

Sensor sensitivity in several settings

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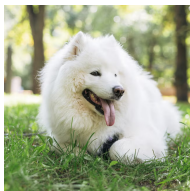
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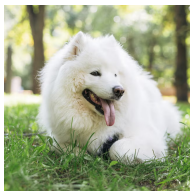
Compression and Reconstruction



Underlying arbitrary
image

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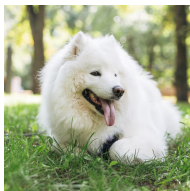
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Incomplete
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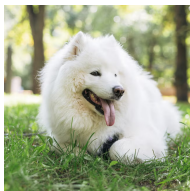
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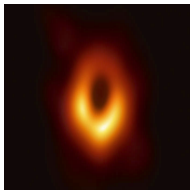
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- ▶ Suppose we are given an arbitrary image generated by an arbitrary function.
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- ▶ A standard reconstruction algorithm uses the observed measurements to estimate the missing values.
- ▶ The reconstruction does not look good because the initial image is not spectrally compressible.

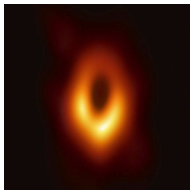
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Compression and Reconstruction



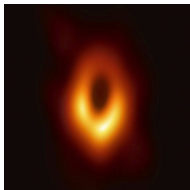
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Compression and Reconstruction



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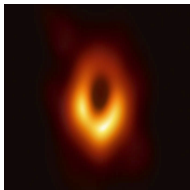
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- ▶ Now what if we assume that the image is generated by an underlying smooth continuous signal.
- ▶ We observe only a discretized version of the signal, and some sampled values are missing. The white squares represent unknown measurements.
- ▶ A standard reconstruction algorithm uses the known measurements to estimate the missing values.
- ▶ The reconstruction closely resembles the complete discretized image that would have been observed if no measurements were missing because this initial signal is indeed spectrally compressible.

Definitions

- For a function $g : \mathbb{Z}_N^2 \rightarrow \mathbb{C}$, define its Fourier ratio by

$$\text{FR}(g) = \frac{\|\widehat{g}\|_{\ell^1}}{\|\widehat{g}\|_{\ell^2}}.$$

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- ▶ The finite Fourier transform of g is

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- ▶ The corresponding norms are

$$\|\widehat{g}\|_{\ell^1} = \sum_{m \in \mathbb{Z}_N^2} |\widehat{g}(m)| \quad \text{and} \quad \|\widehat{g}\|_{\ell^2} = \left(\sum_{m \in \mathbb{Z}_N^2} |\widehat{g}(m)|^2 \right)^{1/2}.$$

The Fourier Ratio

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- ▶ A small ratio means the Fourier mass behaves as if it is concentrated.
- ▶ A large ratio means the spectrum is spread out and harder to recover from few samples.
- ▶ We find bounds for certain families of functions across several geometries.

Reconstruction

The first and most important result to come from the Fourier Ratio is a recovery theorem.

Theorem

Let X be the unit square, the unit sphere, or a compact Riemannian manifold. Then

$$q \gtrsim \max\{1, D_R L\} \frac{\text{FR}_R(f)^2}{\varepsilon^2} \log^2 \left(2 + \frac{\text{FR}_R(f)}{\varepsilon} \right) \log(2D_L),$$

then random point samples give stable ℓ^1 reconstruction.

The consequences of this result is that once the Fourier Ratio of a family of functions is known, then we know that standard signal recovery tools can be applied.

Previous Results

In addition to the previous prior result, previous work shows that for the Unit Square, Unit Sphere, and Compact Riemannian Manifolds, the following sensor sensitivity theorems apply

Fourier Ratio Bounds

$$\text{FR}_R(f) \lesssim_X \frac{|\int_X f|}{\|f\|_{L^2(X)}} + \sqrt{1 + \log R} \frac{\|\nabla_X f\|_{L^2(X)}}{\|f\|_{L^2(X)}}.$$

Sensor Perturbations

If $d_X(x_i, y_i) \leq \delta$, then $\left(\frac{|X|}{q} \sum_{i=1}^q |f(y_i) - f(x_i)|^2\right)^{1/2} \lesssim_X \delta \|\nabla_X f\|_{L^\infty(X)}.$

Random Sensor Sampling

If X is partitioned into K equal-measure cells of diameter at most h_K , then

$$\mathbb{E}[T_q] = K(H_K - H_{K-q}), \quad E_{\text{place}} \lesssim_X h_K \|\nabla_X f\|_{L^\infty(X)}.$$

Three Settings

Unit Square	Unit Sphere	Manifold
Fourier modes	spherical harmonics	Laplace–Beltrami eigenfunctions
uniform grid cells	equal-area cells	equal-volume cells
Euclidean perturbations	geodesic perturbations	geodesic perturbations

Important Result

In the proof for Compact Riemannian Manifolds was an underutilized formulation.

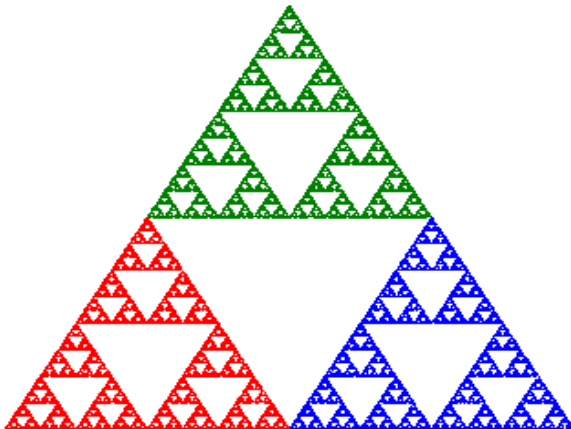
Theorem (Spectral Fourier Ratio)

Let $A^{s/2}$ be a non-negative, self-adjoint generator with compact resolvent and let L be the spectral cutoff of the orthonormal basis. Then

$$\text{FR}_L(f) \leq \frac{|\int_X f d\mu|}{\|f\|_{L^2}} + \left(\sum_{0 < \lambda_j \leq L} \lambda_j^{-s} \right)^{1/2} \frac{\|A^{s/2} f\|_{L^2}}{\|f\|_{L^2}}.$$

The natural setting to test the application of this discovery was, of course, fractals.

The Sierpiński Gasket



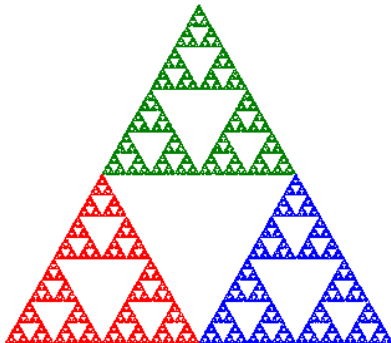
A finite approximation of the Sierpiński-gasket.

Fractals

Resistance Geometry

A resistance space comes with an energy $\mathcal{E}$ and metric

$$R(x, y) = \sup_{\mathcal{E}(u, u) > 0} \frac{|u(x) - u(y)|^2}{\mathcal{E}(u, u)}.$$



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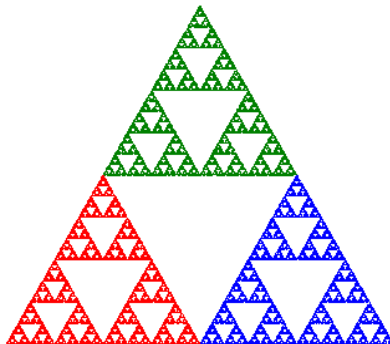
Fourier analysis

The associated Laplacian has eigenfunctions

$$-\Delta_\mu e_j = \lambda_j e_j,$$

and

$$\widehat{f}(j) = \langle f, e_j \rangle.$$



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Spectral dimension of resistance fractals

Theorem

Let X be a compact measured resistance space whose eigenvalue counting function satisfies

$$N_A(\Lambda) = (G(\log \Lambda) + o(1))\Lambda^{d_s/2},$$

where G is bounded above and bounded away from zero. Then

$$d_s < 2 \quad \text{so} \quad \sum_{j \geq 1} \frac{1}{\lambda_j} < \infty.$$

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3. Therefore $d_s/2 < 1$, hence $d_s < 2$.
4. The reciprocal eigenvalue sum is therefore finite.

Fourier Ratio on self-similar fractals

Theorem

Let X be the Sierpiński gasket, Sierpiński tetrahedron, or Minkowski curve. Then for every spectral cutoff L and every nonzero $f \in V_L$,

$$\text{FR}_L(f) \leq \frac{\left| \int_X f \, d\mu \right|}{\|f\|_{L^2(X,\mu)}} + C_X \frac{\mathcal{E}(f, f)^{1/2}}{\|f\|_{L^2(X,\mu)}},$$

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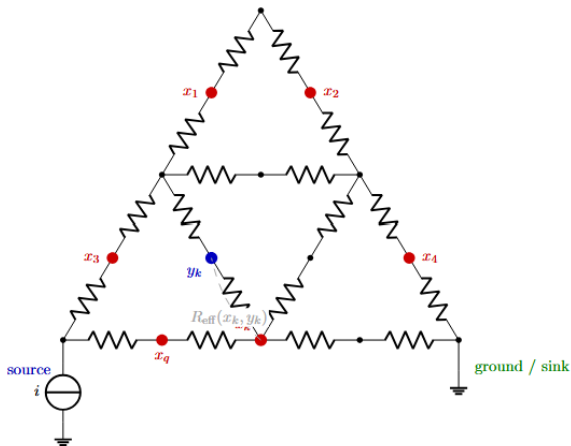
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2. For the Sierpiński tetrahedron, $d_s = \frac{2 \log 4}{\log 6} < 2$.
3. For the Minkowski curve, $d_s = 1 < 2$.
4. Thus the Fourier Ratio remains uniformly bounded as $L \rightarrow \infty$.

Electric Circuits



An electrical circuit using a finite approximation of the Sierpiński-gasket.

Effective resistance and the Kirchhoff index

Theorem

Let $G = (V, E, c)$ be a connected conductance network with $n = |V|$. Then

$$\sum_{j=1}^{n-1} \frac{1}{n\nu_j} = \frac{Kf(G)}{n^2} = \frac{1}{2n^2} \sum_{x,y \in V} R_{\text{eff}}(x,y),$$

Hence the resistance constant is $C_R = \frac{\sqrt{Kf(G)}}{n}$.

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2. Dirichlet energy becomes dissipated electrical power.
3. The reciprocal Laplacian spectrum is determined by the Kirchhoff index.

Fourier Ratio of a voltage profile

Theorem

Every nonzero voltage profile v satisfies

$$\text{FR}_G(v) \leq \frac{|V_{\text{mean}}|}{V_{\text{RMS}}} + \frac{\sqrt{Kf(G)}}{n} \frac{P_{\text{diss}}(v)^{1/2}}{V_{\text{RMS}}}.$$

In the mean-zero voltage gauge,

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2. $P_{\text{diss}}(v)$ measures the energy required to sustain the voltage profile.
3. Together they directly control the spectral complexity of the voltage.

The network as a spectral filter

Theorem

Let i be a mean-zero current injection and let $L_G v = i$. Then $v = L_G^+ i = \sum_{j=1}^{n-1} \frac{\tilde{i}(j)}{\nu_j} \phi_j$, and therefore

$$\text{FR}_G(v) = \frac{\sum_{j=1}^{n-1} \frac{|\tilde{i}(j)|}{\nu_j}}{\left(\sum_{j=1}^{n-1} \frac{|\tilde{i}(j)|^2}{\nu_j^2} \right)^{1/2}}.$$

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3. A resistive network acts as an inverse-Laplacian spectral filter.

Voltage reconstruction from incomplete measurements

Theorem

Let $v \in W_\Lambda$ If

$$q \gtrsim \max\{1, nD_R\Lambda\} \frac{r_{\text{el}}(v)^2}{\varepsilon^2} \log^2 \left(2 + \frac{r_{\text{el}}(v)}{\varepsilon} \right) \log(2D_\Lambda^G),$$

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2. Low Fourier Ratio voltage profiles require fewer measurements.

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1. The required number of sensors depends on electrical geometry.
2. Low Fourier Ratio voltage profiles require fewer measurements.
3. With high probability,

$$\|v^* - v\|_{L^2(V, \mu)} \lesssim \varepsilon V_{\text{RMS}}.$$

Selected references

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Thanks ^_^