

Pascal to Calculus: Steven Miller
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*Or, who cares?

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Solve Equations

$$2x + y = 0 \Rightarrow x$$

$$2x = -y$$

$$x = -y/2 = -2$$

$$3x - 12 = 0$$

$$3x = 12$$

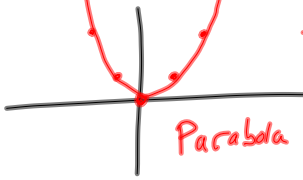
$$x = 12/3 = 4$$

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$ax + b = 0$ a, b numbers
 $ax = -b$
 $x = -b/a$

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$3x^2 - 8x + 13 = 0$ what is x ?
 Solve: $x^2 = 0$: solns: $0, 0$
 Plot $f(x) = x^2$
 Parabola



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Try $x^2 - 1 = 0$
 $x^2 = 1$ roots 1 and -1
 Try $x^2 - 3 = 0$
 $x^2 = 3$ roots $\sqrt{3}$, $-\sqrt{3}$

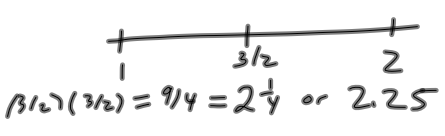
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$x^2 - c = 0$
 $x^2 = c$ so $x = \sqrt{c}$ or $-\sqrt{c}$
 $ax^2 - c = 0$
 $ax^2 = c$
 $x^2 = c/a$ so $x = \sqrt{c/a}$ or $-\sqrt{c/a}$


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$a(x-h)^2 + c = 0$ ← Vertex form
 $a(x-h)^2 = -c$
 $(x-h)^2 = -c/a$
 $x-h = \sqrt{-c/a}$ or $-\sqrt{-c/a}$
 $x = h + \sqrt{\frac{-c}{a}}$ or $h - \sqrt{\frac{-c}{a}}$
 $ax^2 + bx + c = a(x-h)^2 + d$
 for some h, d

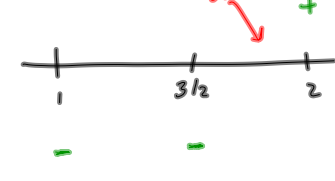
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What is $\sqrt{3}$ approximately?
 $1 \leq \sqrt{3} \leq 2$ Why?
 $1^2 = 1 \leq \sqrt{3} \sqrt{3} = 3 \leq 4 = 2 \cdot 2$


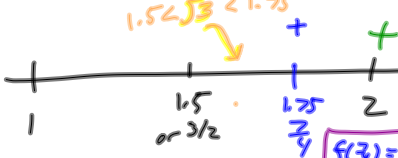
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Divide and Conquer

 $f(x) = x^2 - 3$
 $f(\sqrt{3}) = \sqrt{3} \sqrt{3} - 3 = 3 - 3 = 0$
 If $f(x) > 0$ then $x^2 > 3$ or $x > \sqrt{3}$
 If $f(x) < 0$ then $x^2 < 3$ or $x < \sqrt{3}$

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 $f(x) = x^2 - 3$
 $f(2) = 1$ pos
 $f(1) = -2$ neg
 $f(\frac{3}{2}) = 2.25 - 3$ neg
 Approximate $\sqrt{3}$

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Each time check midpoint: want neg on the left and pos on the right
 $1.5 < \sqrt{3} < 1.75$

 $f(x) = x^2 - 3$
 $f(\frac{7}{4}) = \frac{7}{4} \cdot \frac{7}{4} - 3 = \frac{49}{16} - 3 = \frac{1}{16}$ pos

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Table 7: 1.731 to 1.735
 Table 4: $\frac{55}{32} \leq \sqrt{3} \leq 1.75$
 Table 6: Upper bound 1.73
 Table 3: 1.732 to 1.733
 Table 2: 1.72 to 1.735

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$X_0 = 2$ is my start
 Sequence: each number gives me the next number:
 Ex: $a_{n+1} = 2 \cdot a_n$
 If $a_0 = 1$, $a_1 = 2 \cdot a_0 = 2 \cdot 1 = 2$
 $a_2 = 2 \cdot a_1 = 2 \cdot 2 = 4$ $a_3 = 2 \cdot a_2 = 2 \cdot 4 = 8$
 $a_4 = 2 \cdot a_3 = 2 \cdot 8 = 16$

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Consider: $X_0 = 2$
 and $X_{n+1} = \frac{1}{2} \left(X_n + \frac{3}{X_n} \right)$
 So $X_1 = \frac{1}{2} \left(X_0 + \frac{3}{X_0} \right)$
 $= \frac{1}{2} \left(2 + \frac{3}{2} \right)$
 $= \frac{1}{2} \left(2 \cdot \frac{2}{2} + \frac{3}{2} \right) = \frac{1}{2} \left(\frac{4}{2} + \frac{3}{2} \right)$
 $= \frac{1}{2} \left(\frac{7}{2} \right) = \frac{7}{4}$ or 1.75 $X_1 = 7/4$

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Rule: $X_{n+1} = \frac{1}{2} \left(X_n + \frac{3}{X_n} \right)$
 $X_0 = 2$ $X_1 = 7/4$
 Find X_2 . If you can find X_3
 and if **BRAVE** find X_4
 and why do I care?!

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$X_1 = \frac{1}{2} \left(X_0 + \frac{3}{X_0} \right)$
 $X_2 = \frac{1}{2} \left(X_1 + \frac{3}{X_1} \right)$
 $X_3 = \frac{1}{2} \left(X_2 + \frac{3}{X_2} \right)$
 $X_4 = \frac{1}{2} \left(X_3 + \frac{3}{X_3} \right)$
 $X_{n+1} = \frac{1}{2} \left(X_n + \frac{3}{X_n} \right)$

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$X_{n+1} = \frac{1}{2} \left(X_n + \frac{3}{X_n} \right)$ and $X_0 = 2$, $X_1 = 7/4$
 $X_2 = \frac{1}{2} \left(\frac{7}{4} + \frac{3}{7/4} \right)$
 $= \frac{1}{2} \left(\frac{7}{4} + 3 \cdot \frac{4}{7} \right)$
 $= \frac{1}{2} \left(\frac{7}{4} + \frac{12}{7} \right) = \frac{1}{2} \left(\frac{7 \cdot 7}{4 \cdot 7} + \frac{12 \cdot 4}{7 \cdot 4} \right)$
 $= \frac{1}{2} \left(\frac{49}{28} + \frac{48}{28} \right) = \frac{97}{56}$

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$X_3 = \frac{18547}{10864}$ or $\frac{18817}{10864}$ $\frac{18467}{10864}$
 $X_4 =$
 $X_5 = \frac{1002978273411373057}{579069776145402304}$

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$$\begin{aligned}
 X_3 &= \frac{1}{2} (X_2 + 3/X_2) \quad X_2 = 97/56 \\
 &= \frac{1}{2} \left(\frac{97}{56} + \frac{3}{97/56} \right) \\
 &= \frac{1}{2} \left(\frac{97}{56} + \frac{3 \cdot 56}{97} \right) \\
 &= \frac{1}{2} \left(\frac{97 \cdot 97}{56 \cdot 97} + \frac{3 \cdot 56 \cdot 56}{97 \cdot 56} \right)
 \end{aligned}$$

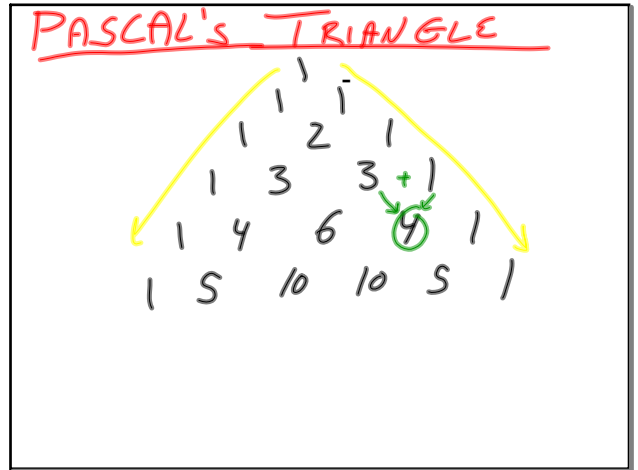
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WHO CARES!

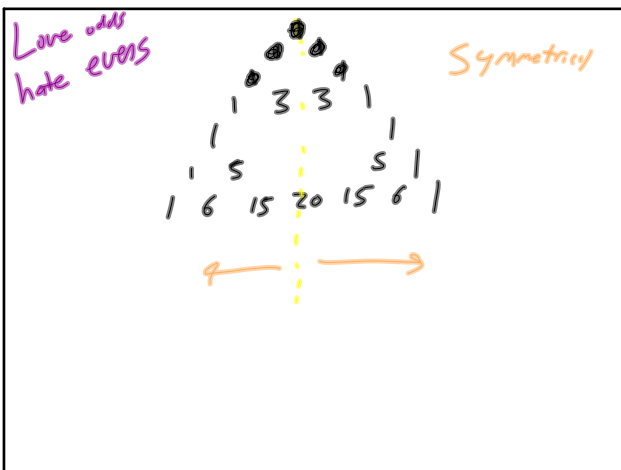
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$$\begin{aligned}
 \sqrt{3} &= 1.732050807568877 \\
 X_0 &= 2.000 \\
 X_1 &= \frac{3}{2} = 1.75000 \\
 X_2 &= \frac{97}{56} = 1.73214285714 \dots \\
 X_3 &= \frac{18917}{10884} = 1.7320508000
 \end{aligned}$$

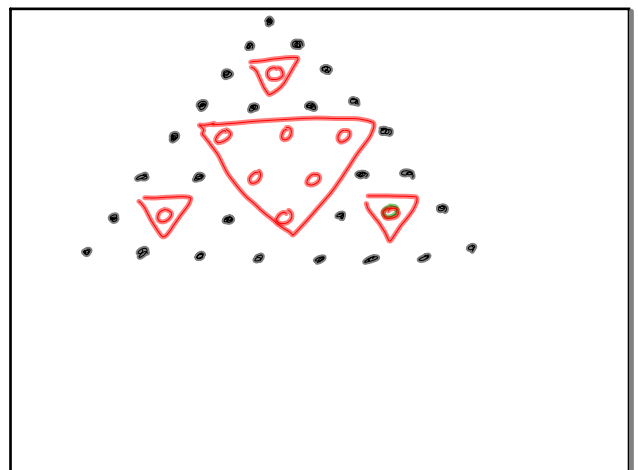
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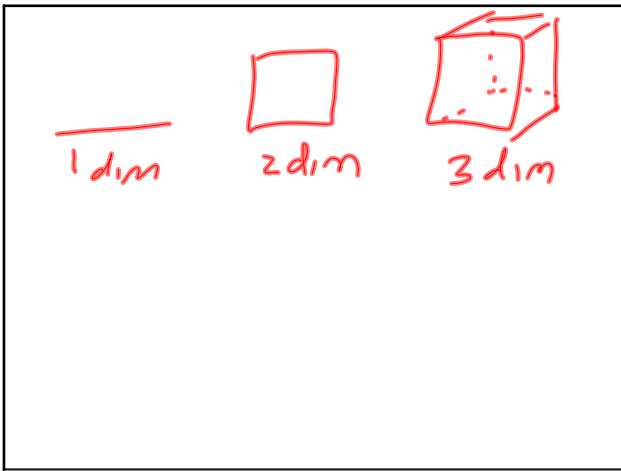
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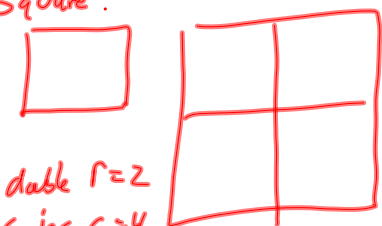
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Dimension: if I rescale by r and get C copies, dimension is the number d so that $r^d = C$

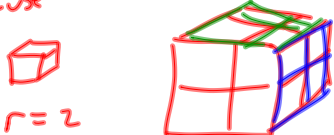
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Line: 1 dim
 double $r=2$ copies $C=2$
 Solve $r^d = C$
 or $2^d = 2 \Rightarrow d=1$

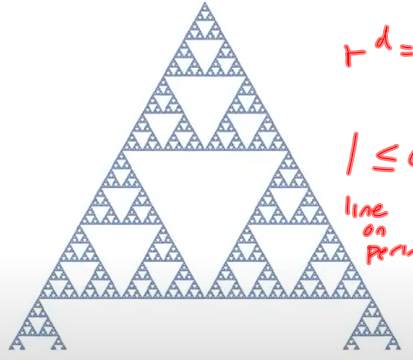
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Square:

 double $r=2$ copies $C=4$
 Solve $r^d = C$
 $2^d = 4 \Rightarrow d=2$

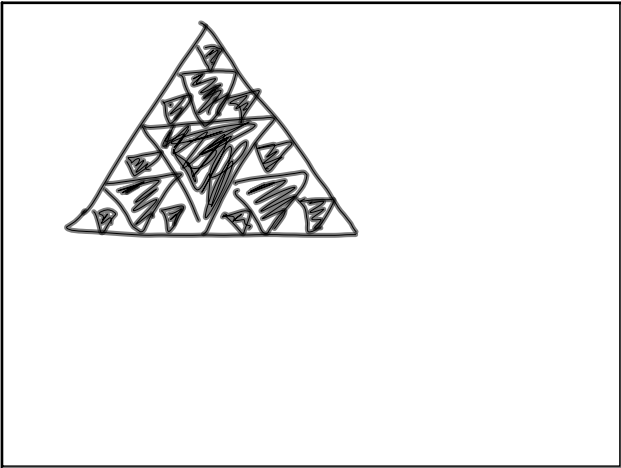
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Cube

 $r=2$
 $C=8$
 Solve $r^d = C$
 $2^d = 8$
 $d=3$

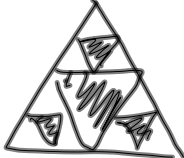
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 $r^d = C$
 $1 \leq d \leq 2$
 line on perimeter lives in 2-dim

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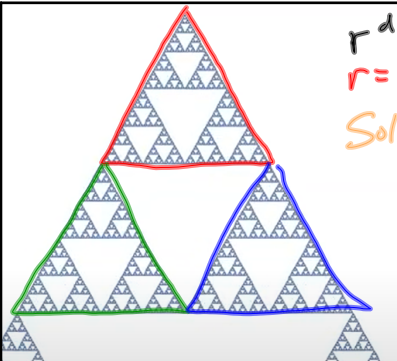


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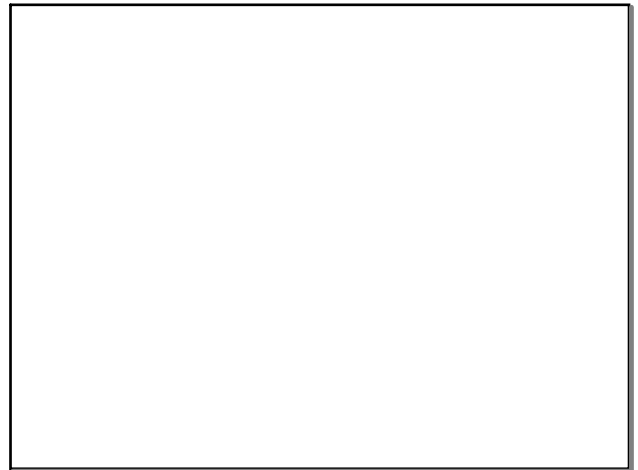
% Throw away 25%
 % Keep 75% or $\frac{3}{4}$
 after 2 levels throw away
 25% each time, keep
 75% each time, have
 $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

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$r^d = c$
 $r = 2 \quad c = 3$
 Solve $2^d = 3$
 $1 < d < 2$
 $d = \log_2 3 = \frac{\log_{10} 3}{\log_{10} 2}$

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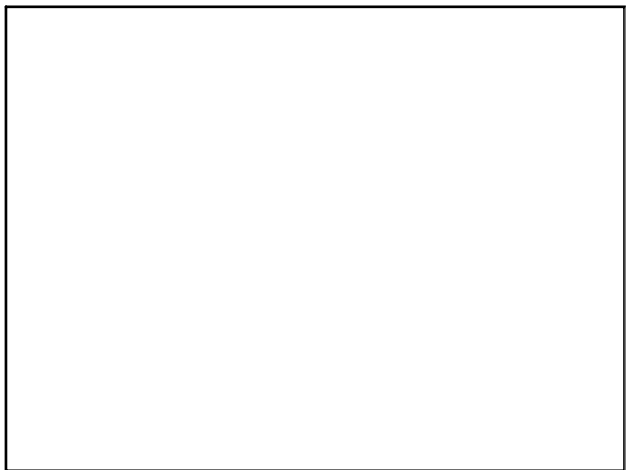
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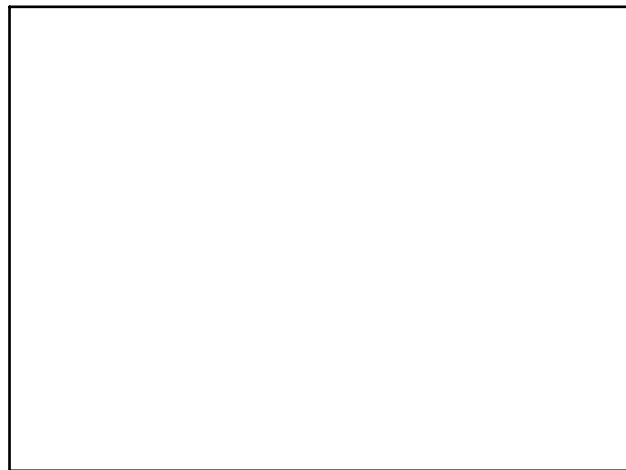
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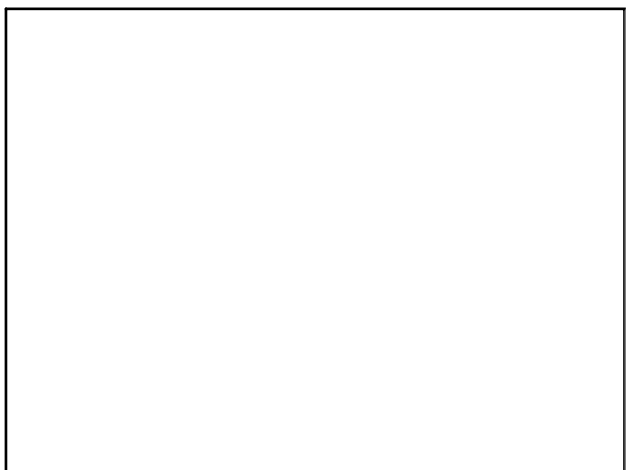
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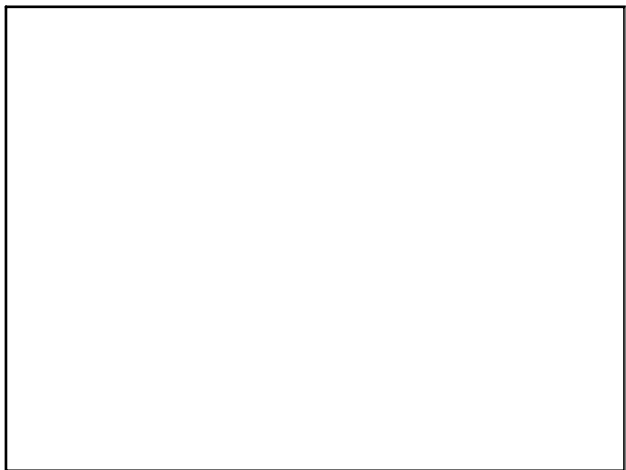
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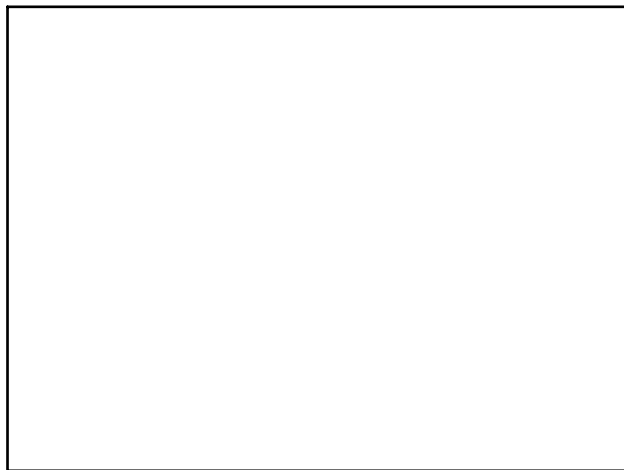
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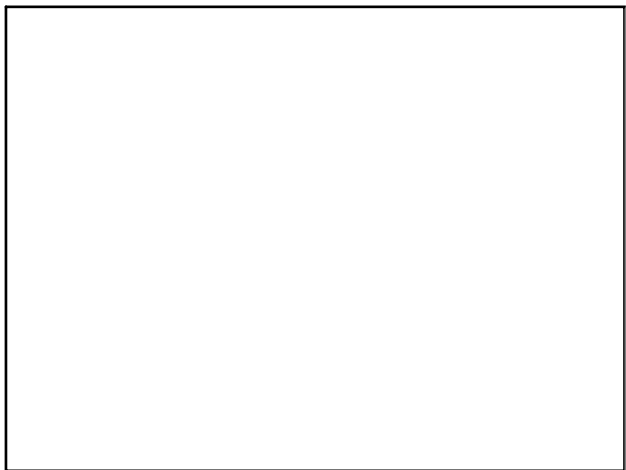
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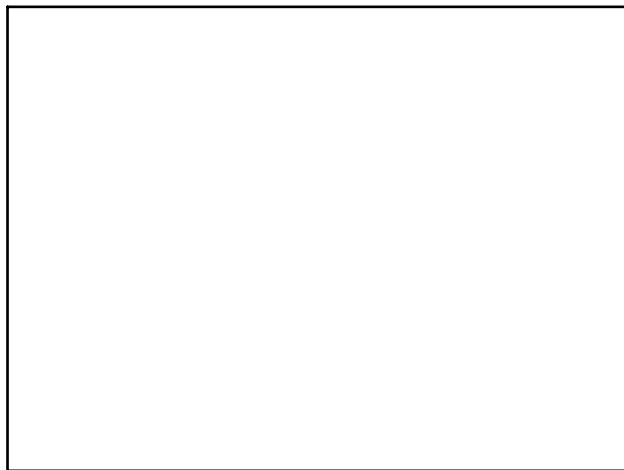
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