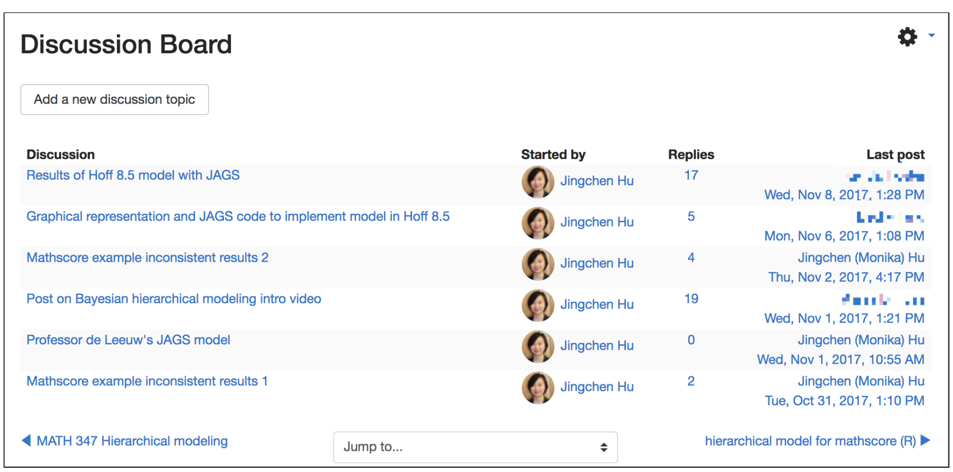


**Figure 14.** Discussion board for a guest introductory video on the Bayesian hierarchical modeling topic.

4.5. **A learning environment for guest introductory videos.** In traditional face-to-face courses, inviting colleagues or professionals in the community for a guest lecture is a great way to connect course material to real applications. For a hybrid/online course, such guest lecture can be created into a video and moved online. Additionally, a learning environment for the online guest lecture should be created to foster students' learning and engagement.

Bayesian hierarchical modeling[10] is a Bayesian approach that is the basis of a large amount of applied scientific work, which makes the teaching and learning of this topic potentially interactive and application-oriented. I have been lucky to have a colleague in the Cognitive Science Department at Vassar College who uses Bayesian hierarchical modeling extensively for his research. In particular, he focuses on the development and application of Internet-based tools for the study of human cognition. He had kindly agreed to create a 20-min introductory video to Bayesian hierarchical modeling based on his applied work for my students, so off we started our teaching collaboration.

Creating and tailoring a video for another class proved to be a very challenging task. For a couple of months, my colleague and I met for an hour every other week to discuss the topics, the material, the format of the video, et cetara. It took me a while to grasp his experimental settings as well as the learning theories in cognitive science. Then it was the period of figuring out a common language in two separate fields (statistics and cognitive science) for presenting the material. There had also been heated discussions about the level of the material and the use of programming languages. Though challenging, the entire process turned out to be a great learning and collaborative experience for me as a statistician. Curious readers could watch the introductory video at https://www.youtube.com/watch?v=eFmZV67wf4I&index=21&list=PL_lWxa4iVNt3XZg4dEoN-mwGt2_VbXrOs&t=2s.

To engage students' watching of the video and connect the video's content to our course material, a Discussion Board on Moodle was created and a request of one pre-class post and another post-class post was given to all students. Going through the pre-class posts before class helped me greatly to set up the background in class, go about the lecture material and make reference to the video content when possible.

In addition to posting on the introductory video, there were other topics created for the Bayesian hierarchical modeling module (see Figure 14). For example, my colleague had kindly shared his JAGS[11] script and it was posted on the discussion board with his permission. Our course just started covering JAGS to facilitate model estimation (as opposed to regular R script that people need to write by themselves), so topics such as sharing JAGS script to replicate results obtained using regular R code from the textbook were also posted. Sometimes, there were inconsistent or non-replicable results that I produced and I could not figure out why. I then went ahead and posted those questions on the discussion board and asked for help, and amazingly students came to rescue. For that, I appreciated the creation of the online learning environment that makes me

[10] For a quick overview, visit https://en.wikipedia.org/wiki/Bayesian_hierarchical_modeling

[11] Short for Just Another Gibbs Sampler. For more information, visit http://mcmc-jags.sourceforge.net/

comfortable enough to share my own obstacles and concerns. I also believe that comparing to traditional face-to-face in-class discussion, the online learning environment made students more comfortable to challenge each other or me in an online environment and gave us all more time and space to ponder and discuss.

4.6. **Summary.** In a hybrid/online course, creating and fostering an online learning community is crucial. I have introduced the list of tools that I have found useful: self-introduction posts, project introduction video posts, a learning environment for reading a research paper, and a learning environment for guest introductory videos. I believe many other tools can be developed and proved to be useful.

Based on my experience, the online discussion board and required students' accountability are central to the success. Only if students are actively engaged in the material and the learning community can they connect to each other and to the instructor. I believe the optimal way of enforcing students' accountability highly depends on the instructor's style and the student group. It can be an ongoing, sometimes frustrating, but ultimately rewarding experience for the instructor to create the right dynamic in the online learning community.

## 5. Future Work

We have run several hybrid/online courses and now have a good sense of what technology is available and how we can use it to engage students from across the country. The next step is to increase the diversity of course offerings. There are several issues that must be resolved. Chief among these are ensuring that faculty and students receive credit for their work and coordinating students at diverse campuses simultaneously enrolling in the same class. Both of these problems, while surmountable, take work.

At many institutions one needs the faculty to vote on course credit for students. The departments or the administration also need to support these efforts. The latter is often challenging, as many schools have strained resources and are concerned that it is too "expensive" to have faculty teach such classes. Fears are often strongest when the course is highly specialized, and involves only a few students from the local campus. There may even be fears that hybrid/online courses will "steal" students away from local courses.

Our hope is that by showing that shared courses can be taught, without an undue amount of work, in a manner compatible with the liberal arts mission that our colleagues will be convinced to add their courses to the mix. Doing so spreads the work across many institutions, and would increase the diversity of course offerings enormously. In such a situation the greater opportunities for students would be clear and help make the case that it is worth the additional faculty time required.

In addition to the more standard advanced classes found at our institution, we hope that a collaborative environment would exist to facilitate creating and offering new courses. One example we have discussed is a class on Financial Mathematics. From numerous conversations with students it is clear that there is an enormous demand for such a course, but most of our institutions do not offer it. One reason for this is that frequently no school has someone who does this type of mathematics/statistics, and creating such a new course is an enormous amount of work. However, if this class were shared across schools, the workload for each involved faculty member would be very manageable. We are exploring a model where 3 or 4 faculty members agree to teach a few weeks, thus making the commitment on course development modest and allowing us to build on what we have learned to address a need on our campuses.

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