Pirates and Math:
Dividing the Spoils and Game Theory

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https://web.williams.edu/Mathematics/sjmiller/public_html/

https://www.shutterstock.com/image-vector/cartoon-pirate-walking-plank-260nw-715720567.jpg
shutterstock.com 715720567

AEIOU

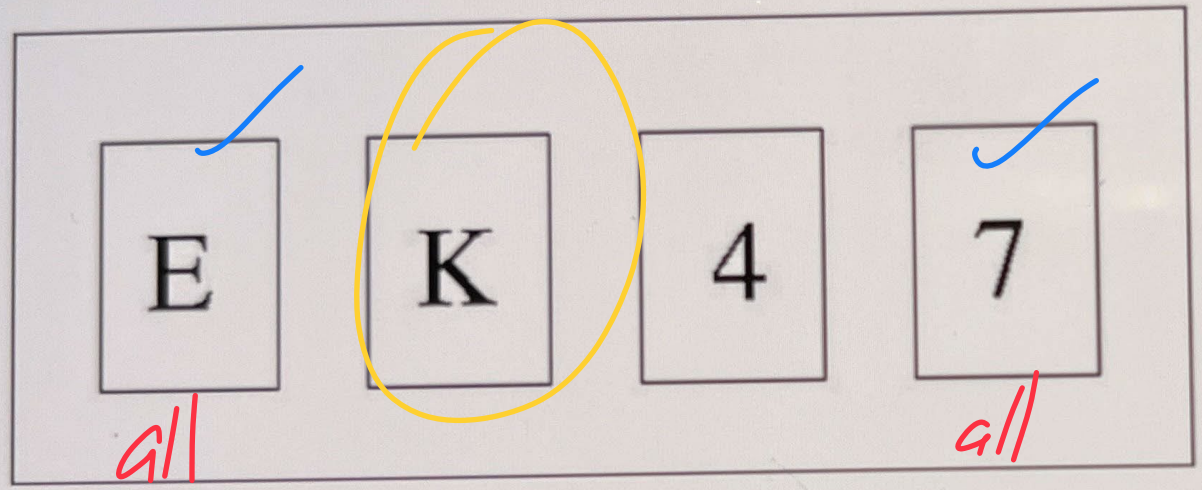


Figure 2.3. *The Wason 4-card task.* Which card(s) must you turn over to verify the rule that if a card shows a vowel on one face, then it has an even number on the other?

Pirate Riddle

- Five pirates, named 1, 2, 3, 4 and 5.
- Must split 100 gold coins.
- The lowest named pirate alive proposes a division.
- If 50% or more vote for the plan it passes and is done.
- If 50% or more vote against the plan it fails, the proposer walks the plank and dies, and the lowest named surviving pirate proposes a plan.
- Repeat till a plan is accepted.
- Assume the pirates are intelligent – what is the final division?

Questions

- Is the game play clear? Can it be interpreted more than one way? ✓
- How should you try to analyze the game? Start from Back
- Have you seen anything like this before?
- Anything special about 5 pirates and 100 coins?

Game Rules (again)

- Five pirates, named 1, 2, 3, 4 and 5.
- Must split 100 gold coins.
- The lowest named pirate alive proposes a division.
- If 50% or more vote for the plan it passes and is done.
- If 50% or more vote against the plan it fails, the proposer walks the plank and dies, and the lowest named surviving pirate proposes a plan.
- Repeat till a plan is accepted.
- Assume the pirates are intelligent – what is the final division?

Play smaller game and work up....

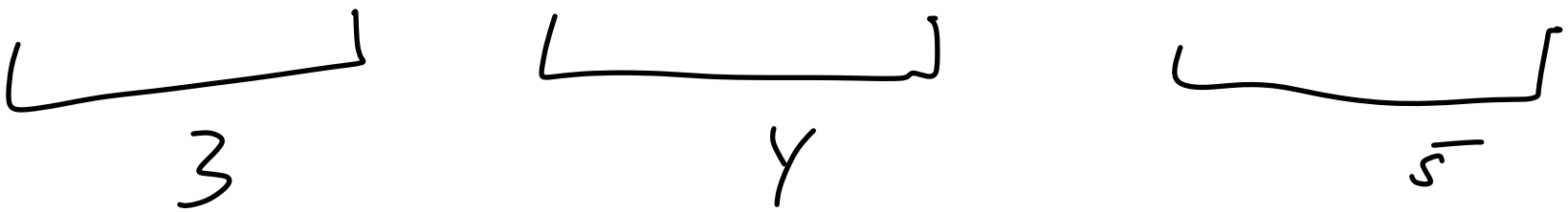
One pirate Game: 5 gets 100 : ✓

Two pirate Game: 4 proposes:

YES NO

3 Picks:

120

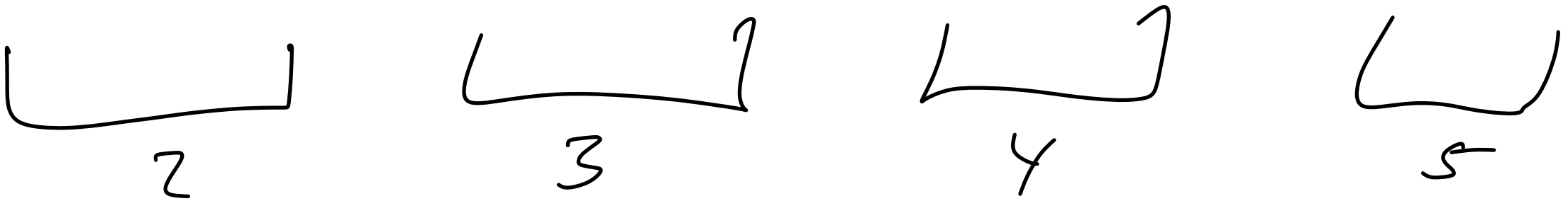


99 0 1

YES NO YES

not $2 > \frac{1}{2}(3)$ So passes

4 Pirates



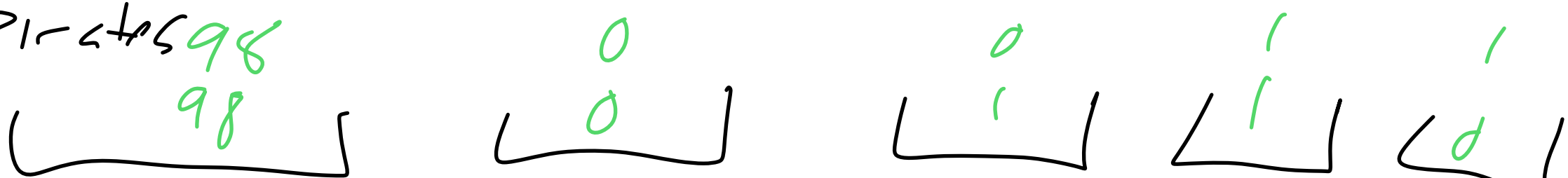
2 3 4 5

99 0 1 0

YES NO YES NO Passes

5 Pirates 98

98 0 0 1 1

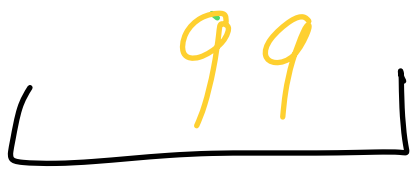


1 2 3 4 5

98 0 1 0 1

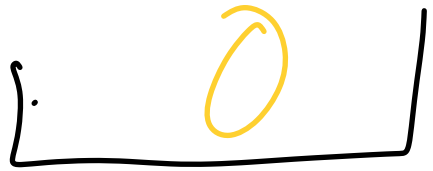
YES NO YES NO YES

3 Picks



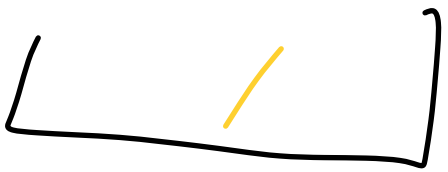
3

0
80
0
80
50
49



4

0
0
0
10
0
0



5

100
20
100
10
50
51

5 Pirates Find Gold

https://www.ba-bamail.com/riddles/

Riddle:

5 pirates are parting ways after finding a treasure of 100 pieces of gold. The pirates decide to split it based on a vote. Each pirate, from oldest to youngest, gets to propose a plan on how to split the gold.

If at least 50 percent of the other remaining pirates agree on the plan, that is how they will split the gold. If less than 50 percent of the pirates agree, the pirate who came up with the plan will be thrown overboard. Each pirate is smart, greedy, and wants to throw as many others overboard as possible without reducing the amount of gold they get.

What plan can the first (oldest) pirate propose to live and get as much gold as possible?

Answer:

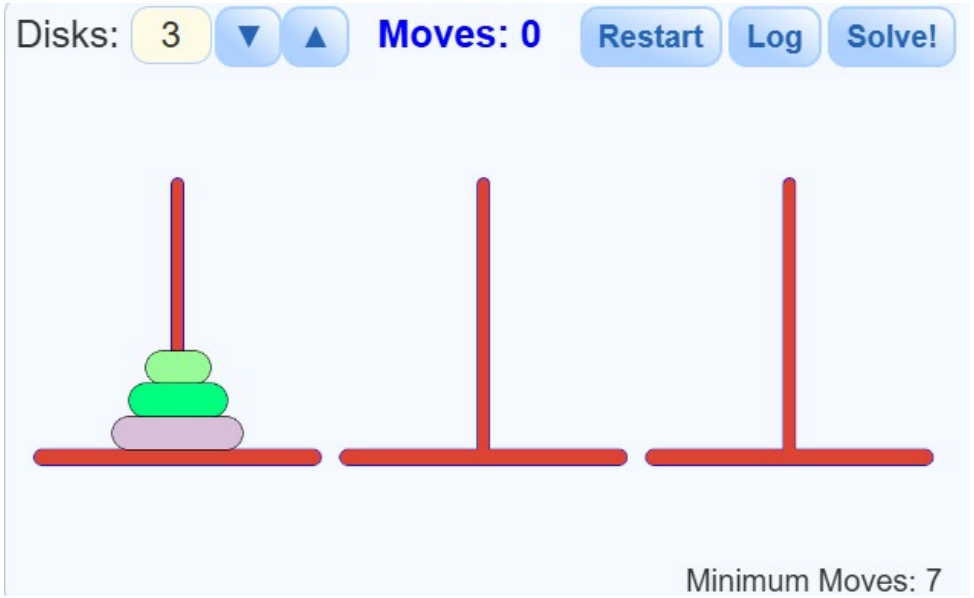
He can propose a plan that he gets 98 pieces of gold, the 3rd pirate gets 1 piece, and the 5th pirate gets 1 as well. If there were just 2 pirates the younger pirate would definitely deny the plan so he could get all of the gold. If there were 3 pirates the first pirate can offer the second pirate 1 piece of gold and take the rest himself because the second pirate wouldn't get anything if he has to propose a plan himself. If there were 4 pirates the first pirate could take 99 for himself and offer 1 to the youngest pirate. They would both agree. If the youngest disagrees then he won't get any gold in the next plan. So when there are 5 pirates it is in the interest of the 3rd and 5th pirate to accept 1 piece, because if they don't they won't get anything in the next plan.

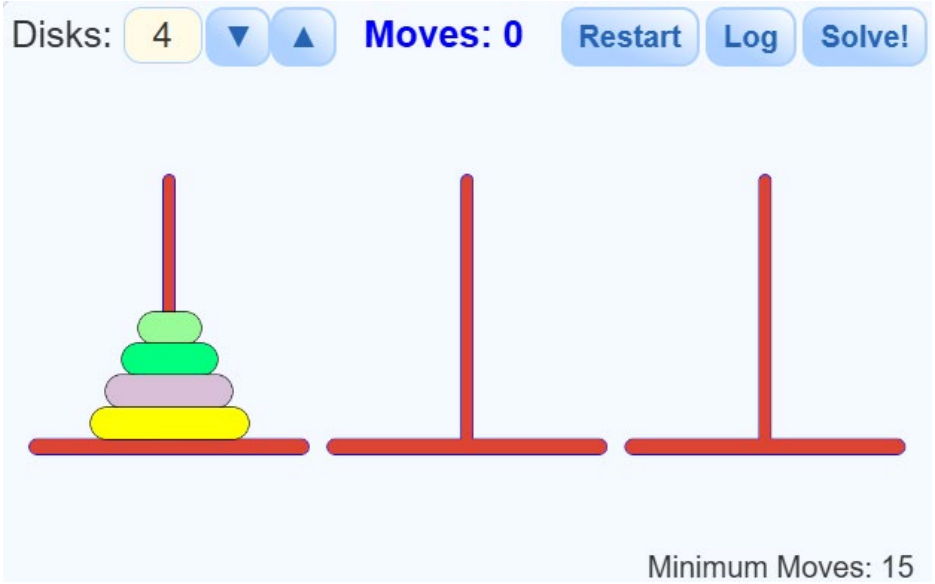
TOWERS OF HANOI:

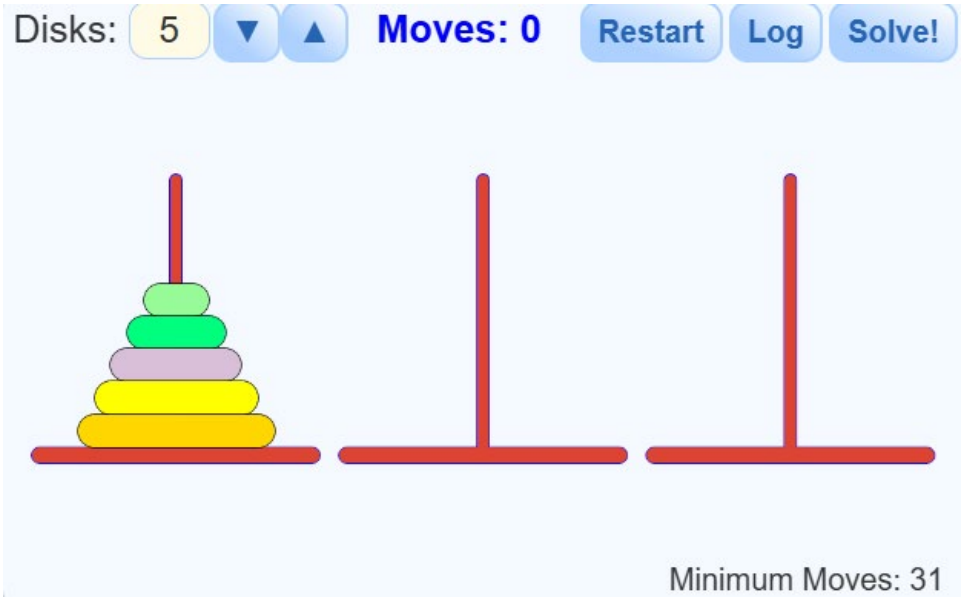
1, 3, 7, 31, 63

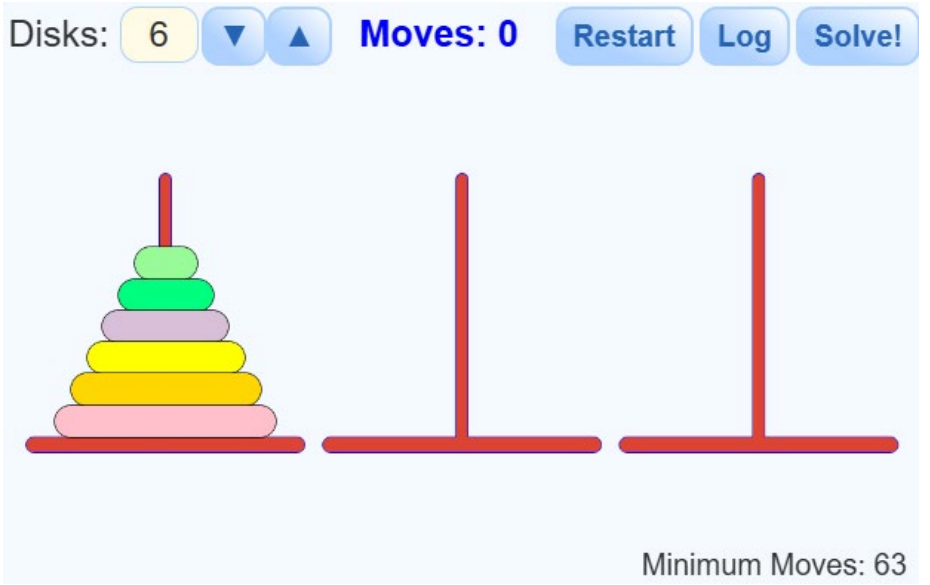
Goal is to move all the disks over to Tower 3 (drag and drop).

But you cannot place a larger disk onto a smaller disk. https://www.mathsisfun.com/games/towerofhanoi.html









On-Line Encyclopedia of Integer Sequences (OEIS)

https://oeis.org/

Search: **seq:1,3,7,15,31,63,127,255**

Displaying 1-10 of 24 results found. page 1 2 3

Sort: relevance | references | number | modified | created Format: long | short | data

A000225 a(n) = 2^n - 1. (Sometimes called Mersenne numbers, although that name is usually reserved for A001348.) +30 1334
(Formerly M2655 N1059)

0, **1, 3, 7, 15, 31, 63, 127, 255**, 511, 1023, 2047, 4095, 8191, 16383, 32767, 65535, 131071, 262143, 524287, 1048575, 2097151, 4194303, 8388607, 16777215, 33554431, 67108863, 134217727, 268435455, 536870911, 1073741823, 2147483647, 4294967295, 8589934591

(list; graph; refs; listen; history; text; internal format)

OFFSET 0,3

COMMENTS
This is the Gaussian binomial coefficient [n,1] for q=2.
Number of rank-1 matroids over S_n.
Numbers k such that the k-th central binomial coefficient is odd: A001405(k) mod 2 = 1. - Labos Elemer, Mar 12 2003
This gives the (zero-based) positions of odd terms in the following convolution sequences: A000108, A007460, A007461, A007463, A007464, A061922.
Also solutions (with minimum number of moves) for the problem of Benares Temple, i.e., three diamond needles with n discs ordered by decreasing size on the first needle to place in the same order on the third one, without ever moving more than one disc at a time and without ever placing one disc at the top of a smaller one. - Xavier Acloque, Oct 18 2003
a(0) = 0, a(1) = 1; a(n) = smallest number such that a(n)-a(m) == 0 (mod (n-m+1)), for all m. - Amarnath Murthy, Oct 23 2003
Binomial transform of [1, 1/2, 1/3, ...] = [1/1, 3/2, 7/3, ...]; (2^n - 1)/n, n=1,2,3, ... - Gary W. Adamson, Apr 28 2005
Numbers whose binary representation is 111...1. E.g., the 7th term is (2^7) - 1 = 127 = 1111111 (in base 2). - Alexandre Wajnberg, Jun 08 2005
Number of nonempty subsets of a set with n elements. - Michael Somos, Sep 03 2006
For n >= 2, a(n) is the least Fibonacci n-step number that is not a power of 2. - Rick L. Shepherd, Nov 19 2007
Let P(A) be the power set of an n-element set A. Then a(n+1) = the number of pairs of elements {x,y} of P(A) for which x and y are disjoint and for which either x is a subset of y or y is a subset of x. - Ross La Haye, Jan 10 2008
A simpler way to state this is that it is the number of pairs (x,y) where at least one of x and y is the empty set. - Franklin T. Adams-Watters, Oct 28 2011
2^n-1 is the sum of the elements in a Pascal triangle of depth n. - Brian Lewis (bsl04(AT)uark.edu), Feb 26 2008
Sequence generalized: a(n) = (A^n -1)/(A-1), n >= 1, A integer >= 2. This sequence has A=2; A003462 has A=3; A002450 has A=4; A003463 has A=5; A003464 has A=6; A023000 has A=7; A023001 has A=8; A002452 has A=9; A002275 has A=10; A016123 has A=11; A016125 has A=12; A091030 has A=13; A135519 has A=14; A135518 has

0, 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047, 4095, 8191, 16383, 32767, 65535, 131071, 262143, 524287, 1048575, 2097151, 4194303, 8388607, 16777215, 33554431, 67108863, 134217727, 268435455, 536870911, 1073741823, 2147483647,

A123121 Length of the n-th Zimin word (A082215(n)). +30 5

1, 3, 7, 15, 31, 63, 127, 255, 511, 1024, 2050, 4102, 8206, 16414, 32830, 65662, 131326, 262654, 525310, 1050622, 2101246, 4202494, 8404990, 16809982, 33619966, 67239934, 134479870, 268959742, 537919486, 1075838974, 2151677950, 4303355902, 8606711806

A117060 Mersenne numbers for which the product of the digits is not zero. +30 1

1, 3, 7, 15, 31, 63, 127, 255, 511, 8191, 16383, 32767, 65535, 262143, 524287, 16777215, 33554431, 134217727, 268435455, 2147483647, 4294967295, 8589934591, 17179869183, 34359738367, 68719476735, 137438953471, 549755813887, 562949953421311

(list; graph; refs; listen; history; text; internal format)

OFFSET 1,2

COMMENTS These are all the Mersenne numbers for which no digit is 0. The indices of these numbers are A007377.

LINKS Arkadiusz Wesolowski, Table of n, a(n) for n = 1..35

FORMULA a(n) = 2^(A007377(n)) - 1. [Arkadiusz Wesolowski, Jun 28 2011]

MATHEMATICA
```
Flatten[Table[If[First@Union@IntegerDigits[2^n - 1] != 0, 2^n - 1, {}], {n, 100}]] (* Arkadiusz Wesolowski,
  Sep 04 2011 *)
```

PROG
```
(Perl) use bignum;
for(1..99) {
  if((1<<$_) =~ /^[1-9]+$/) {
    print(((1 << $_) - 1) . ", ")
  }
} # Charles R Greathouse IV, Jun 30 2011
(PARI) for(n=1, 99, if(vecmin(eval(Vec(Str(2^n)))), print1(2^n-1", "))) \\ Charles R Greathouse IV, Jun 30
  2011
```

CROSSREFS Cf. A000225, A007377.

KEYWORD base,nonn

AUTHOR Luc Stevens (lms022(AT)yahoo.com), Apr 16 2006

How can we generalize the Towers of Hanoi Problem?

More Towers

More Pegs

Different height rules

Multiple Starts

Can only move certain distances

Remove weight condition: within an amount

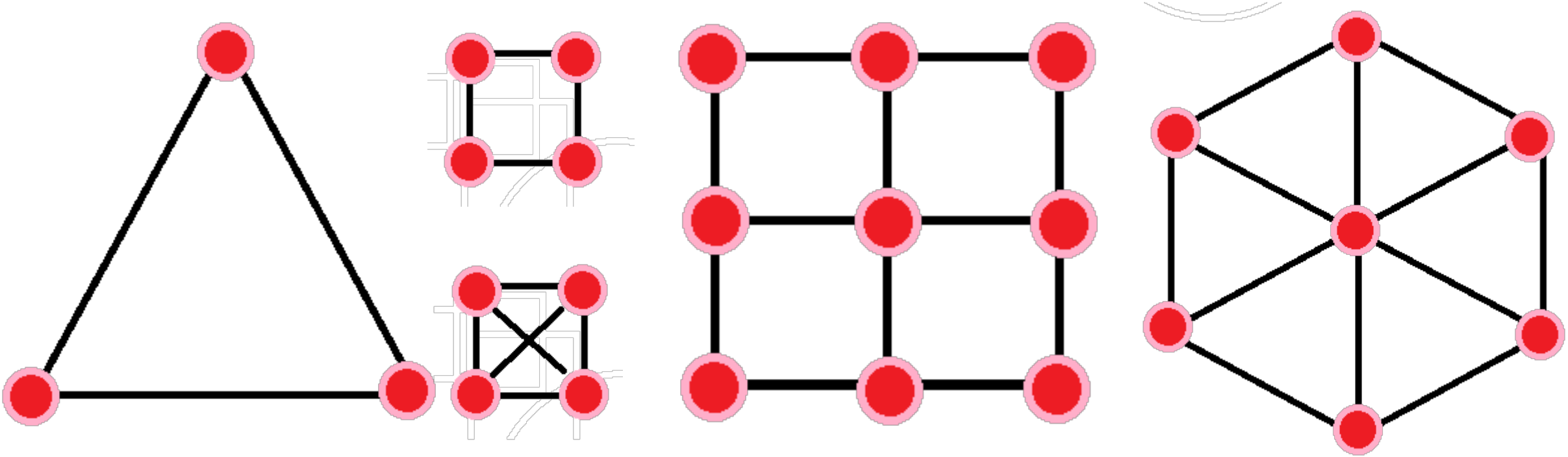
Do mod 3, cannot have certain residues touch

How can we generalize the Towers of Hanoi Problem?

- More towers….
- Can support any disk that is less and any that is the next heaviest, but not two heavier.
- Can support up to T times its weight above.

How can we generalize the Towers of Hanoi Problem?

- More towers… …but can only move to adjacent.



AI Summary

The Towers of Hanoi with four pegs can be solved more efficiently than with three pegs. The minimum number of moves required to solve the puzzle with \(n \) disks and \(k \) pegs is not straightforward, but it can be estimated using the Frame-Stewart algorithm.

Key Points:

- **Minimum Moves:** The minimum number of moves for \(n \) disks with four pegs is generally less than \(2^n \) but more than \(n^2 \).
- **Frame-Stewart Algorithm:** This algorithm provides a recursive method to determine the optimal number of moves. It involves splitting the disks into two groups and solving for each group recursively.
- **Example Calculation:** For 4 disks, the minimum moves with four pegs is 15, while with three pegs it is 7.

Efficiency:

- The four-peg version significantly reduces the number of moves compared to the traditional three-peg version, especially as the number of disks increases.

For precise calculations, especially for larger numbers of disks, using the Frame-Stewart algorithm or computational tools is recommended.

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$\frac{16}{64} = \frac{1}{4}$ $\frac{15}{75} = \frac{1}{5}$ $\frac{49}{98} = \frac{1}{2}$ $\frac{12}{24} = \frac{1}{4}$