Sequences of Sum and Difference Dominated Sets

Yorick Herrmann[1], Connor Hill[2], Merlin Phillips[3], Daniel Flores[4], Steven J. Miller[5], and Steven Senger[6]

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[1]yherrman@uci.edu, [2]hill.connor.03@gmail.com, [3]merlin.phillips216@gmail.com, [4]flore205@purdue.edu, [5]sjm1@williams.edu, [6]stevensenger@missouristate.edu

Sumsets and Difference sets

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We say A is sum-dominated or a **More Sums than Differences** (MSTD) set if $|A + A| > |A - A|$.

Example: The Conway Set

Let $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$.

- $A + A \;=\; [0, 28] \setminus \{1, 20, 27\}, \quad |A + A| \;=\; 26$
- $A - A \;=\; [-14, 14] \setminus \{\pm 6, \pm 13\}, \quad |A - A| \;=\; 25$

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This set A is called the **Conway set**, and it is the smallest MSTD set in terms of both cardinality and diameter.

Known Results

- MSTDs should be rare since $a + b = b + a$, but $a - b \neq b - a$, for all $a, b \in \mathbb{Z}$ with $a \neq b$.

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- If there exists an $a^* \in \mathbb{Z}$ such that $A = \{a^*\} - A$, then A is *symmetric* with respect to a^* and A is *sum-difference balanced* ($|A + A| = |A - A|$).

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- If there exists an $a^* \in \mathbb{Z}$ such that $A = \{a^*\} - A$, then A is *symmetric* with respect to a^* and A is *sum-difference balanced* ($|A + A| = |A - A|$).
- Several methods for constructing MSTD sets exist.

Nathanson's Construction

Nathanson provides one such construction for MSTD sets.

Theorem (Nathanson, 2006)

Let m, d, k be integers such that $m \geq 4$, $1 \leq d \leq m-1$, $d \neq \frac{m}{2}$, and

$$k \geq 3 \text{ if } d < \frac{m}{2}, \quad k \geq 4 \text{ if } d > \frac{m}{2}.$$

Define

$$\begin{aligned} B &= [0, m-1] \setminus \{d\}, \\ L &= \{m-d, 2m-d, \ldots, km-d\}, \\ a^* &= (k+1)m - 2d, \\ A^* &= B \cup L \cup (a^* - B), \\ A &= A^* \cup \{m\}. \end{aligned}$$

Then A is an MSTD set of integers.

Example

Let $m = 4$, $d = 1$, and $k = 3$. Then:

- $B = [0, m-1] \setminus \{d\} = \{\mathbf{0}, \mathbf{2}, \mathbf{3}\}$
- $L = \{m-d, 2m-d, 3m-d\} = \{\mathbf{3}, \mathbf{7}, \mathbf{11}\}$
- $a^* = (k+1)m - 2d = \mathbf{14}$
- $A^* = B \cup L \cup (a^* - B) = \{\mathbf{0}, \mathbf{2}, \mathbf{3}, \mathbf{7}, \mathbf{11}, \mathbf{12}, \mathbf{14}\}$
- $A = A^* \cup \{m\} = \{\mathbf{0}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{7}, \mathbf{11}, \mathbf{12}, \mathbf{14}\}$

This is the Conway set!

Problem Statement

At the recent CANT (Combinatorial and Additive Number Theory Conference), Samuel Alexander posed the following: **Find a sequence of sets with $A_{i-1} \subset A_i$ that alternate being sum and difference dominated.**

Filling In

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- For a set $A \subset [a, b]$, *filling in A* refers to the process of adding elements in $[a, b] \setminus A$ to A.
- We arrive at trivial methods of obtaining the desired sequences by noticing that an interval of integers $[a, b]$ is symmetric with respect to $a + b$.
- Thus, between each step in the sequence we fill in the sets to 'reset' our sum and difference counts.

Filling Method 1

Lemma

Let $[a, b]$ be an interval of integers where $a, b \in \mathbb{Z}$, and $a < b$. Let $p > b + 1$, $p \in \mathbb{Z}$. Then

$$A := [a, b] \cup \{p\}$$

is difference-dominated.

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Lemma

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is difference-dominated.

Consider any MSTD set A_1, where $A_1 \subset [0, n]$, $n \in \mathbb{N}$. From the lemma, $A_2 := [0, n] \cup \{p\}$ is difference dominated and contains A_1 if $p > n + 1$.

Filling Method 1

Next, we obtain an MSTD set A_3 from A_2:

- Let $m = p + 2$ if p is odd, $m = p + 5$ if p is even
- Set $2 \leq d \leq m - 3$
- Set $k \geq 2$, and $a^* = (k + 3)m - 2d$.

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We apply Nathanson's Construction for MSTD sets:

- $B = [0, m-1] \setminus \{d\}$.
- $A_3^* = B \cup \{2m - d, 3m - d, \dots, (k+1)m - d\} \cup (a^* - B)$.
- $A_3 = A_3^* \cup \{m\}$.

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- $A_3 = A_3^* \cup \{m\}$.

Thus, A_3 is MSTD. We have $A_1 \subset A_2 \subset A_3$, and we can extend this sequence infinitely by setting $n = \max(A_3)$ and repeating the steps used to generate A_2 and A_3.

Filling Method 1 Example

- $A_1 = \{0, 2, 3, 4, 7, 11, 12, 14\}, p = 17$
- $A_2 = [0, 14] \cup \{17\}$
- $A_3 = [0, 18] \setminus \{16\} \cup \{22, 41\} \cup [45, 63] \setminus \{47\} \cup \{19\}$
- $A_4 = [0, 63] \cup \{65\}$

Filling Method 1 Example Sequence

Table: Filling in Method 1 Example Sequence

Set	$\|A_i + A_i\|$	$\|A_i - A_i\|$	Cardinality	Diameter	Density
A_1	26	25	8	14	0.571
A_2	33	35	16	17	0.941
A_3	126	125	39	63	0.619
A_4	130	131	65	65	1.000
A_5	414	413	135	207	0.652
A_6	418	419	209	209	1.000
A_7	1278	1277	423	639	0.662
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.667

Sets Not Contained in $[0, n]$

Since $x \cdot A + \{y\}$ is MSTD for any MSTD set A, we can translate *any* MSTD set to fit within $[0, n]$ and use it to start this method.

Filling Method 2

Definition (P_n Set)

A set of integers is called a ***P_n* set** if both its sumset and difference set contain all possible elements except for the first and last n.
Equivalently, for a set A with $a = \min A$ and $b = \max A$,

- $[2a + n,\ 2b - n] \subset A + A$
- $[-(b - a) + n, (b - a) + n] \subset A - A$

For instance, if $A = \{0, 1\}$, then $|A + A| = \{0, 1, 2\}$ and $|A - A| = \{-1, 0, 1\}$. As such, A is both a P_0 and a P_1 set.

Filling Method 2

Theorem

Let $A_1 = L \cup R$ *where* $L \subset [1, n]$ *and* $R \subset [n+1, 2n]$, *with* $1, 2n \in A_1$ *and* $n \notin A_1$. *Suppose that* A_1 *is a* P_n *set and MSTD.*

For $l \geq 1$, *define*

$$\begin{aligned} A_{2l} &= ([(1-l)n, (l+1)n] \setminus \{n\}) \cup \{(l+2)n\} \\ A_{2l+1} &= (L - ln - 1) \cup ([(1-l)n, (l+1)n] \setminus \{n\}) \cup (R + ln). \end{aligned}$$

Then the sequence of sets $A_1 \subset A_2 \subset \cdots$ *alternates between being MSTD and MDTS.*

Filling Method 2 Example Sequence

- $A_1 = \{1,3,4,8,9,12,13,15,18,19,20\}$ is P_{10} and MSTD, and contains 1 and 20 but not 10
- $A_2 = [0,20] \setminus \{10\} \cup \{30\}$
- $A_3 = \{-10,-8,-7,-3,-2\} \cup [0,20] \setminus \{10\} \cup \{22,23,25,28,29,30\}$

Filling Method 2 Example Sequence

Table: Filling in Method 2 Example Sequence

Set	$\lvert A_i + A_i \rvert$	$\lvert A_i - A_i \rvert$	Cardinality	Diameter	Density
A_1	38	37	11	16	0.688
A_2	52	61	21	30	0.700
A_3	80	79	31	40	0.775
A_4	92	101	41	50	0.820
A_5	120	119	51	60	0.850
A_6	132	141	61	70	0.871
A_7	160	159	71	80	0.888
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 1.000

Non-Filling Method 1

We now add the constraint that we are not allowed to fill in sets to obtain the desired sequence.

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Theorem

Let A_1 be MSTD with $0 \in A_1$, and $n > \max(A_1)$ satisfying:

- $|(A_1 + A_1) \mod n| = |(A_1 - A_1) \mod n|$
- $2y - x - 1 > |A_1 + A_1| - |A_1 - A_1|$ *where:*
  - $x = |a \in A_1 : n + a \notin A_1 + A_1|$
  - $y = |b \in A_1 : n - b \notin A_1 - A_1|$
- *Then,* $A_2 = A_1 \cup \{n\}$ *is difference-dominated,* $A_3 = A_1 + \{0, n\}$ *is sum-dominated.*

Non-filling Method 1

- We have $A_1 \subset A_2 \subset A_3$.
- For $l \geq 2$, let

$$\begin{aligned} A_{2l} &= A_{2l-1} \cup \{ln\} \\ A_{2l+1} &= A_{2l-1} \cup (A_1 + ln) \end{aligned}$$

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- Clearly, $A_{2l} \subset A_{2l+1}$. We proved A_{2l} and A_{2l+1} continue to alternate being sum- and difference-dominated.
- Using these constructions, we are able to generate the desired infinite sequence.

Non-filling Method 1 Example

$A_1 = \{0, 2, 3, 4, 7, 11, 12, 14\}$, $n = 17$.

- $A_2 = \{0, 2, 3, 4, 7, 11, 12, 14, 17\}$
- $A_3 = \{0, 2, 3, 4, 7, 11, 12, 14, 17, 19, 20, 21, 24, 28, 29, 31\}$
- $A_{2l+1} = 17 \cdot [0, l] + \{0, 2, 3, 4, 7, 11, 12, 14, 17\}$
- $A_{2l} = A_{2l-1} \cup \{ln\}$

Non-filling Method 1 Example

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A_1	26	25	8	14	0.571
A_2	30	33	9	17	0.529
A_3	60	59	16	31	0.516
A_4	64	67	17	34	0.500
A_5	94	93	24	48	0.500
A_6	98	101	25	51	0.490
A_7	128	127	32	65	0.492
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.471 (8/17)

Non-Filling Method 2

Theorem

Suppose there are sets $L, R \subset [0, n]$ *such that*

- $0,n \in L,R$
- $[0, n-1] \subset (L+L)$
- $[0, n-1] \subset (R+R)$
- $[0, n-1] \not\subset [R+L]$

Non-Filling Method 2

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Then for sufficiently large $m \geq n$ *and for all* $k \geq 1$*, set*

$$A_{2k-1} \;=\; L \;\cup\; [n, m] \;\cup\; n \cdot [1, k] + (m - R).$$

Then A_{2k-1} *is MSTD, and there may exist* A_{2k} *which is MDTS such that* $A_{2k-1} \subset A_{2k} \subset A_{2k+1}$.

Non-Filling Method 2

- The sumset of $A_{2k-1} + A_{2k-1}$ is

$$\begin{aligned}
&[0, 2m+n] \;\cup\; n \cdot [2, 2k] + (m-R) + (m-R) \\
&= [0, 2m+n] \;\cup\; n \cdot [2, 2k] + 2m - (R+R) \\
&= [0, 2m+n] \;\cup\; [2m+n+1, 2m+2kn] \\
&= [0, 2m+2kn] \;=\; [0, 2 \cdot \max A_{2k-1}]
\end{aligned}$$

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&= \ [0, 2m+n] \ \cup \ n \cdot [2, 2k] + 2m - (R+R) \\
&= \ [0, 2m+n] \ \cup \ [2m+n+1, 2m+2kn] \\
&= \ [0, 2m+2kn] \ = \ [0, 2 \cdot \max A_{2k-1}]
\end{aligned}
$$

- Since we have all possible sums, it suffices to have one element missing from $A_{2k-1} - A_{2k-1}$.

Non-Filling Method 2

- We know $A_{2k-1} - A_{2k-1} \subseteq [-(m+nk), m+nk]$.
- Elements of $A_{2k-1} - A_{2k-1}$ which are larger than $m+(k-1)n+1$ must belong to $(nk+m-R) - L \; = \; nk+m-(R+L)$.
- Due to $[0, n-1] \not\subset [R+L]$, at least one such element is missing.

Non-Filling Method 2 Example

For $l \geq 1$, let

$$\begin{aligned} A_{2l-1} &= \{0,1,2,5,8,9,10\} \cup (8 \cdot [1,l] + \{6,7,9,10\}), \\ A_{2l} &= A_{2l-1} \cup \{8l+14\} \end{aligned}$$

This corresponds to $n = 8$, $L = \{0,1,2,5,8\}$, $R = \{0,1,3,4,8\}$, and $m = 10$.

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$$A_{2l-1} = \{0,1,2,5,8,9,10\} \cup (8 \cdot [1,l] + \{6,7,9,10\}),$$
$$A_{2l} = A_{2l-1} \cup \{8l+14\}$$

This corresponds to $n = 8$, $L = \{0,1,2,5,8\}$, $R = \{0,1,3,4,8\}$, and $m = 10$.

- $A_1 = \{0,1,2,5,8,9,10,14,15,17,18\}$
- $A_2 = \{0,1,2,5,8,9,10,14,15,17,18,22\}$
- $A_3 = \{0,1,2,5,8,9,10,14,15,17,18,22,23,25,26\}$

Non-Filling Method 2 Example

Table: Non-Filling in Method 2 Example Sequence

Set	$\lvert A_i + A_i \rvert$	$\lvert A_i - A_i \rvert$	Cardinality	Diameter	Density
A_1	36	35	11	18	0.611
A_2	40	41	12	22	0.545
A_3	52	51	15	26	0.577
A_4	56	57	16	30	0.533
A_5	68	67	19	34	0.559
A_6	72	73	20	38	0.526
A_7	84	83	23	42	0.548
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.500

Non-filling Method 3

Theorem

Let A be an MSTD set built via Nathanson's construction, with the additional constraints that $m \equiv 0 \pmod 4$ *and* $d \in \{\frac{m}{4}, \frac{3m}{4}\}$. *Then*

$$A_1 = A \cup \{-d, (k+1)m - d\}$$

is MSTD. For $r \geq 1$, *define*

$$A_{2r} := A_{2r-1} \cup \{(k+r+1)m - d\},$$
$$A_{2r+1} := A_{2r-1} \cup \{-rm - d, (k+r+1)m - d\}.$$

Then A_{2r} *is MDTS,* A_{2r+1} *is MSTD, and*

$$A_1 \subset \cdots \subset A_{2r-1} \subset A_{2r} \subset A_{2r+1} \subset \cdots$$

forms the desired alternating sequence.

Non-filling Method 3 Example

- $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$ can be expressed as $\{0, 2, 3\} \cup \{3, 7, 11\} \cup \{11, 12, 14\} \cup \{4\}$.

Non-filling Method 3 Example

- $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$ can be expressed as $\{0, 2, 3\} \cup \{3, 7, 11\} \cup \{11, 12, 14\} \cup \{4\}$.
- By extending the arithmetic progression $\{3, 7, 11\}$, we are able to generate the desired sequence.

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- By extending the arithmetic progression $\{3, 7, 11\}$, we are able to generate the desired sequence.
- $A_1 = A \cup \{-1, 15\}$ is MSTD. $A_2 = A_1 \cup \{19\}$ is difference-dominated. $A_3 = A_2 \cup \{-5\}$ is MSTD.

Non-filling Method 3 Example

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- By extending the arithmetic progression $\{3, 7, 11\}$, we are able to generate the desired sequence.
- $A_1 = A \cup \{-1, 15\}$ is MSTD. $A_2 = A_1 \cup \{19\}$ is difference-dominated. $A_3 = A_2 \cup \{-5\}$ is MSTD.
- For $l \geq 2$, $A_{2l} = A_{2l-1} \cup \{4l + 15\}$ is difference-dominated, and $A_{2l+1} = A_{2l} \cup \{-4l - 1\}$ is MSTD.

Non-filling Method 3 Example

Table: Non-Filling in Method 3 Example Sequence

Set	$\lvert A_i + A_i \rvert$	$\lvert A_i - A_i \rvert$	Cardinality	Diameter	Density
A_1	32	31	10	16	0.625
A_2	36	37	11	20	0.550
A_3	40	39	12	24	0.500
A_4	44	45	13	28	0.464
A_5	48	47	14	32	0.437
A_6	52	53	15	36	0.416
A_7	56	55	16	40	0.400
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.250

Non-filling Method 3 Extension

We extend the general idea behind Non-filling Method 3 to create an even more efficient method.

Non-filling Method 3 Extension

We extend the general idea behind Non-filling Method 3 to create an even more efficient method.

- For $k \geq 0$, let

$$\begin{aligned} A_{4k+1} &= (5 \cdot [-k-5, k+6] + \{1,2\}) \\ &\quad \cup \{-7, 0, 5, 8, 15\} \setminus \{-9, 1, 2, 6, 7, 17\} \\ A_{4k+2} &= A_{4k+1} \cup \{5k+36\} \\ A_{4k+3} &= A_{4k+1} \cup \{-5k-28, 5k+36\} \\ A_{4k+4} &= A_{4k+1} \cup \{-5k-28, 5k+36, 5k+37\} \end{aligned}$$

Non-Filling Method 3 Extension Table

Table: Non-Filling in Method 3 Sequence

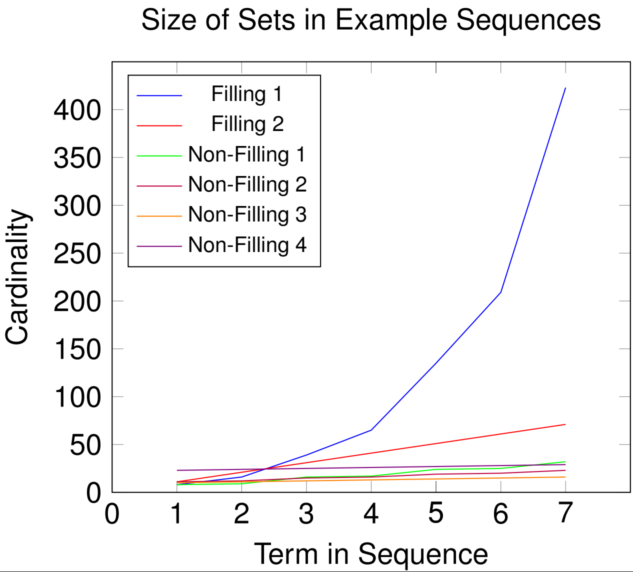
Set	$\|A_i + A_i\|$	$\|A_i - A_i\|$	Cardinality	Diameter	Density
A_1	98	97	23	56	0.410
A_2	102	103	24	60	0.400
A_3	106	105	25	64	0.391
A_4	110	111	26	65	0.400
A_5	114	113	27	66	0.409
A_6	118	119	28	70	0.400
A_7	122	121	29	74	0.392
⋮	⋮	⋮	⋮	⋮	⋮

Limiting MSTD density: 0.400

Table: Comparison of MSTD set growth characteristics

Method	$\lvert A_1 \rvert$	A_1 Diam.	Growth
Filling 1	≥ 8	≥ 14	Exponential
Filling 2	≥ 10	≥ 17	Linear
Non-Filling 1	≥ 8	≥ 14	Linear
Non-Filling 2	≥ 8	≥ 14	Linear
Non-Filling 3	≥ 11	≥ 18	Linear
Non-Filling 4	≥ 13	≥ 33	Linear

Growth Rates



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References I

P. V. Hegarty.
Some explicit constructions of sets with more sums than differences.
Acta Arith., 130(1):61–77, 2007.

M. B. Nathanson.
Sets with more sums than differences.
Integers, 7:A5, 24 pp., 2007.

G. Martin and K. O'Bryant.
Many sets have more sums than differences.
In *Additive combinatorics*, CRM Proc. Lecture Notes, vol. 43, pp. 287–305. Amer. Math. Soc., Providence, RI, 2007.

S. J. Miller, B. Orosz, and D. Scheinerman.
Explicit constructions of infinite families of MSTD sets.
J. Number Theory, 130(5):1221–1233, 2010.

M. B. Nathanson.
Problems in additive number theory, V: affinely inequivalent MSTD sets.
North-West. Eur. J. Math., 3:123–141, 2017.

References 2

G. Iyer, O. Lazarev, S. J. Miller, and L. Zhang.
Generalized more sums than differences sets.
J. Number Theory, 132(5):1054–1073, 2012.

J. Marica.
On a conjecture of Conway.
Canad. Math. Bull., 12:233–234, 1969.

S. J. Miller, L. Robinson, and S. Pegado.
Explicit constructions of large families of generalized more sums than differences sets.
Integers, 12(5):935–949, 2012.

Y. Zhao.
Constructing MSTD sets using bidirectional ballot sequences.
J. Number Theory, 130(5):1212–1220, 2010.

Y. Zhao.
Sets characterized by missing sums and differences.
J. Number Theory, 131(11):2107–2134, 2011.