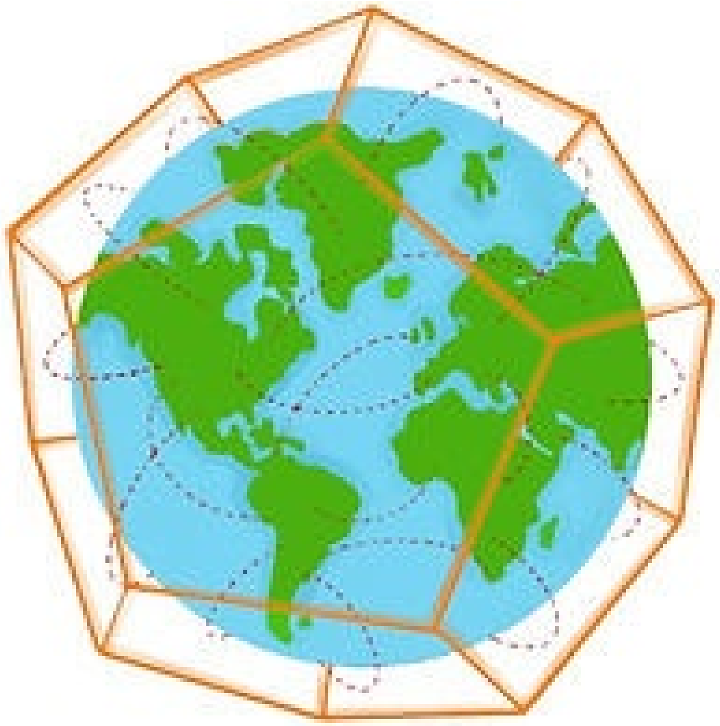
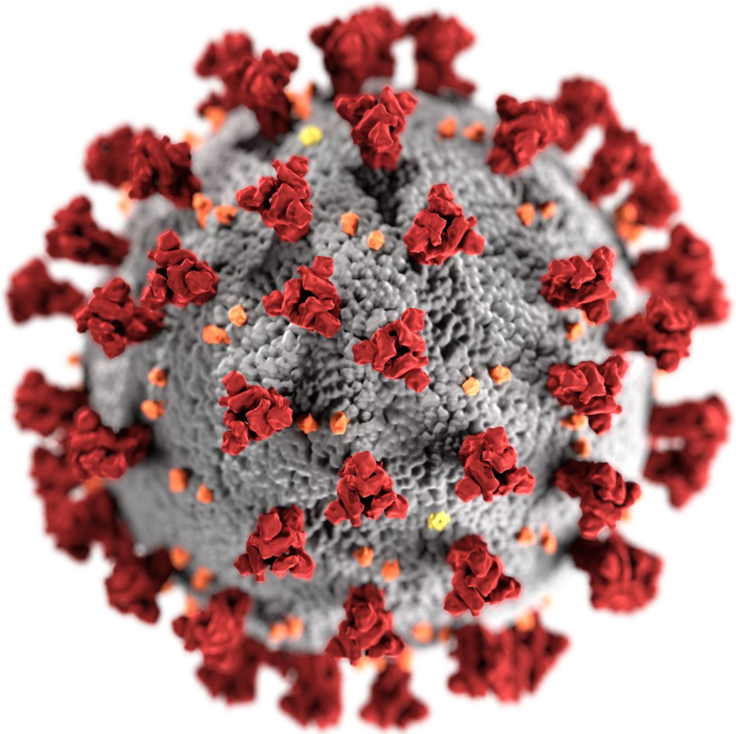
# The Polymath Jr Program: An Invitation to Online Collaborative Research

Wael El Khateeb <Wael.ElKhateeb@rockets.utoledo.edu> and Steven J. Miller <sjm1@williams.edu>

Conversations on Participating as Students and Mentors: Tuesday, February 17, 2026





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# Outline

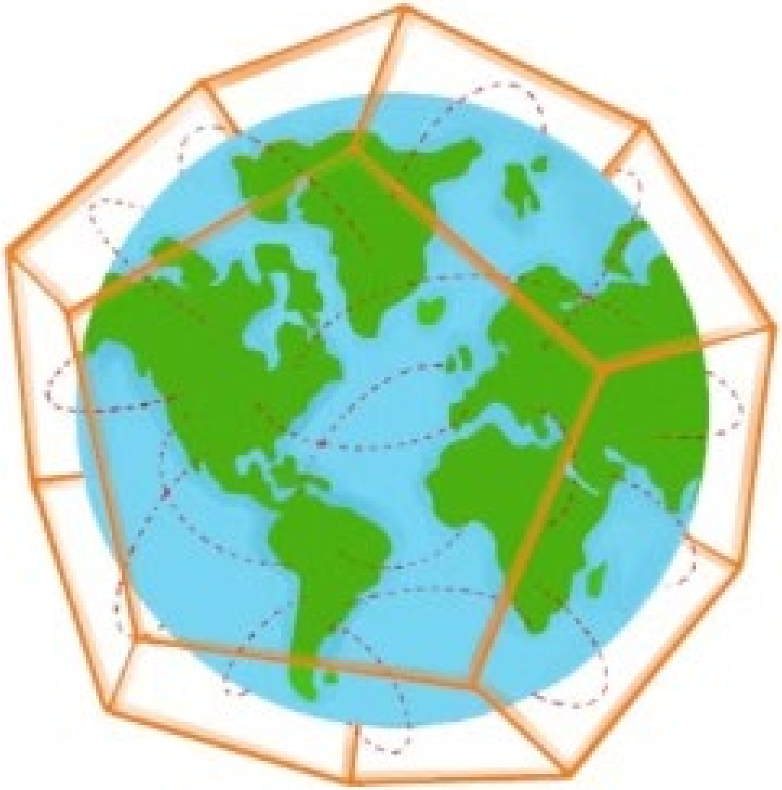
- History.
- Program mechanics.
- Open discussion and opportunities:
    - Professors / postdocs to run groups.
    - Graduate students to assist and mentor.
    - Undergraduates to research.

**HISTORY:**

# Polymath Jr.

https://geometrynyc.wixsite.com/polymathreu

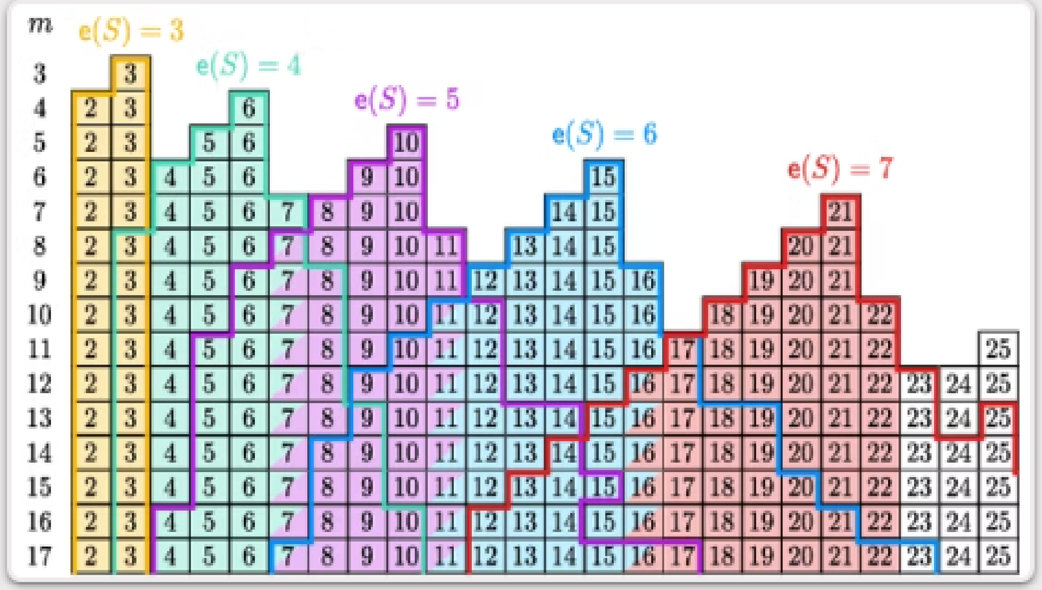
Collaborative mathematical research for undergraduate students



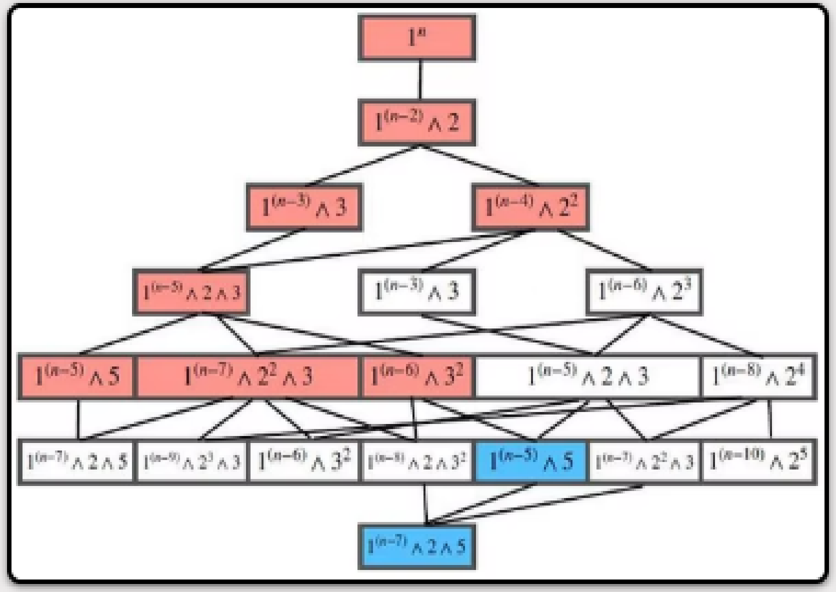
Applications open for summer of 2026; deadline is **April 1**. Apply through mathprograms.org (check for programs under Williams College.)

https://geometrynyc.wixsite.com/polymathreu

Our goal is to provide research opportunities to every undergraduate who wishes to explore advanced mathematics. This online program consists of research projects in a variety of mathematical topics and runs in the spirit of the Polymath Project. Each project is mentored by an active researcher with experience in undergraduate mentoring.




Each project consists of 15-25 undergraduates, a main mentor, and graduate students and postdocs as additional mentors. The group works towards solving a research problem and writing a paper. Each participant decides what they wish to obtain from the program, and participates accordingly.



The program is partially supported by NSF award DMS-2218374.

# MECHANICS: The perfect is the enemy of the good.

- Most groups:
  - 1-3 Professors / Postdocs.
  - 1-3 graduate student mentor.
  - 15-25 undergraduates / high school students.

- Summer structure:
  - Group run independently.
  - Virtual, no student stipends, some $$ for mentors.
  - Background lectures as needed.
  - Frequently sub-problems so big groups subdivide.
  - Several professors give general advice talks to mentors.
  - End of summer virtual conference, special session at JMM.

# DISCUSSION:


- Wael El Khateeb <Wael.ElKhateeb@rockets.utoledo.edu>
- Steven J. Miller <sjm1@williams.edu>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In 2018, Miller et al. introduced a two player game based on the Zeckendorf Decomposition.

- Played on a board of bins labeled with the Fibonacci numbers, where the game begins with $N$ tokens on the $F_1$ bin

| $F_1$ | $F_2$ | $F_3$ | ... | $F_n$ |
| --- | --- | --- | --- | --- |
| N | 0 | 0 | ... | 0 |

- Each turn one of two kinds of moves can be chosen: Split or Combine.
- Players alternate turns and the last player to move wins.

Split Move:

$$S_k : F_k \wedge F_k \mapsto F_{k-2} \wedge F_{k+1} \quad (k > 2)$$

$$S_2 : F_2 \wedge F_2 \mapsto F_1 \wedge F_3$$

| $F_1$ | $F_2$ | $F_3$ | $F_4$ | ... | $F_n$ |
|---|---|---|---|---|---|
| 0 | 0 | 2 | 0 | ... | 0 |

$\mapsto$

| $F_1$ | $F_2$ | $F_3$ | $F_4$ | ... | $F_n$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | ... | 0 |

Combine Move:

$$C_k : F_{k-1} \wedge F_k \mapsto F_{k+1} \quad (k > 1)$$

$$C_1 : F_1 \wedge F_1 \mapsto F_2$$

| $F_1$ | $F_2$ | $F_3$ | $F_4$ | ... | $F_n$ |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | ... | 0 |

$\mapsto$

| $F_1$ | $F_2$ | $F_3$ | $F_4$ | ... | $F_n$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | ... | 0 |

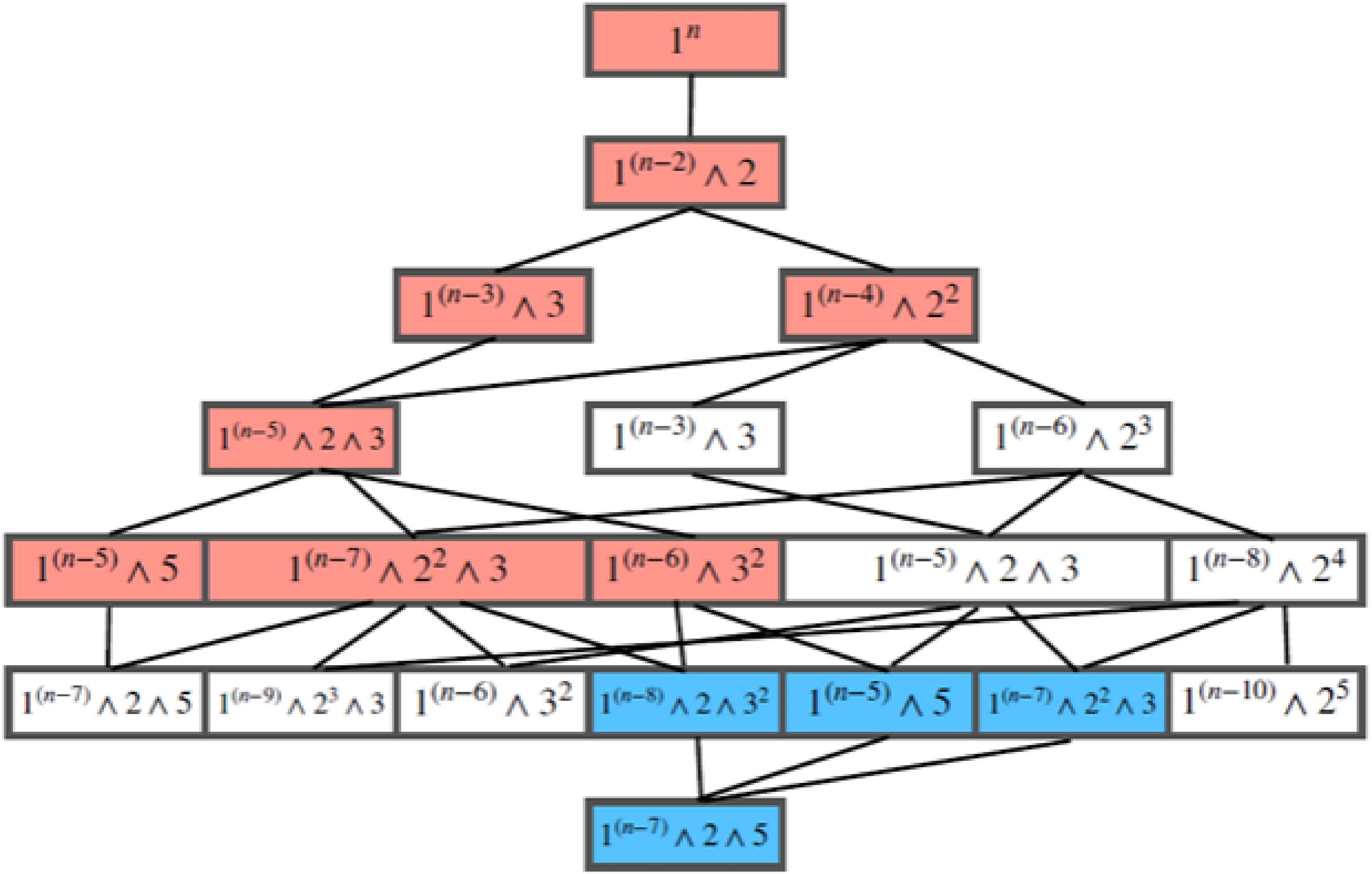



FIGURE 7. Tree depicting the proof of Theorem 1.7. Red boxes have a winning strategy for Player 1, and blue boxes indicate a winning strategy for Player 2.

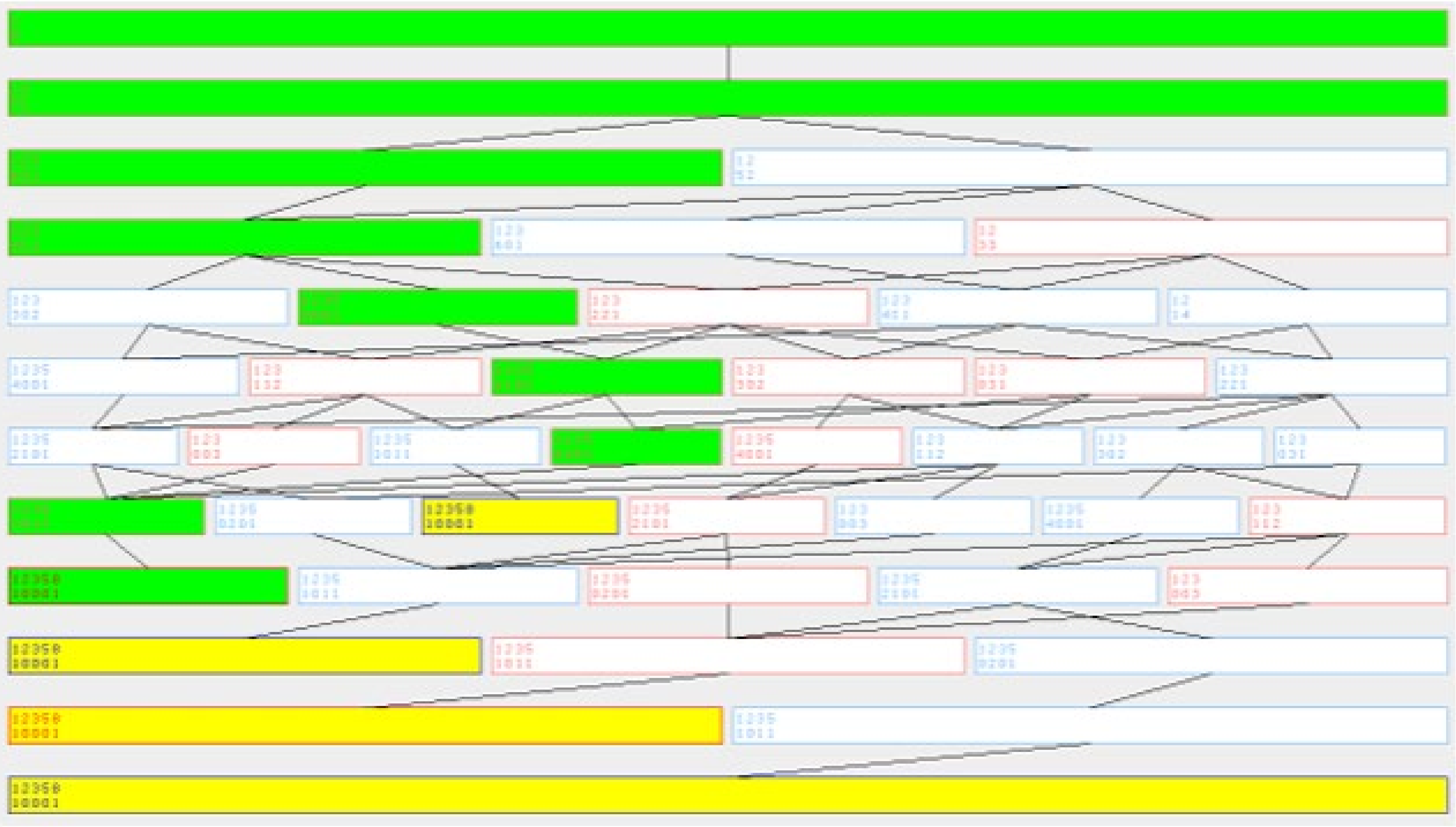


FIGURE 8. Game tree for $n = 9$, showing a winning path in green.

# Meet the Team

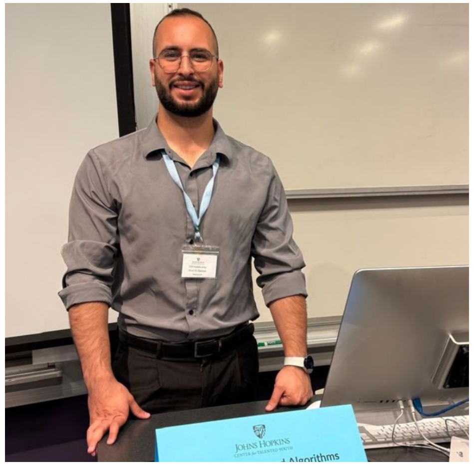
Anh Quynh Bui

Jasmine Pham

Jessie Wang

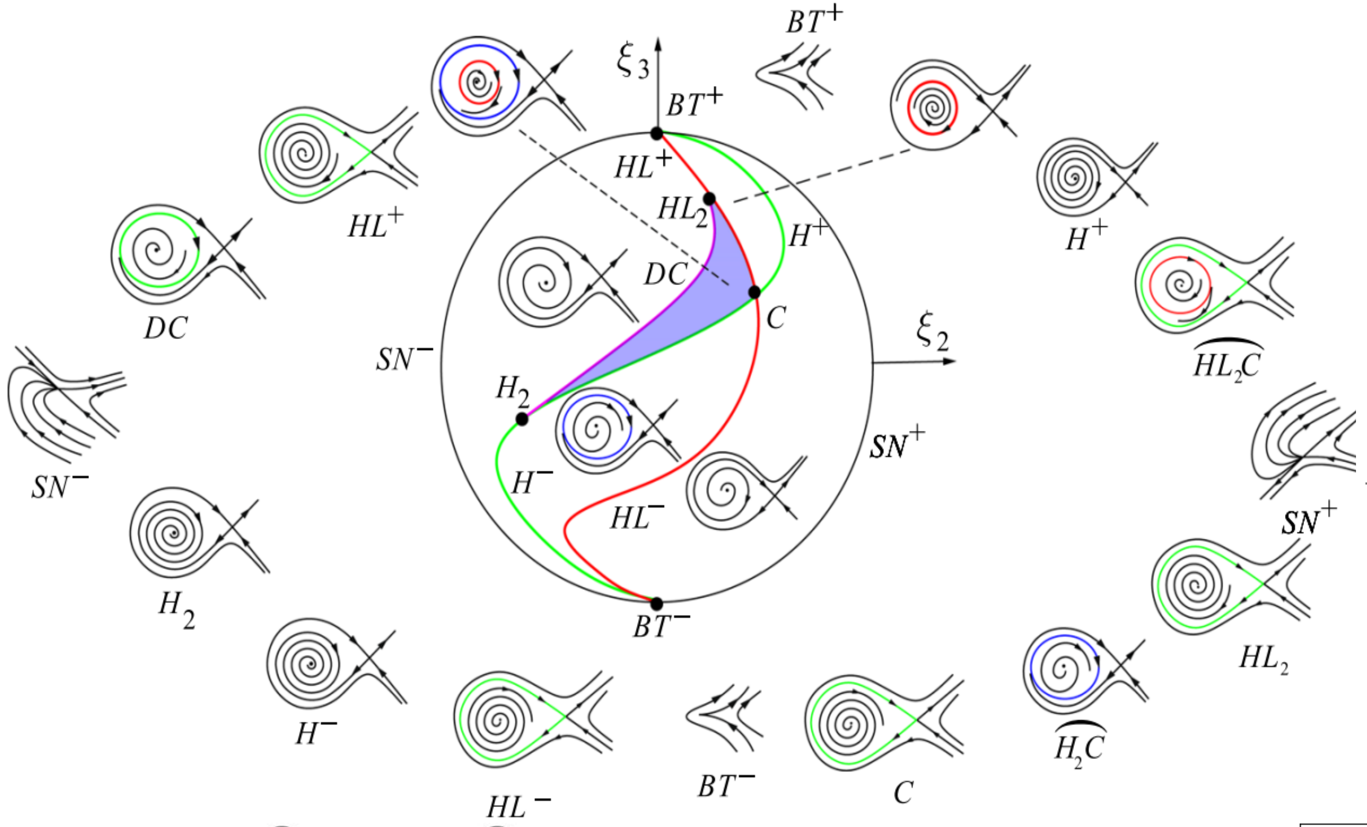
Sangam Dhakal

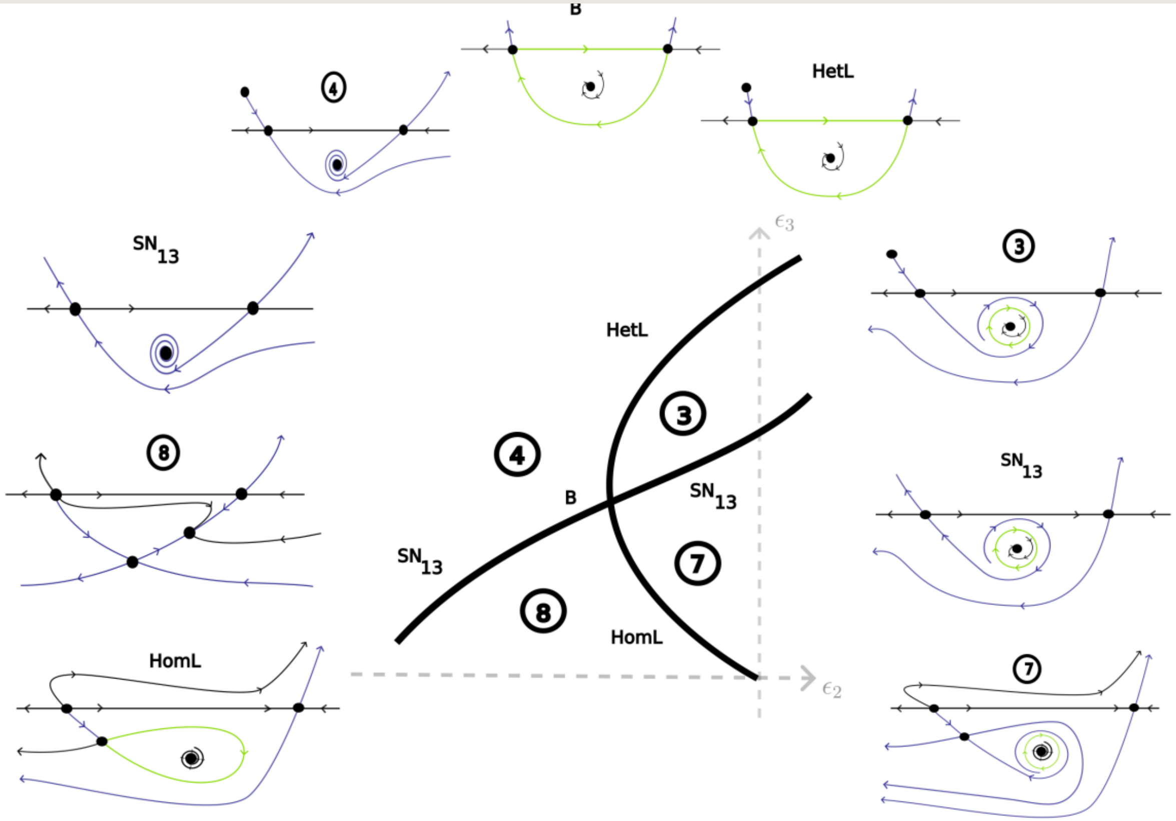
Steven Miller

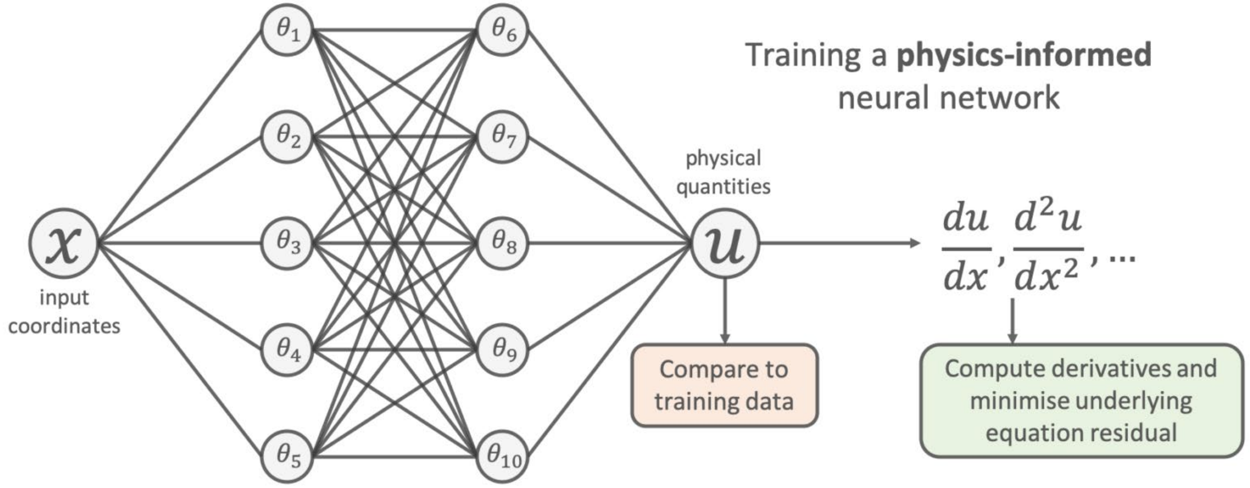


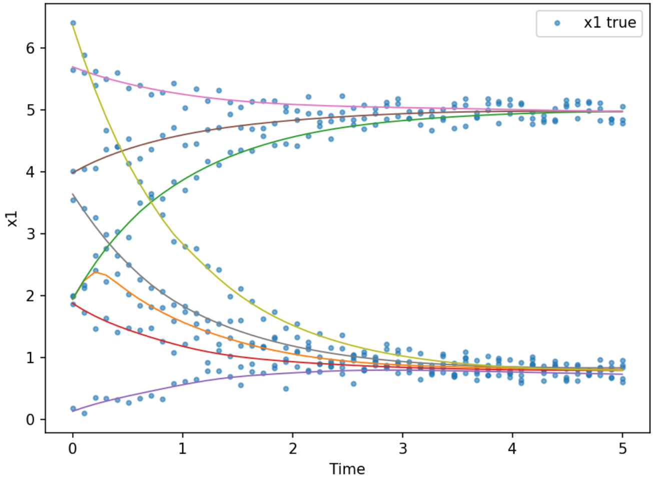
Wael El Khateeb

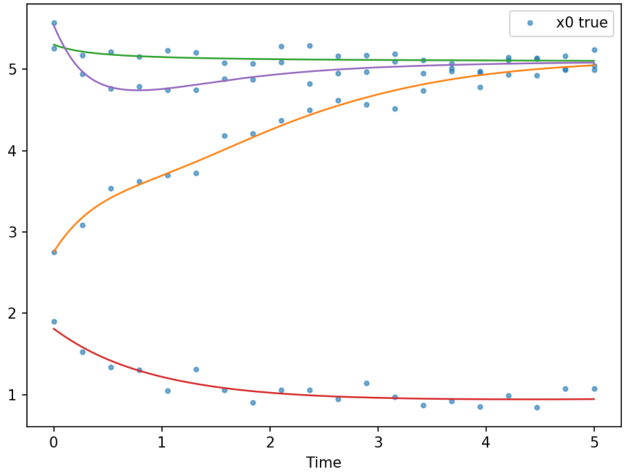
# My Background



**PHASE 1: Introduction**

How to guide students without prior differential equations experience to produce high-quality research in just six weeks?

We established a clear framework built on the following principles.

### Consistent and Timely Communication

- The average email response time, amazingly, did not exceed 30 minutes.

### Resricted Material

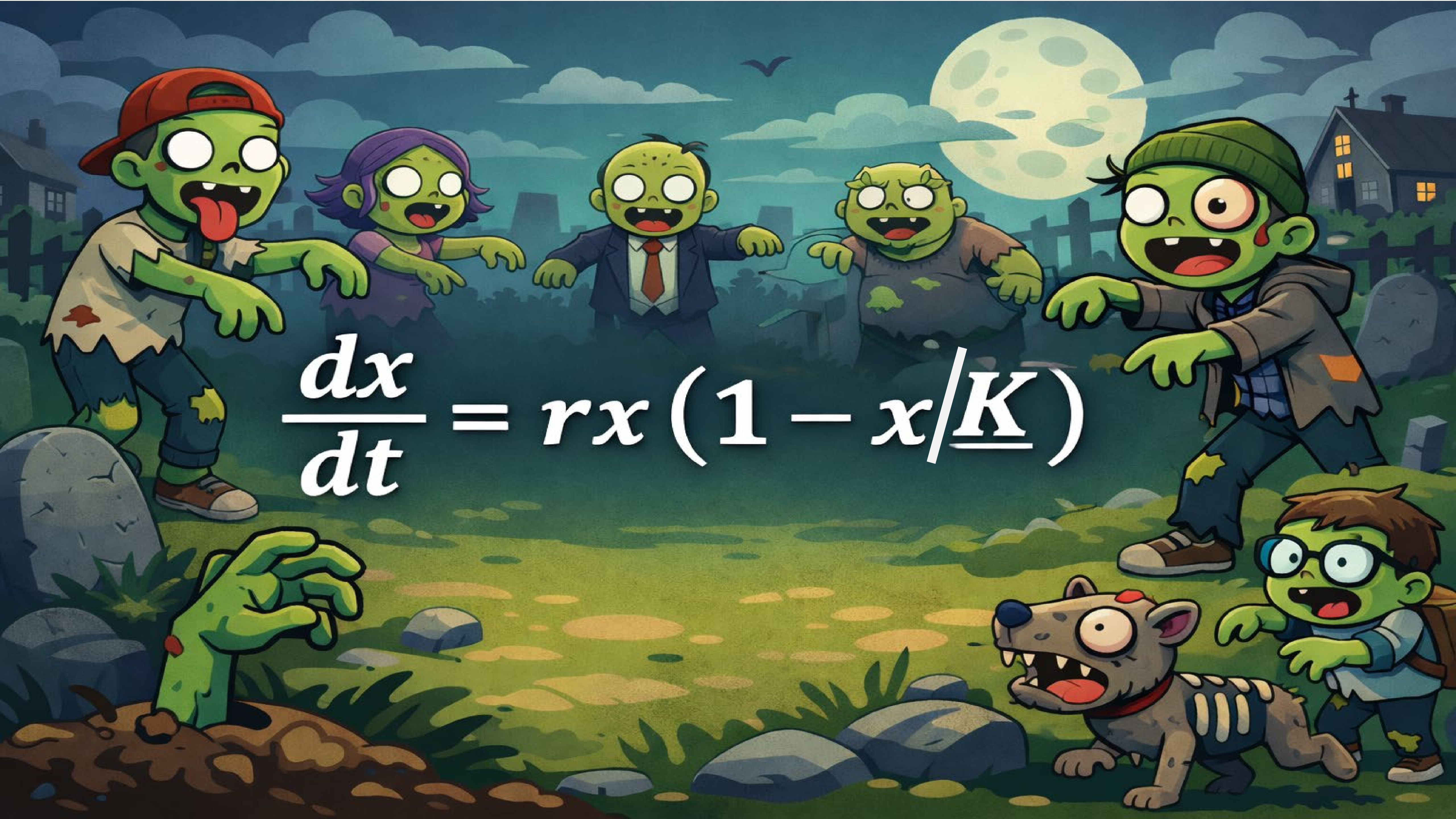
- Restriciting our sharing of knowledge to only the essential tools (System of ODEs, Jacobian, basic bifurcations,...).

### Weekly group meetings

- Complemented by one-on-one office hours.

### Positive Reinforcement and motivation

- Maintain high morale throughout the program to encourage confidence and sustained engagement.
- Motivating goal: All expense paid trip to UT Austin to present at a research conference.


$$\frac{dx}{dt} = rx\left(1 - x/K\right)$$

Find the values of the functions

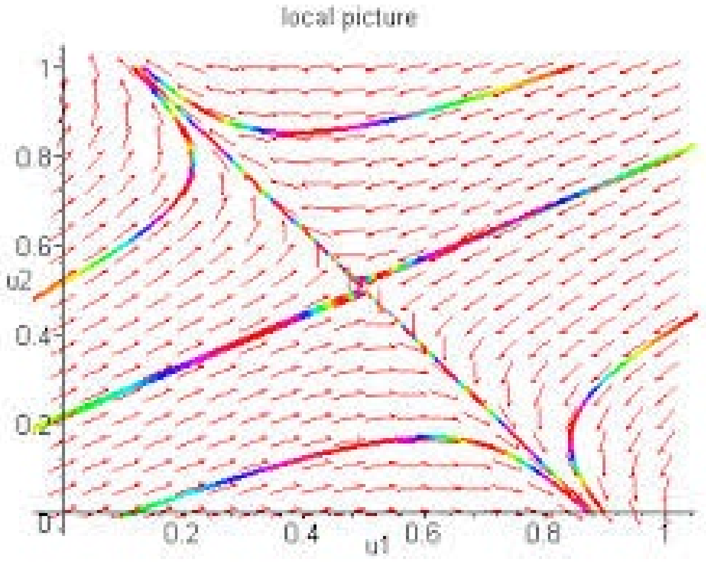
$$f_1(x) = x, f_2(x) = x^2, f_3(x) = x^3$$

near $x = 100, x = 1000, x = 10000$

Now do the same for

$$x = 0.01, x = 0.001, x = 0.0001$$

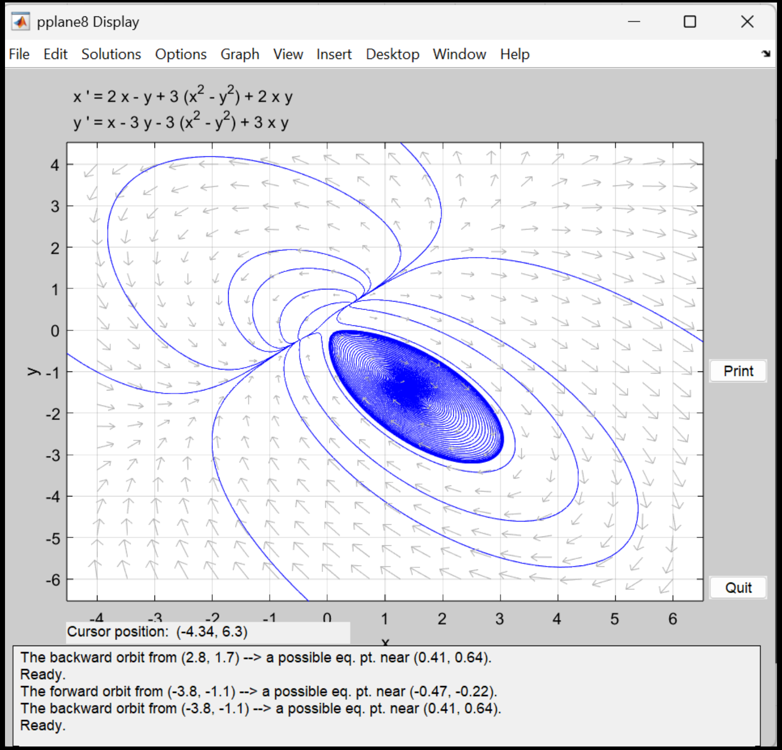
What can you conclude?

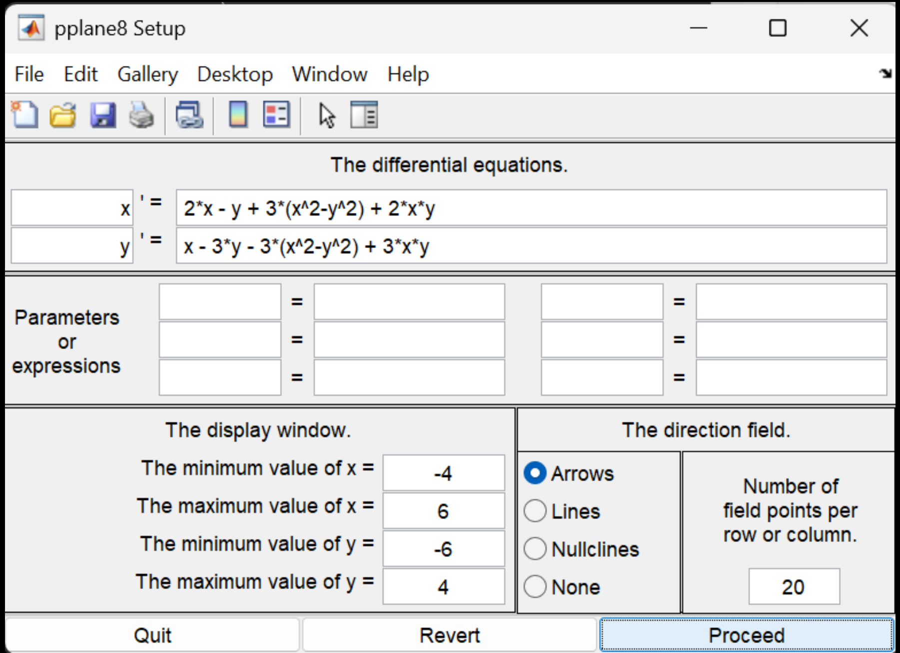


# Software tools

1) pplane
2) ODE 45

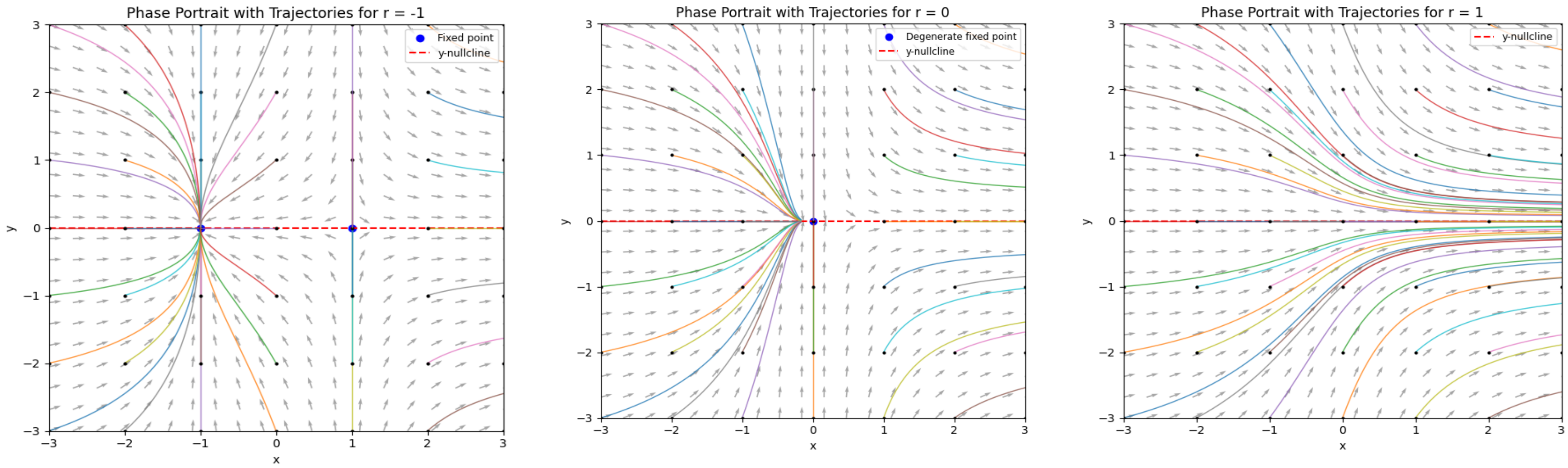
Well written code that students read, analyze, understand, and use instead of writing from scratch.



# Bifurcations: qualitative change in the behavior of a system as a parameter is varied.

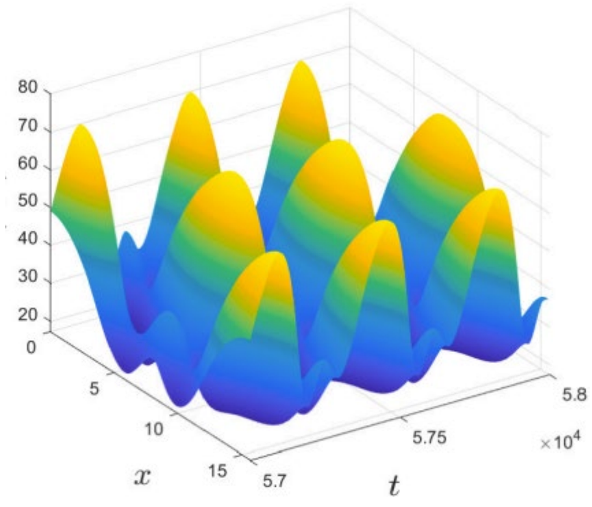
$$\begin{cases} \dfrac{dx}{dt} = r + x^2, \\ \dfrac{dy}{dt} = -y. \end{cases}$$



$$\begin{cases} \dfrac{dx(t)}{dt} = rx(t)\left(1 - \dfrac{x(t)}{K}\right) - \dfrac{mx(t)y(t)}{a + x(t)}, \\ \dfrac{dy(t)}{dt} = -dy(t) + e^{-d\tau}\dfrac{cmx(t-\tau)y(t-\tau)}{a + x(t-\tau)}, \end{cases}$$

Initial condition is a trajectory!

$$\begin{aligned} S_t - r_1\Delta S &= A - dS - \beta SI, \\ I_t - r_2\Delta I &= \beta SI - dI - h(I)I \\ R_t - r_3\Delta R &= h(I)I - dR, \end{aligned}$$



# BACKGROUND (BIOLOGY)

1800s to mid-1900s: Intense hunting for leather and meat.

1950s to 1960s: Populations crash; poaching persists; weak enforcement.

1967: Listed as endangered in the United States.

1970s: States close hunting, protect nests, begin science-based management.

# BACKGROUND (BIOLOGY)

Late 1970s to 1980s: Rebound via protection, anti-poaching, wetland conservation.

1987: Delisted federally; managed as "threatened due to similarity of appearance."

FWC lottery permits; two alligators per permit.

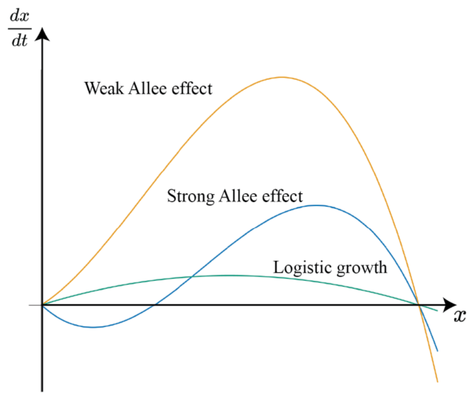

# Burmese Pythons

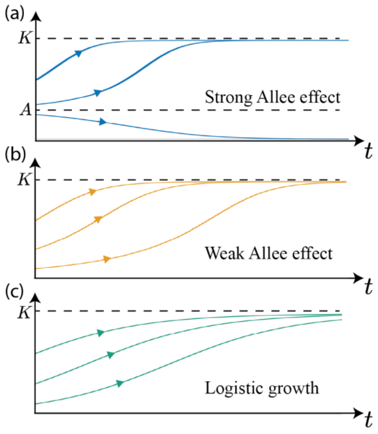
- Intentional releases by owners who could no longer keep large snakes.
- Accidental escapes from pet stores, breeders, and private collections.
- Facility damage during Hurricane Andrew (1992) likely released additional snakes into the Everglades.

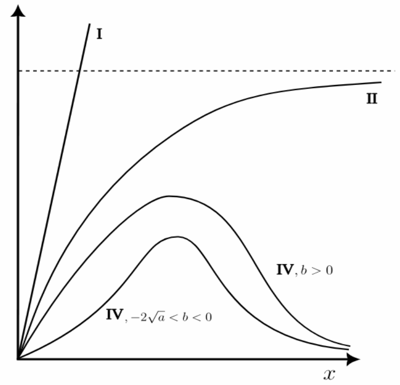
# ECOLOGICAL IMPLICATIONS

- Sharp declines of midsized mammals in parts of the Everglades (raccoons, opossums, marsh rabbits)
- Competition with native apex predators for prey, altering food webs

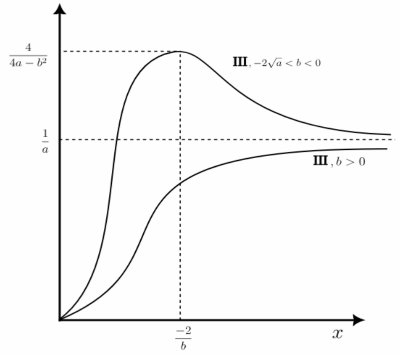
# PP model and extensions



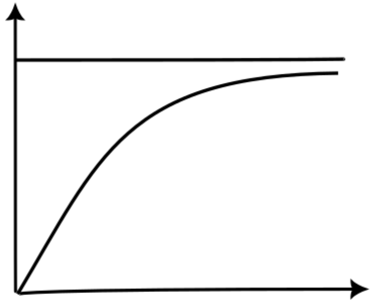

(a) Holling Type I,II, and IV



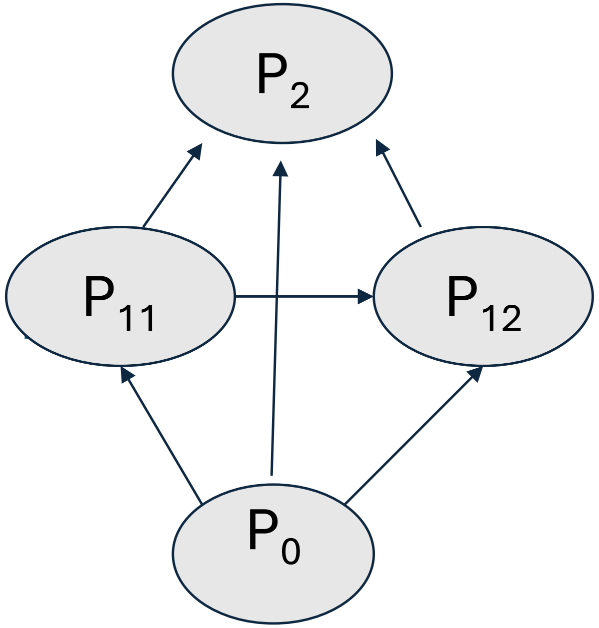

(b) Holling Type III

$$\frac{dx}{dt} = G_1(x) - \alpha_1 F(x)y - H_1(x)$$

$$\frac{dy}{dt} = -G_2(y) + \alpha_2 F(x)y - H_2(y)$$



# Model Illustrated:




$P_0$ - Raccoon/prey population

$P_{11}$ - Juvenile Alligator population

$P_{12}$ - Adult Alligator population

$P_2$ - Burmese Python population

# Initial Model

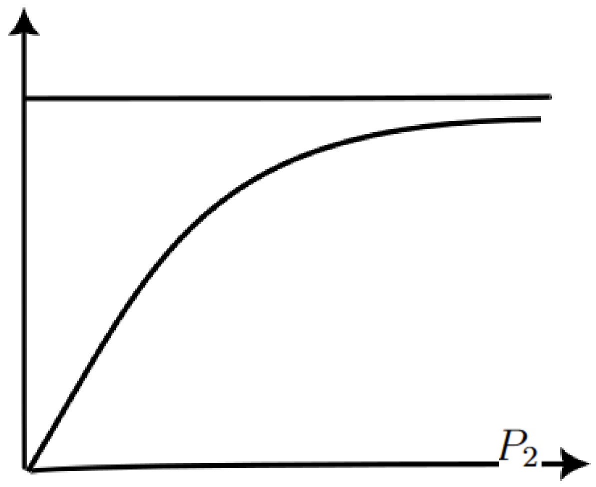
$$\begin{cases} \frac{dP_0}{dt} &= rP_0 - m_{20}P_0P_2 - m_{30}P_0P_2 \\ \frac{dP_1}{dt} &= c_{20}P_0P_2 - m_{31}P_1P_3 - d_{11}P_{11} \\ \frac{dP_2}{dt} &= d_{12}P_1 - d_2P_2 - h_2 \\ \frac{dP_2}{dt} &= c_{30}P_0P_3 + c_{31}P_3P_1 - d_3P_2 - h_3P_2 \end{cases}$$

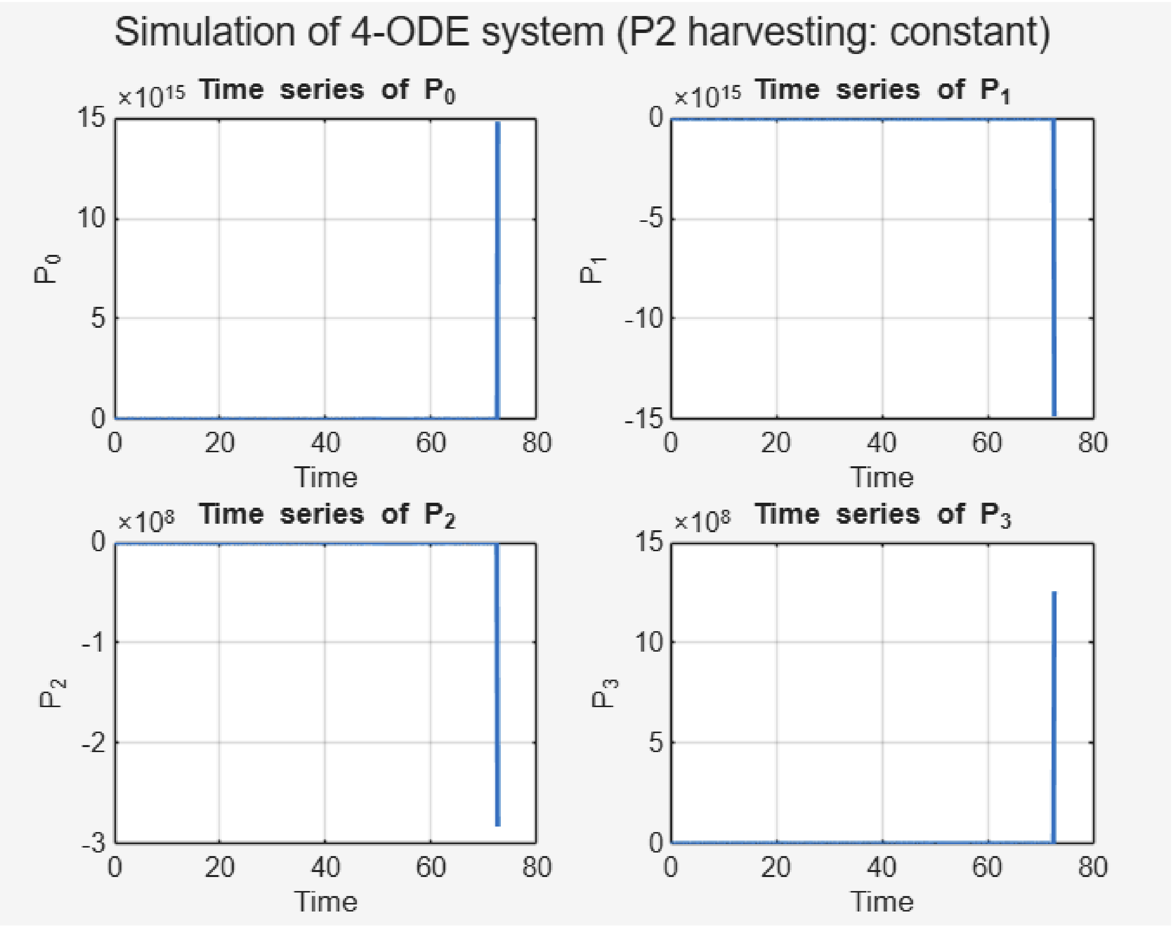
$h_2 \Rightarrow$ Saturated

$$\begin{cases} \frac{dP_0}{dt} &= rP_0 - m_{20}P_0P_{12} - m_{30}P_0P_2 \\ \frac{dP_{11}}{dt} &= c_{20}P_0P_{12} - m_{31}P_{11}P_3 - d_{11}P_{11} \\ \frac{dP_{12}}{dt} &= d_{12}P_{11} - d_2P_{12} - h_2\frac{P_{12}}{a+P_{12}} \\ \frac{dP_2}{dt} &= c_{30}P_0P_2 + c_{31}P_2P_{11} - d_3P_2 - h_3P_2 \end{cases}$$

# Why constant harvesting doesn't work

Applying constant harvesting shows that $P_1$ and $P_2$ eventually become **negative**, which is biologically unrealistic and highlights why **constant harvesting does not work.**



# Final Model

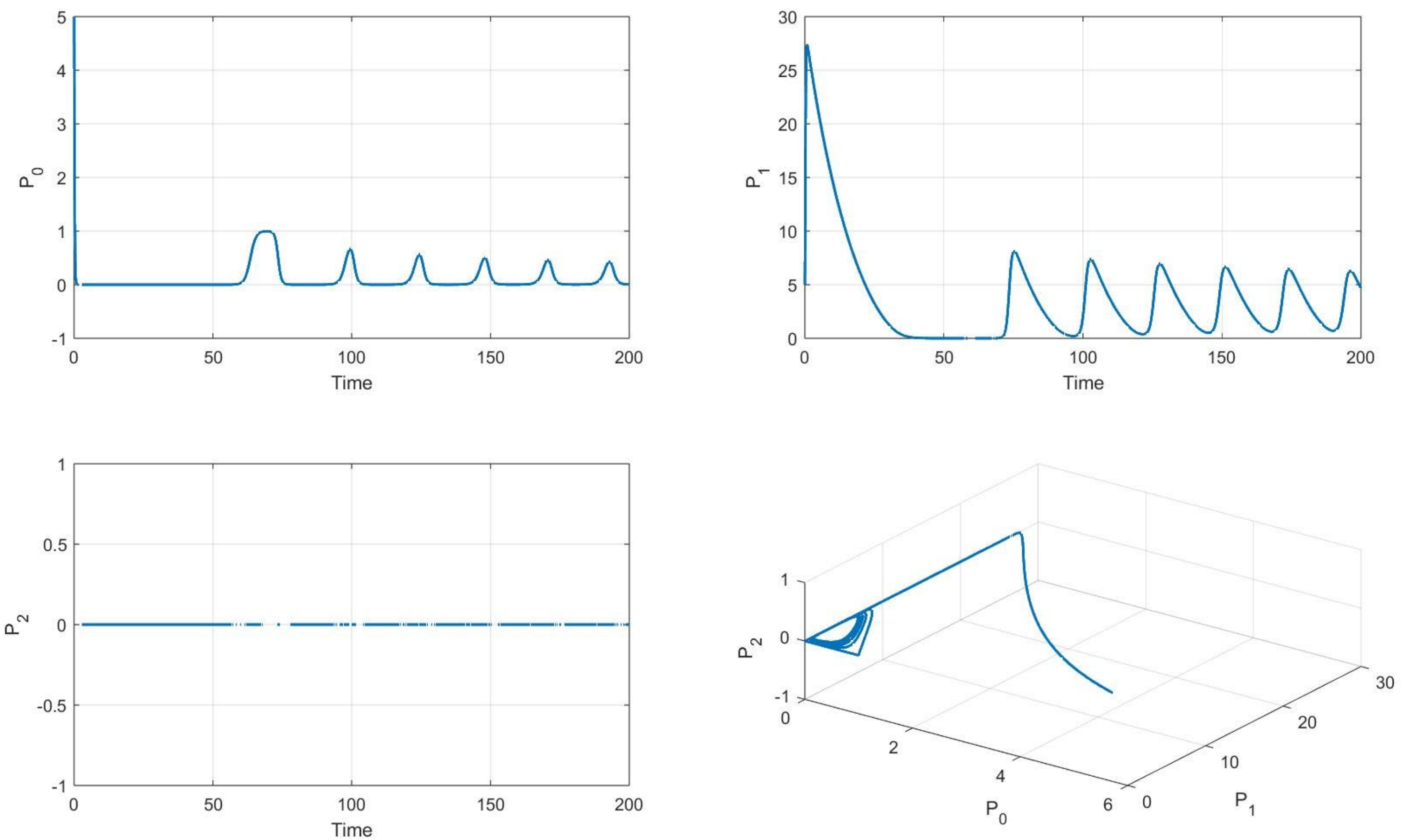
$$\begin{cases} \dfrac{dP_0}{dt} = rP_0\left(1 - \dfrac{P_0}{K}\right) - m_{10}P_0P_1 - m_{20}P_0P_2, \\ \dfrac{dP_1}{dt} = c_{10}P_0P_1 - d_1P_1 - \dfrac{h_1P_1}{a + P_1} - m_{31}P_1P_2, \\ \dfrac{dP_2}{dt} = c_{20}P_0P_2 + c_{21}P_1P_2 - d_2P_2 - h_2P_2. \end{cases}$$

## Equilibria: 7 in total

- 2 Equilibria on the $P_0$ axis.
- 2 equilibria on the $P_0 - P_1$ plane suggesting that the prey can coexist with the alligators.
- 2 non-zero equilibria where all 3 species coexist.
- 1 equilibrium in the $P_0 - P_2$ plane suggesting that python can coexist with the general prey.

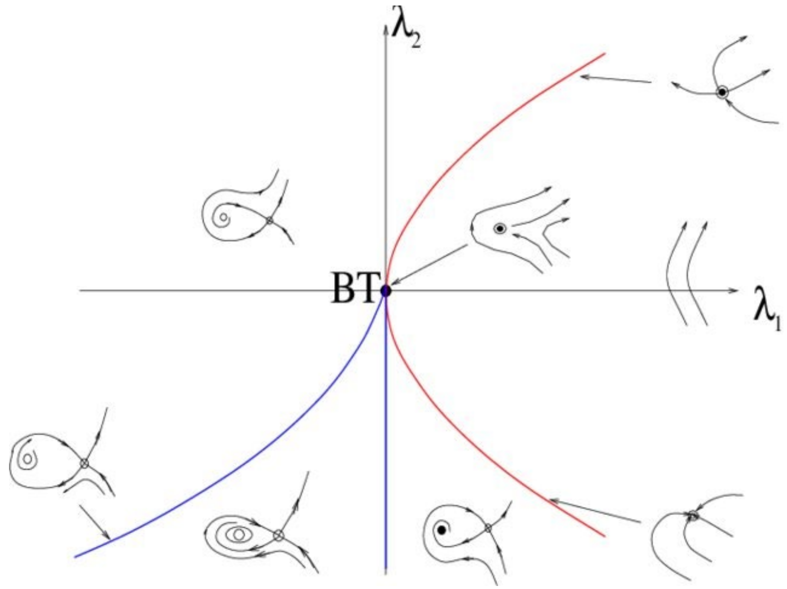
# THE CASE WITH NO BURMESE PYTHONS

- The system provides grounds to modeling the relation between Alligators and general prey in the absence of pythons.



What happens if both Hopf and Saddle node occur simultaneously?

Students were able to do so using Bogdanov takens bifurcations, which explained to formation of limit cycles




1. **Shift the equilibrium:** Find the equilibrium $P^* = (P_0^*, P_1^*, P_2^*)$ and shift variables:

$$u = P_0 - P_0^*, \quad v = P_1 - P_1^*, \quad w = P_2 - P_2^*$$

so that the system becomes $\dot{z} = F(z)$ with $F(0) = 0$.

2. **Linearize and identify eigen-directions:** Compute the Jacobian $J = DF(0)$. At the BT point: $\mathrm{spec}(J) = \{0, 0, \lambda_s\}$ with $\lambda_s < 0$. The eigenvectors corresponding to zero eigenvalues define the center subspace.

3. **Center manifold reduction (3D → 2D):** There exists a smooth invariant manifold $w = h(u, v)$ with $h(0) = 0$, $Dh(0) = 0$. Substituting $w = h(u, v)$ reduces the system to:

$$\dot{u} = \tilde{F}_1(u, v), \quad \dot{v} = \tilde{F}_2(u, v)$$

4. **Reduced dynamics:** Expand $\tilde{F}_1, \tilde{F}_2$ in a Taylor series near the origin and retain quadratic terms:

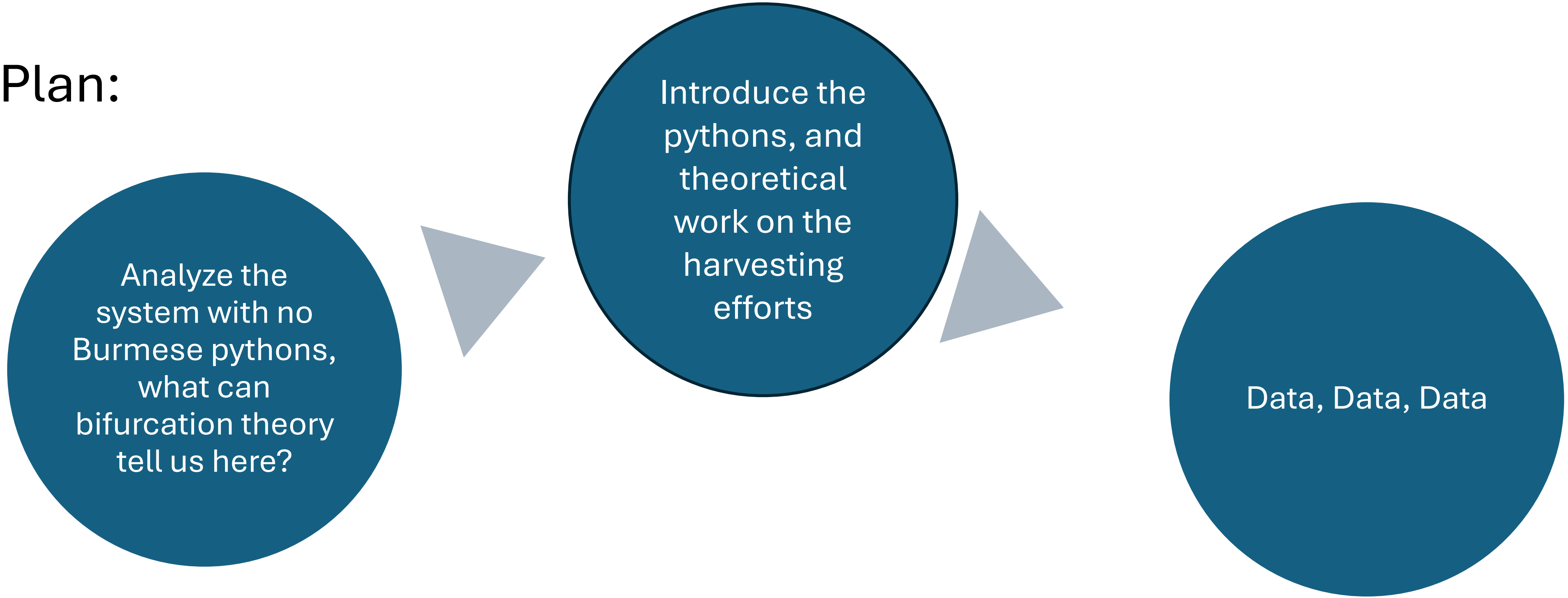
$$\dot{u} = v + \mathcal{O}(2), \quad \dot{v} = au^2 + buv + \cdots$$

5. **Transform to BT normal form:** Apply smooth coordinate and parameter changes to obtain:

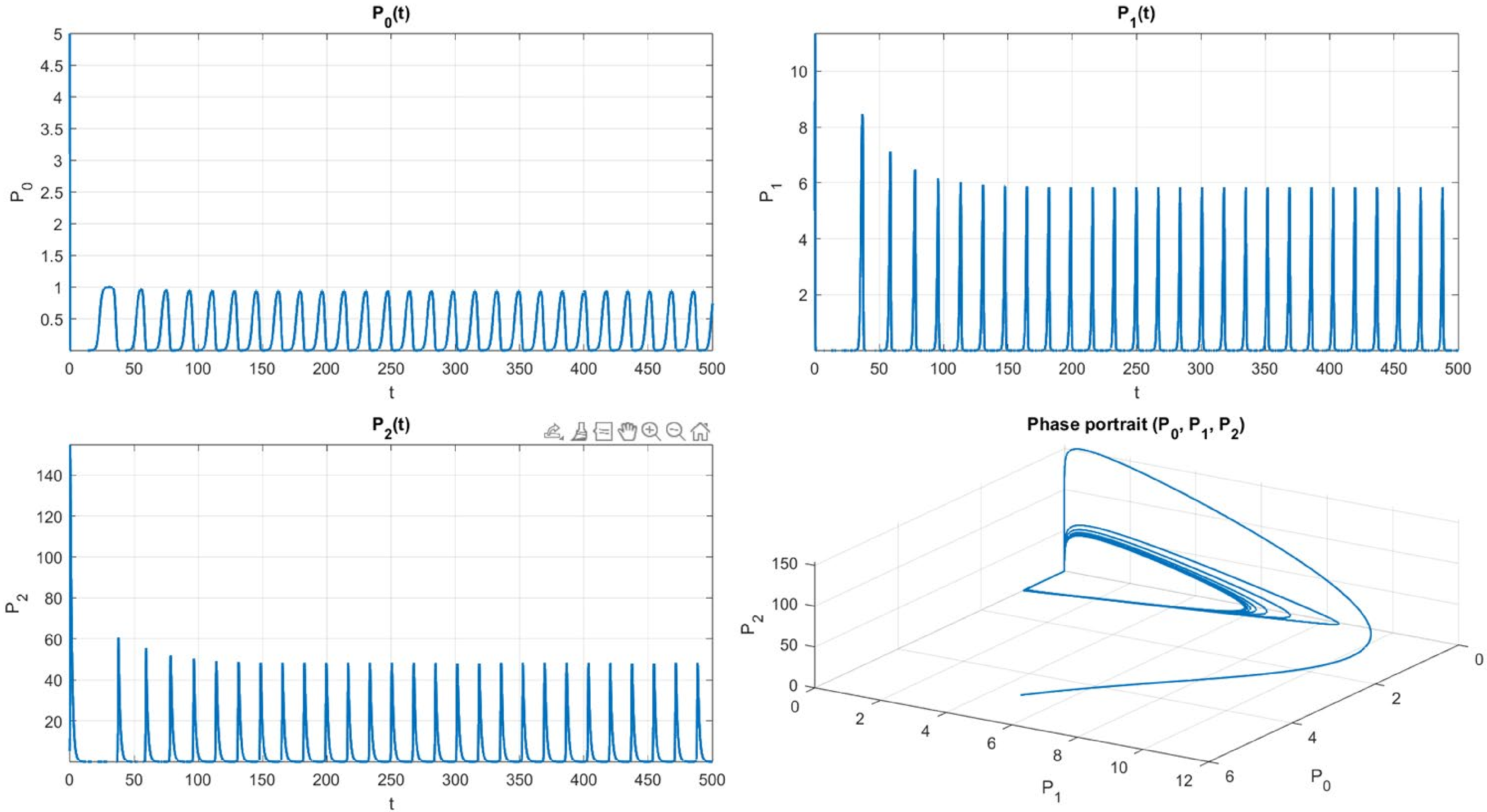
$$\boxed{\dot{x} = y, \quad \dot{y} = \beta_1 + \beta_2 x + ax^2 + bxy + \mathcal{O}(3)}$$

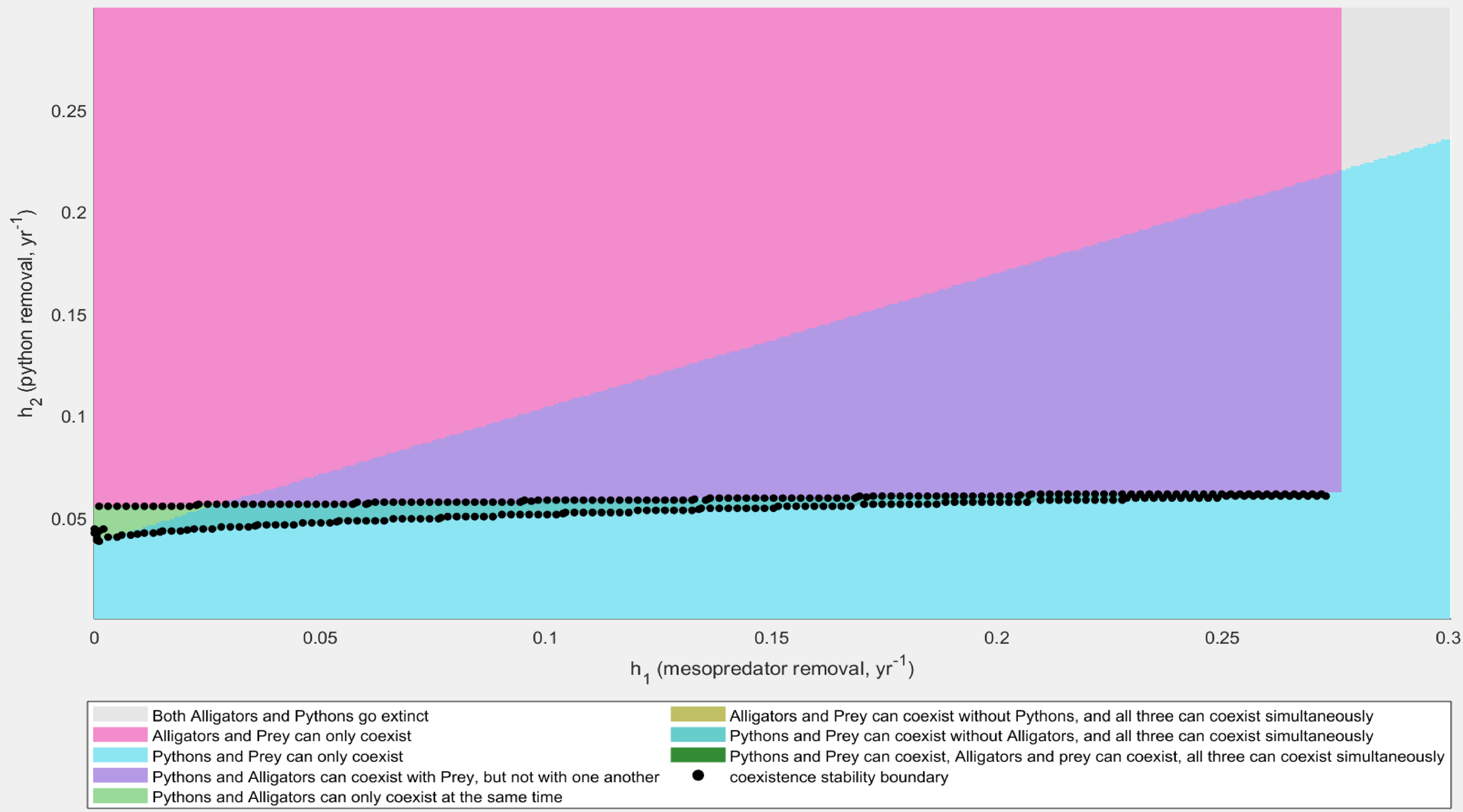
# Ultimate goal: Policy making

- With current harvesting restricted to 2 alligators per hunter, per season, should the hunting policies be altered, especially that the common prey population is decreasing (Racoons drop by 95%...)
- Plan:



# The presence of Burmese Pythons





0.25
0.2
0.15
0.1
0.05
h2 (python removal, yr-1)
0
0.05
0.1
0.15
0.2
0.25
0.3
h1 (mesopredator removal, yr-1)
Both Alligators and Pythons go extinct
Alligators and Prey can only coexist
Pythons and Prey can only coexist
Pythons and Alligators can coexist with Prey, but not with one another
Pythons and Alligators can only coexist at the same time
Alligators and Prey can coexist without Pythons, and all three can coexist simultaneously
Pythons and Prey can coexist without Alligators, and all three can coexist simultaneously
Pythons and Prey can coexist, Alligators and prey can coexist, all three can coexist simultaneously
coexistence stability boundary

The Croc Docs
Wildlife Research Team

UF | IFAS Extension
UNIVERSITY of FLORIDA

Fort Lauderdale
Research and
Education Center

UT
THE UNIVERSITY OF
TOLEDO
1872

# SIAM TX-LA Conference and JMM 2026


# Useful techniques/ Strategies implemented

- **Ownership**
- **Reaching out to contacts**
- **Know your strong points, know your weak points**
- **Always have something to share**
- **Provide opportunities, but students must seize**