DOUBLE PLUS UNGOOD:
From Fibonacci to Roulette and Modeling

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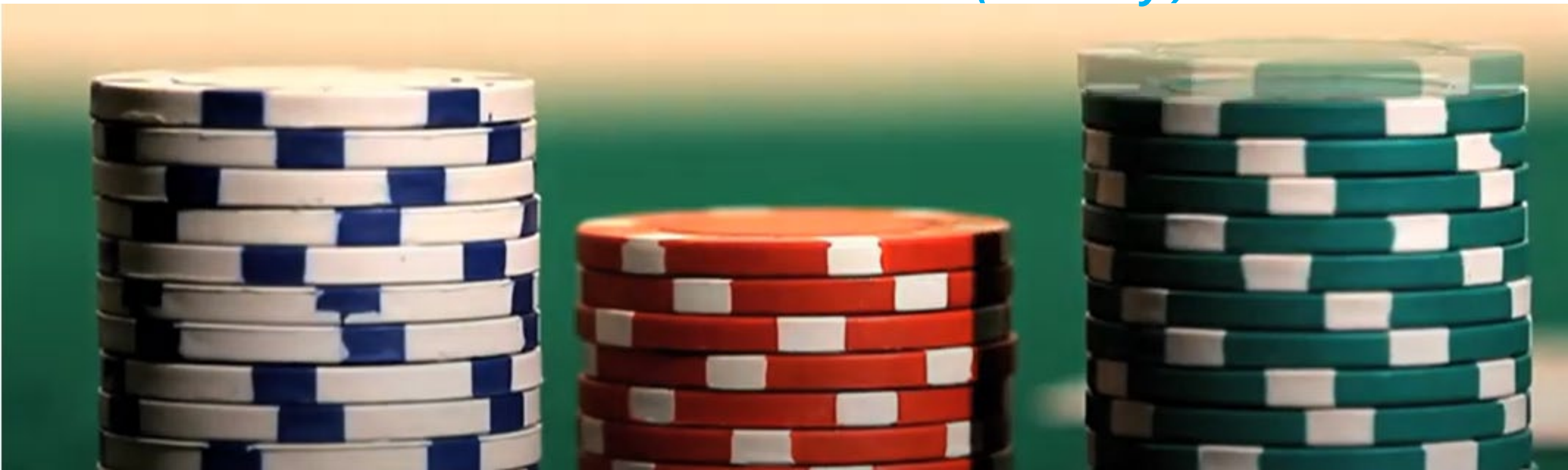
https://web.williams.edu/Mathematics/sjmiller/public_html/

SIMIODE EXPO CONFERENCE 2025 (February)



CREDITS https://www.youtube.com/watch?v=Q10_srZ-pb

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Producer: Adam Wang

Animator: Galen Corey

Cinematographers: Ralston Louie & Dhyan Adler-Belendez

Editor: Ralston Louie

GOALS

- ***Excite students about mathematics.***
- ***Introduce data collection and modeling.***
- ***Discuss simulation versus theory.***

WHAT TO STUDY?

Danger of same examples (pure math, physics, sports).

Widen appeal: last year naval warfare, this year gambling!

https://as1.ftcdn.net/v2/jpg/03/30/51/20/1000_F_330512072_w4tRMY9zKanDHGsGxbP5RttlReZCPGEC.jpg

ROULETTE


0
00
1 to 18
1st 12
2nd 12
3rd 12
EVEN
ODD
19 to 36
2 to 1
2 to 1
2 to 1
0:47 / 6:42 • Introduction >

Double Plus One:

Bet $1 on red, if red up $1, else down $1.

If lost bet $2 red, if red up $1, else down $3.

If lost bet $4 red, if red up $1, else down $4…

Double Plus One: Analysis

Eventually win, what can go wrong?

Let students think….

Write code to explore.

```
probbetexceedlimit[numspins_, p_, targetnum_, numdo_] := Module[{},
  exceed = 0;
  For[n = 1, n ≤ numdo, n++,
   {
    nonred = 0;
    For[s = 1, s ≤ numspins, s++,
     {
      If[Random[] ≤ p, nonred = 0, nonred = nonred + 1];
      If[nonred == targetnum,
       {
        exceed = exceed + 1;
        s = numspins + 10;
       }]; (* end of if statement for hitting targetnum*)
     }]; (* end of spin s loop *)
   }]; (* end of n loop *)
  percentexceed = 100.0 exceed / numdo;
  Print["numspins = ", numspins, " and percent exceed = ", percentexceed, "%."];
  Return[{numspins, percentexceed}];
 ]
```

```
extrapolateprobexceedlimit[start_, numjumps_, pp_, ttargetnum_, nnumdo_] := Module[{},
   results = {};
   For[j = 0, j ≤ numjumps - 1, j++,
    results = AppendTo[results, probbetexceedlimit[start * 2^j, pp, ttargetnum, nnumdo]];
   ]; (* end of j loop *)
   Print["Plot of results"];
   Print[ListPlot[results]];
   Print["Log-Log Plot of results"];
   Print[ListPlot[Log[results]]];
   oneminusresults = {};
   For[k = 1, k ≤ Length[results], k++,
    {
     temp = {results[[k, 1]], Log[100 - results[[k, 2]]]};
     oneminusresults = AppendTo[oneminusresults, temp];
    }]; (* end of k loop*)
   Print["Semi-Log Plot of 100 - results"];
   Print[ListPlot[oneminusresults]];
  ];
```

extrapolateprobexceedlimit[start_, numjumps_, pp_, ttargetnum_, nnumdo_]
extrapolateprobexceedlimit[20,11, .5,**10**,1000]

```
numspins = 20 and percent exceed = 0.4%.
numspins = 40 and percent exceed = 1.4%.
numspins = 80 and percent exceed = 3.5%.
numspins = 160 and percent exceed = 7.2%.
numspins = 320 and percent exceed = 15.8%.
numspins = 640 and percent exceed = 26.9%.
numspins = 1280 and percent exceed = 44.2%.
numspins = 2560 and percent exceed = 72.8%.
numspins = 5120 and percent exceed = 91.8%.
numspins = 10240 and percent exceed = 99.5%.
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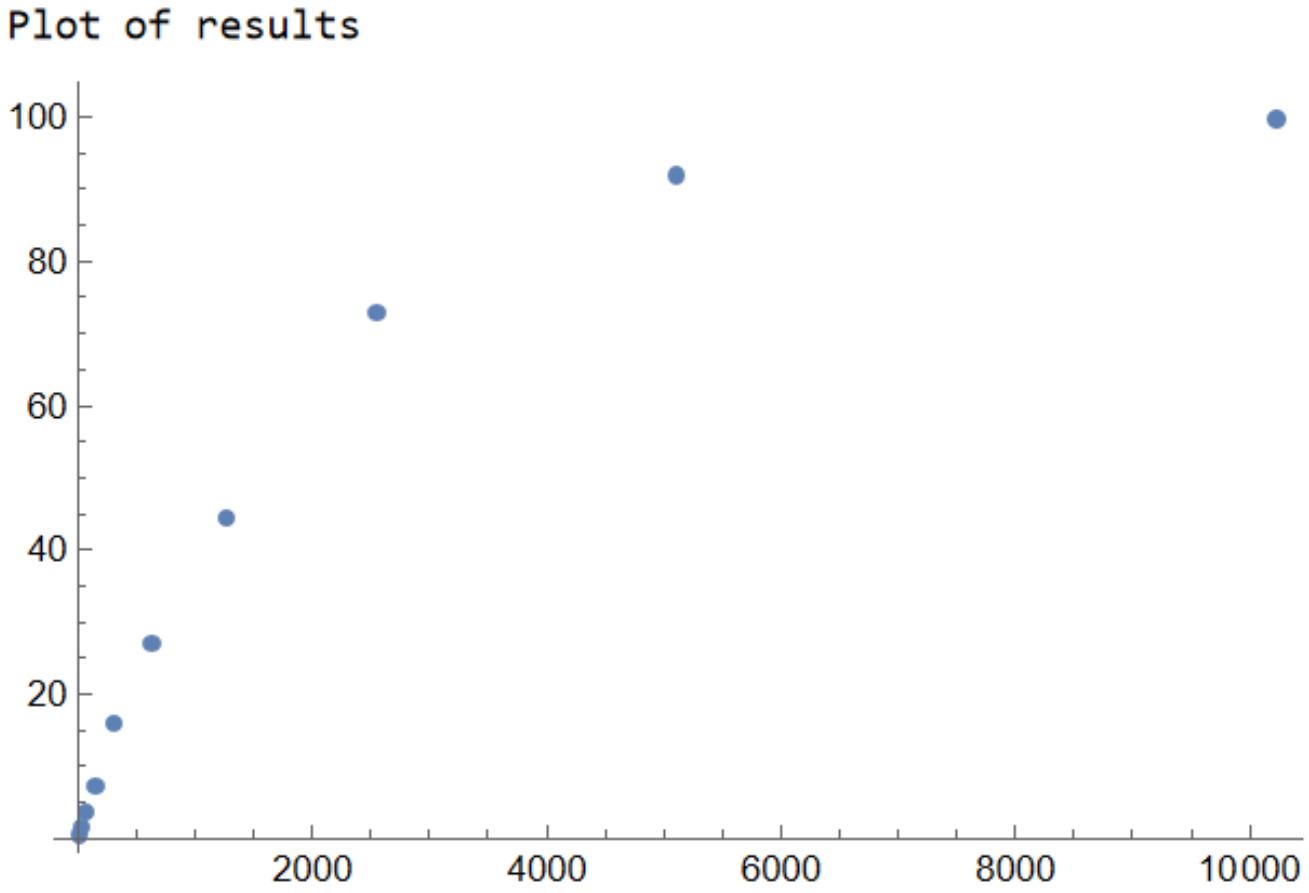

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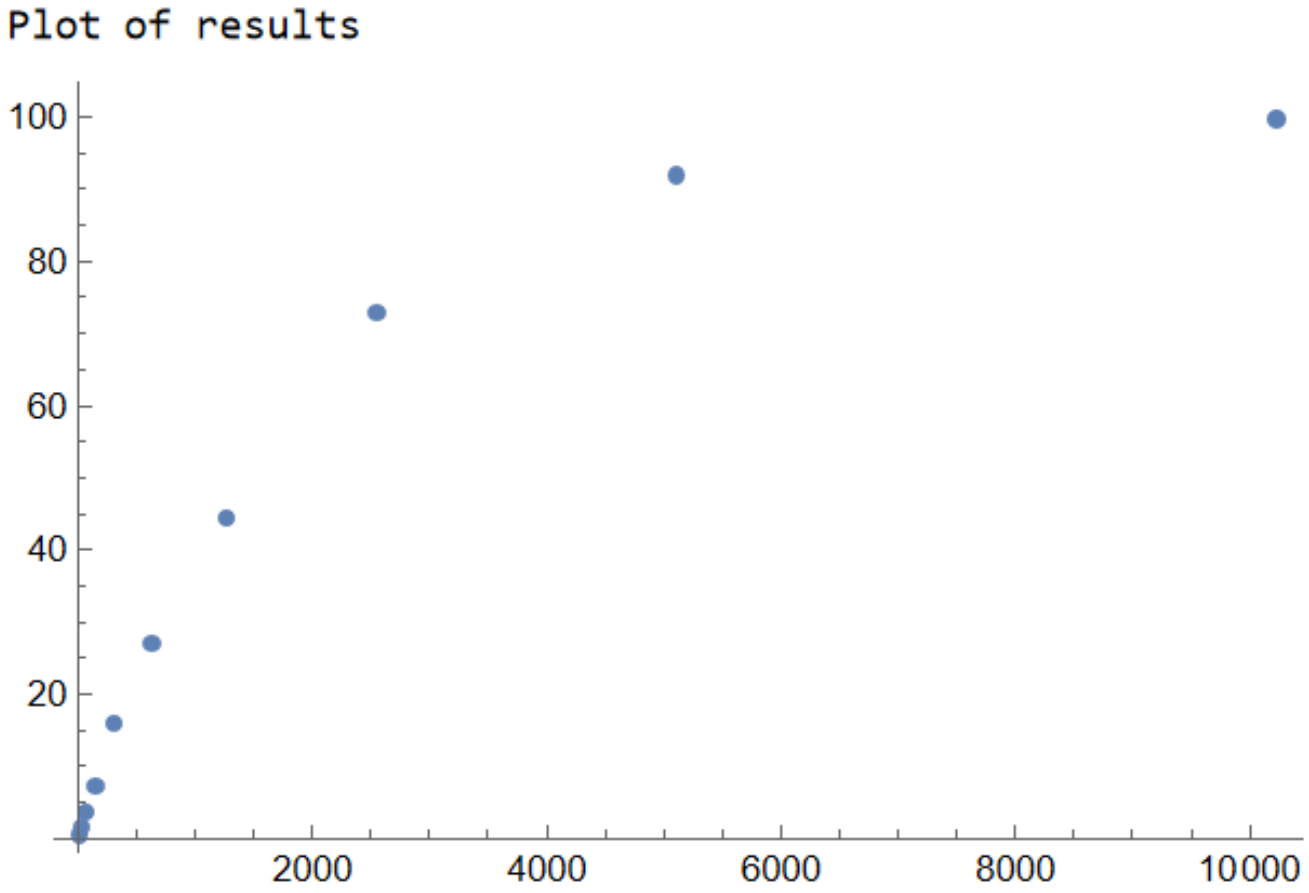

NOTE: Not solving 'right' problem: can exceed the limit if up….

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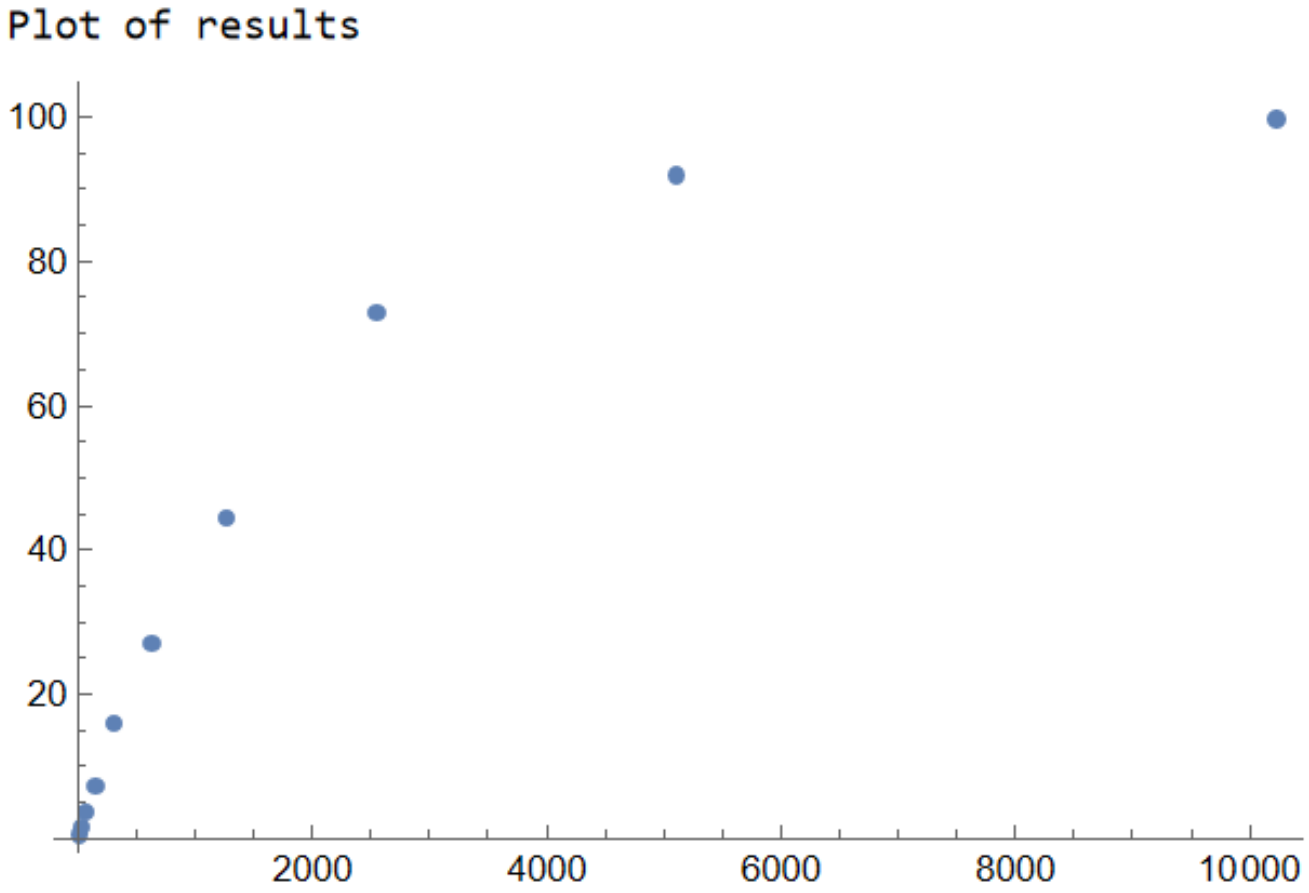

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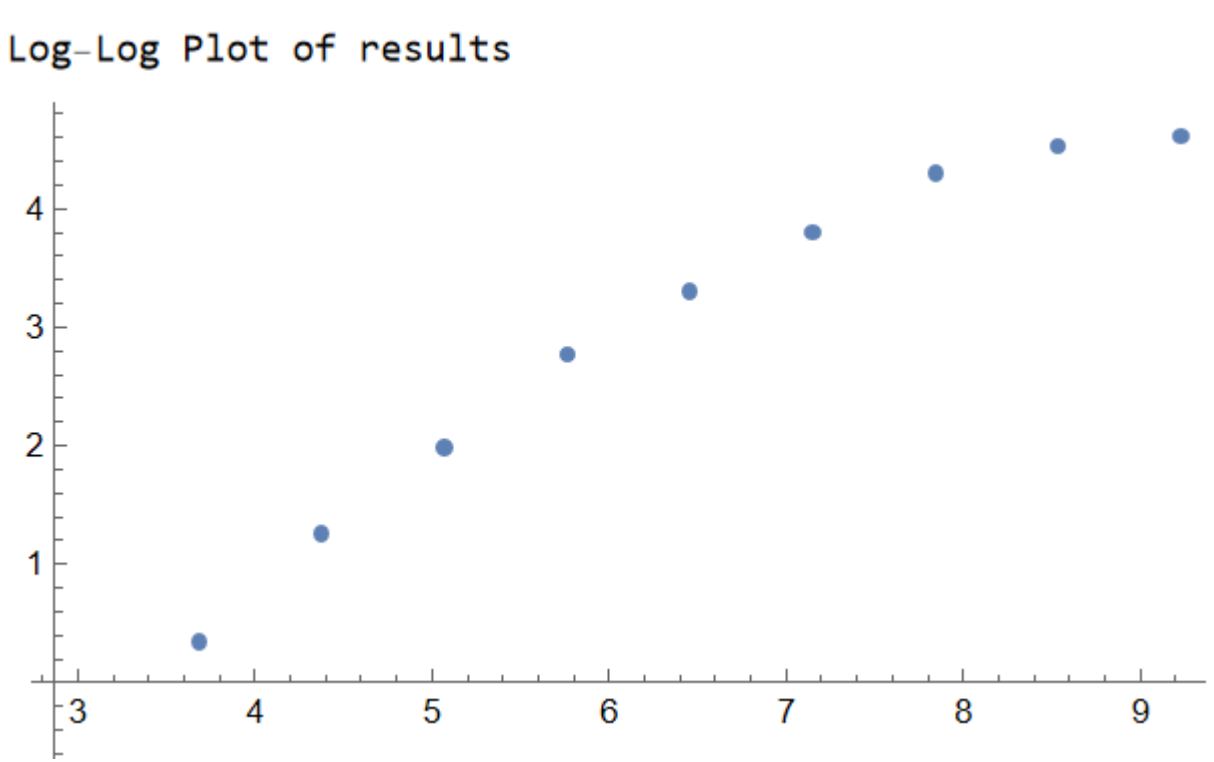
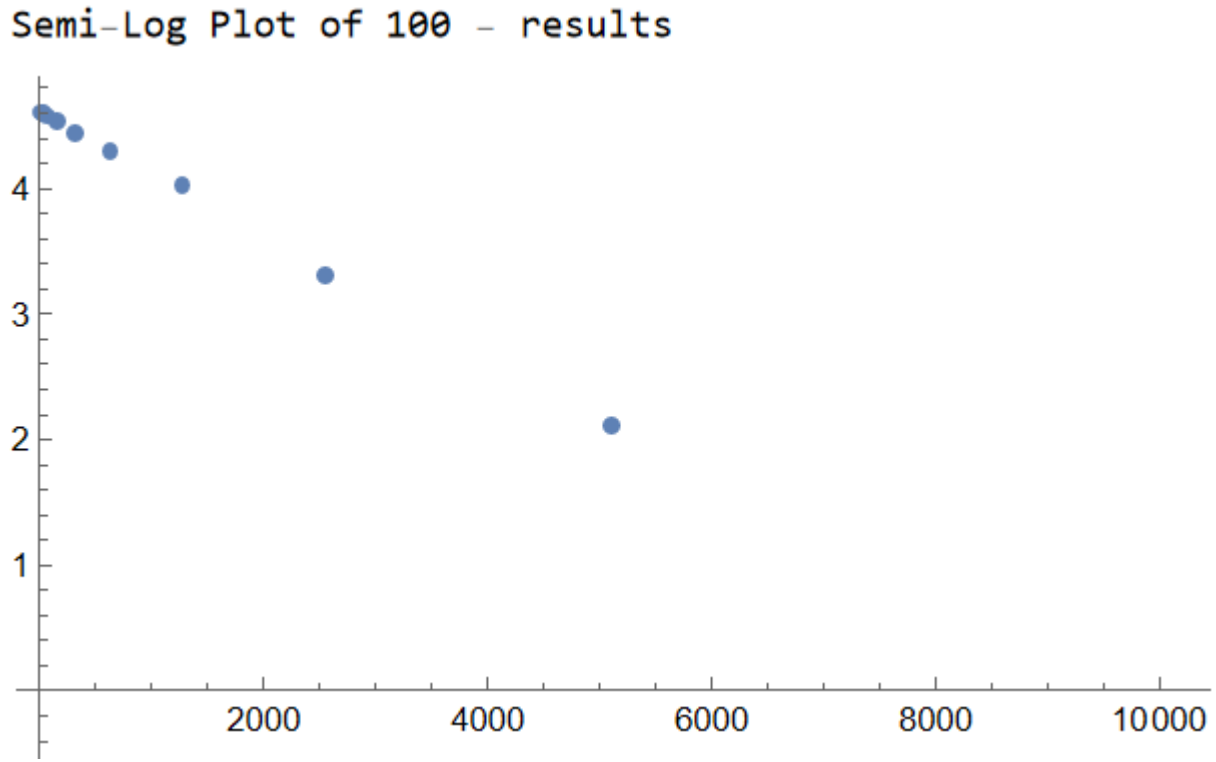
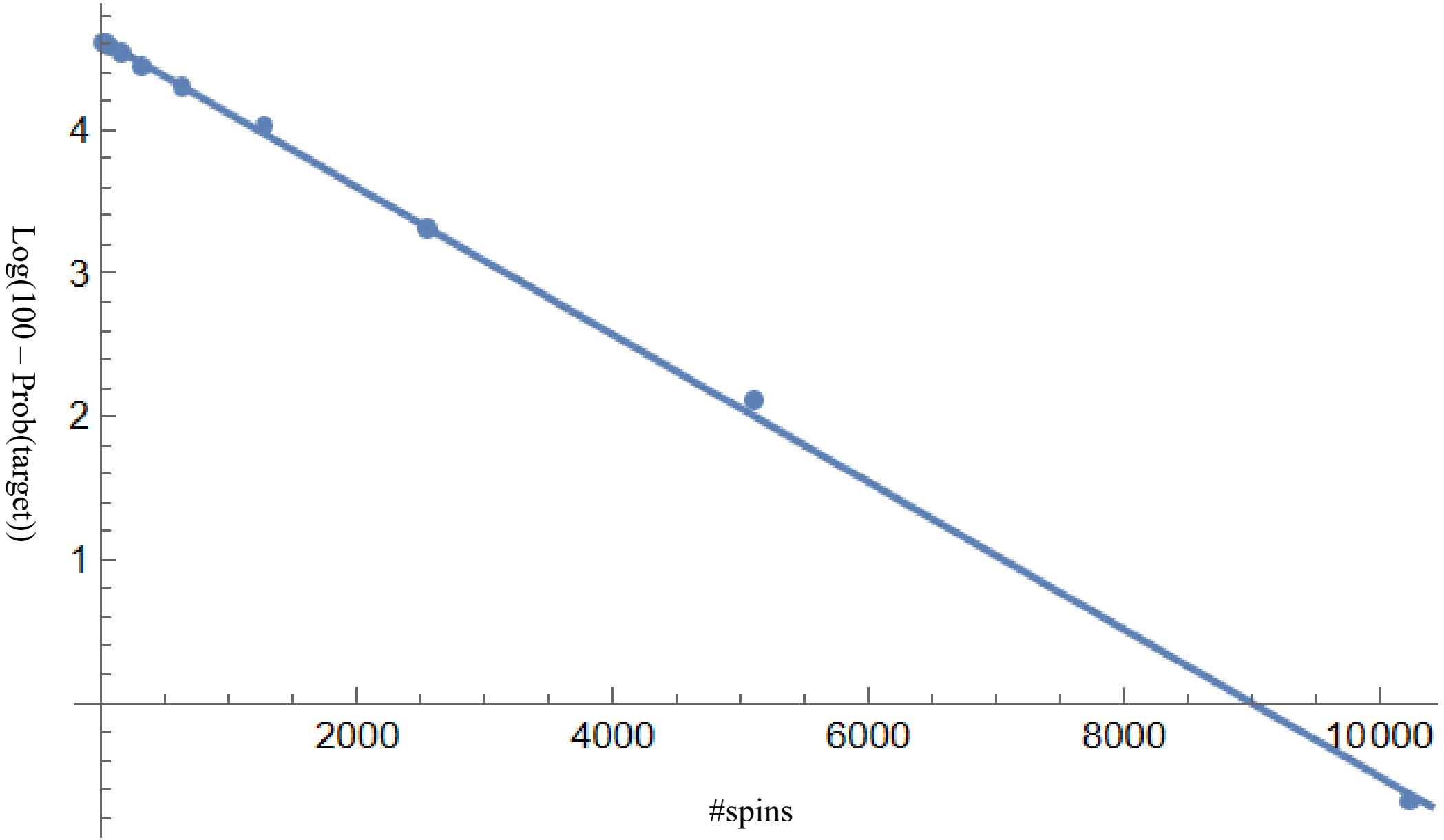


#spins



#spins

LinearModelFit[oneminusresults, x, x] (from 10,000 simulations for 10 geometrically spaced #spins)
Show[ListPlot[oneminusresults], Plot[4.626850562937115 -0.0005139433749086 72 x, {x,0,10400}]]



PREDICTION VS EXPERIMENT

Show[ListPlot[oneminusresults], Plot[4.626850562937115 -0.0005139433749086 72 x, {x,0,10400}]]

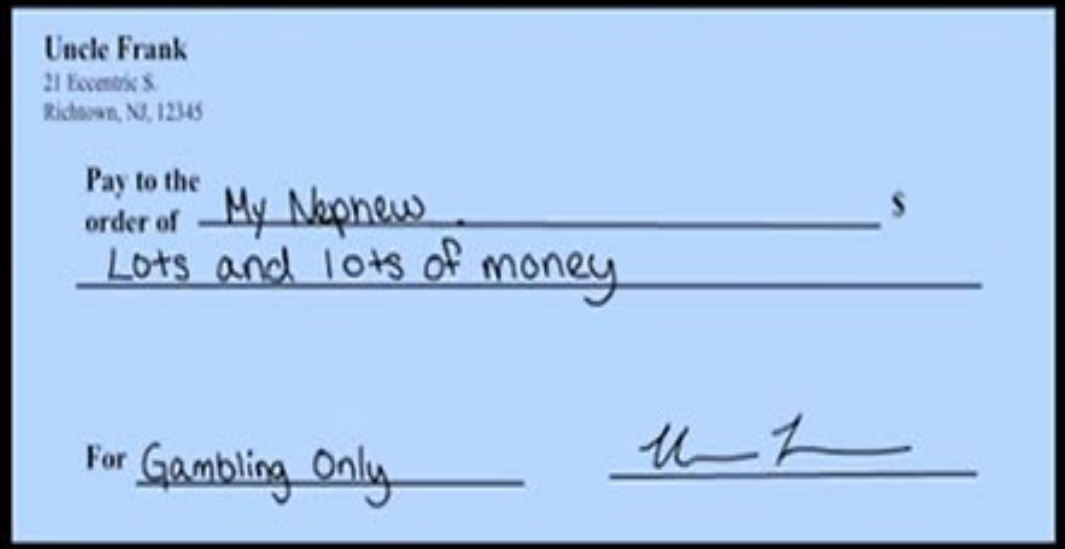
With target of 10 consecutive non-red:

#spins	Simulation	Best-Fit Line
15	0.32%	-1.41%
200	8.93%	7.79%
1000	38.48%	38.88%
20000	99.994%	99.996%

(100,000 simulations for each, used 10,000 to find best fit line, geometrically from 20 to 10,240)

# of times black is Rolled	Next Bet
1	$2
2	$4
3	$8
4	$16
5	$32
6	$64
7	$128
8	$256
9	$512
10	$1024
11	$2048
12	$4096
13	$8192
14	$16384
15	$32768
16	$65536
17	$131072

ERU/AH:
The Eccentric Rich Uncle/Aunt Hypothesis



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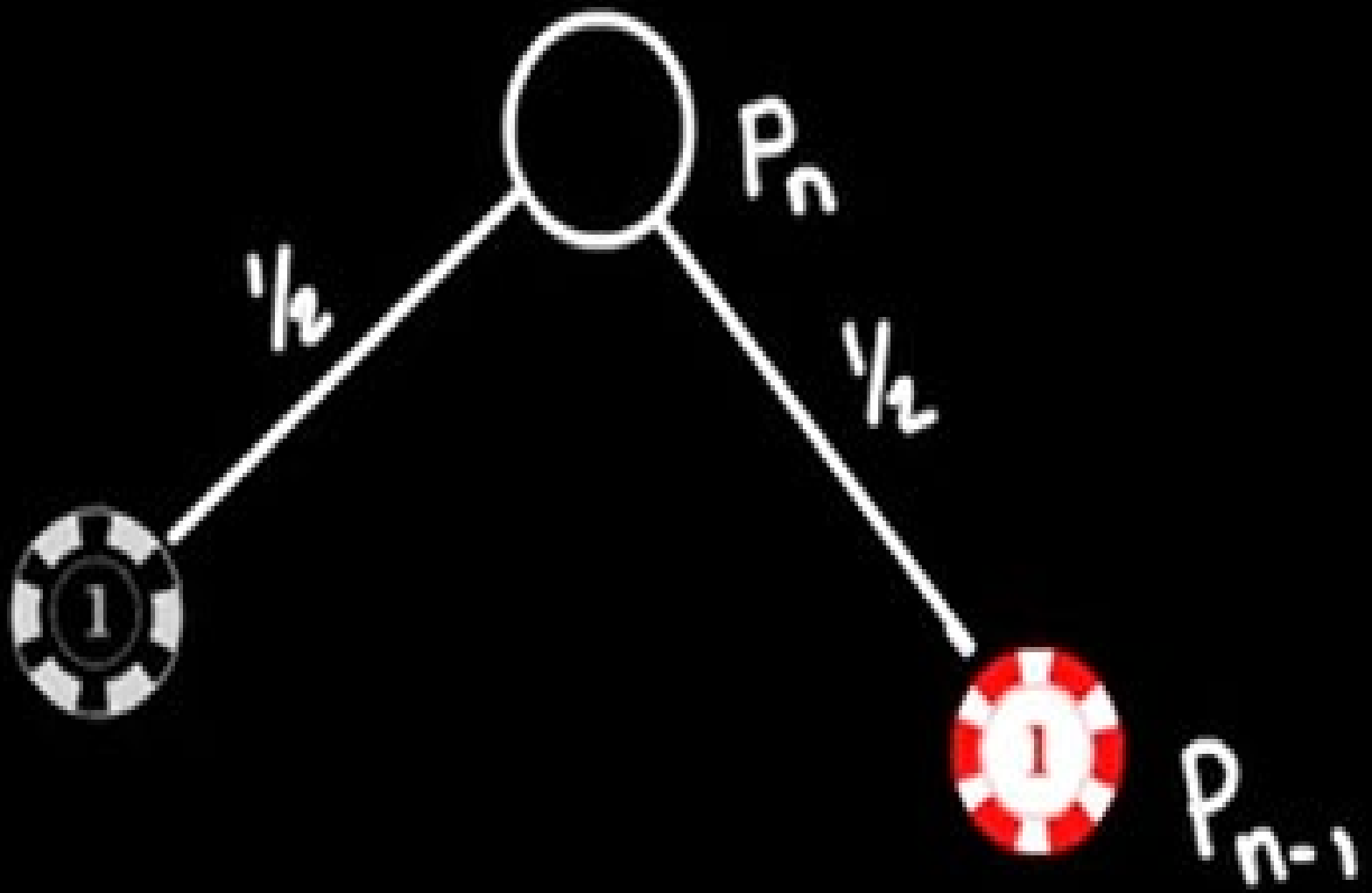
12	$4096
13	$8192
14	$16384
15	$32768
16	$65536
17	$131072
18	$262144
19	$524288

Table Limit

- 5 blacks will make us bankrupt
- P_n = probability we will <u>not</u> get 5 consecutive blacks in N spins
- Q_n = probability we <u>will</u> get 5 consecutive blacks in N spins

$$P_n + Q_n = 1$$


P_n
½
½
1
1
P_{n-1}

- 5 blacks will make us bankrupt
- P_n = probability we will not get 5 consecutive blacks in N spins
- Q_n = probability we will get 5 consecutive blacks in N spins

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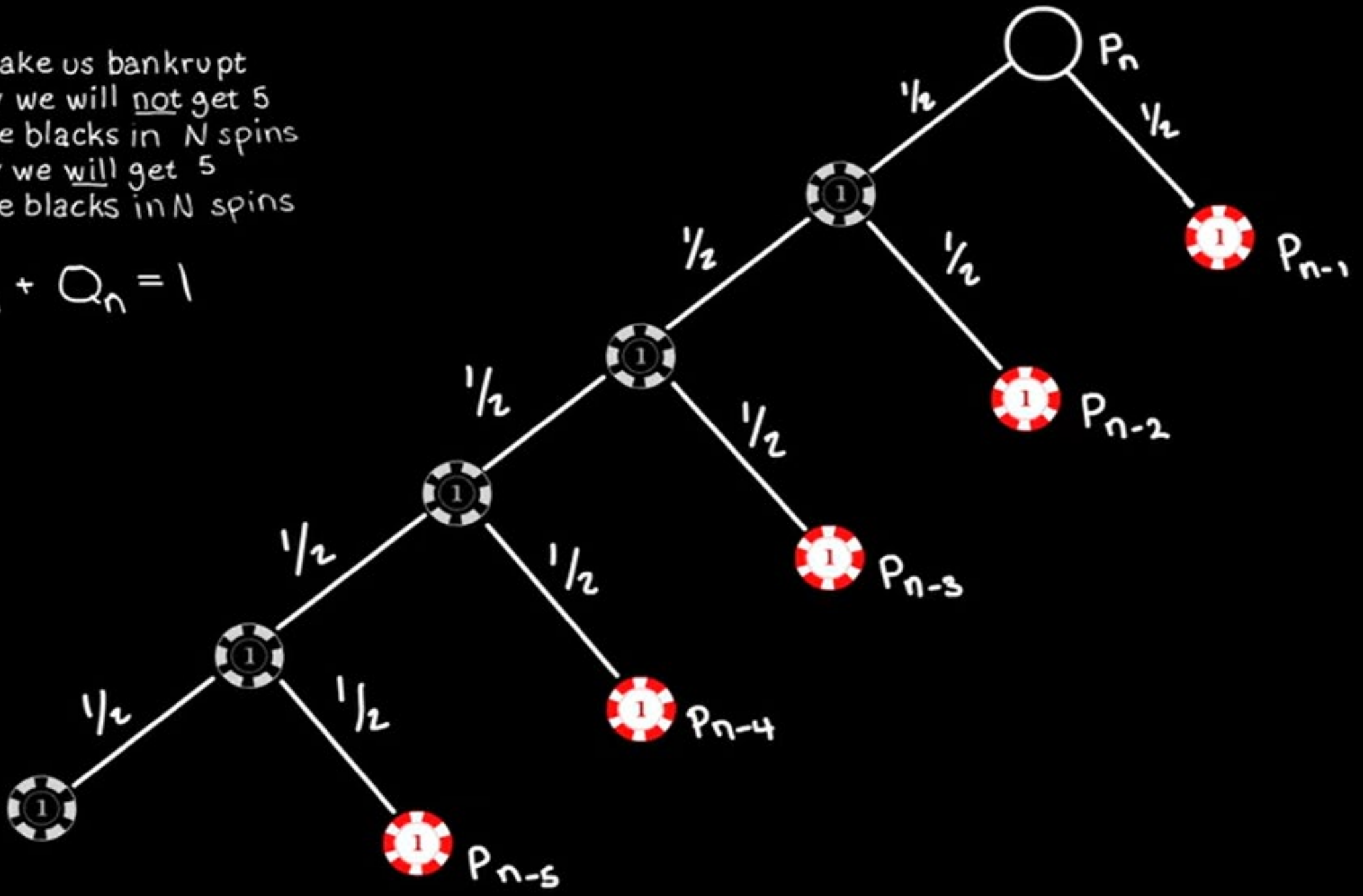

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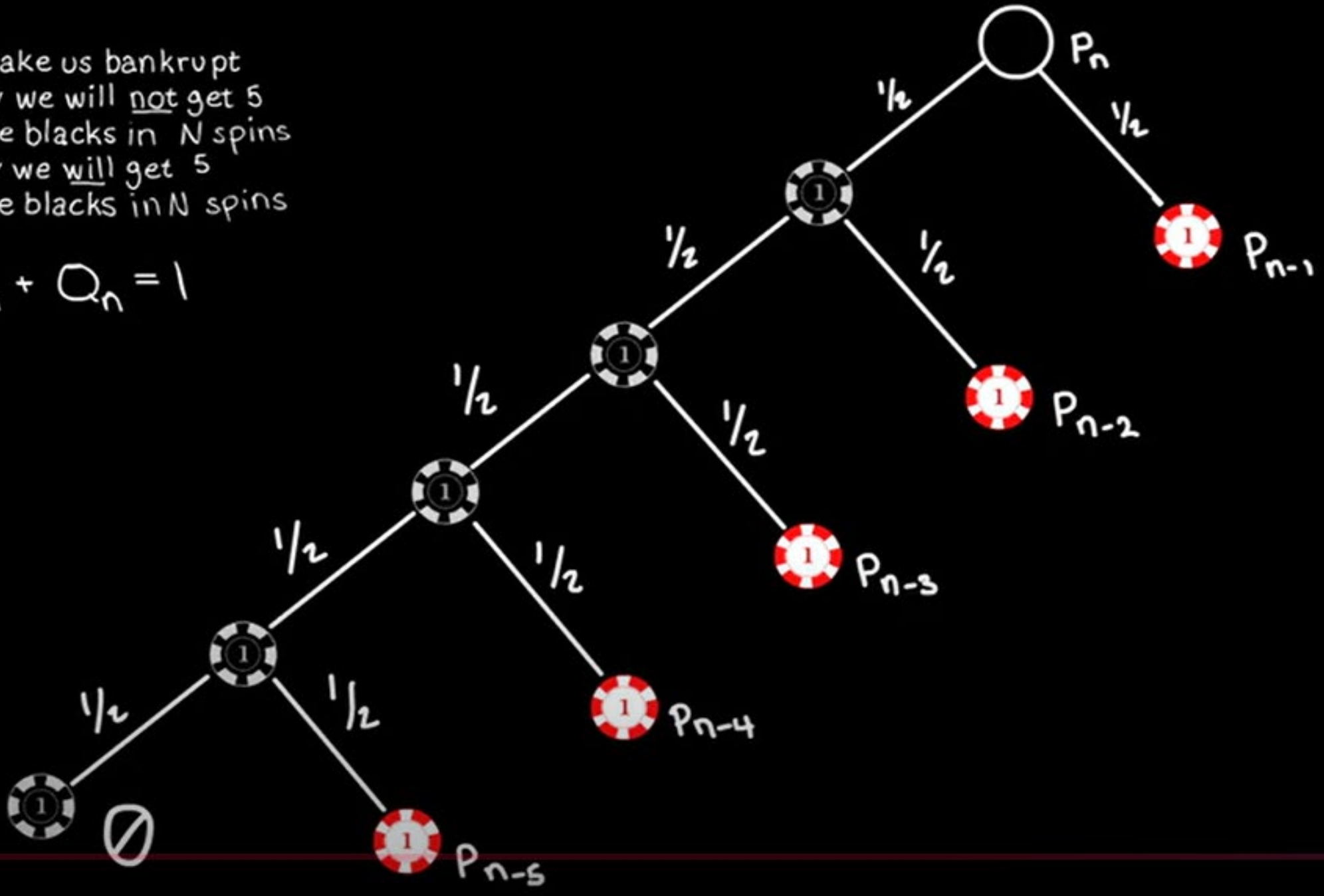
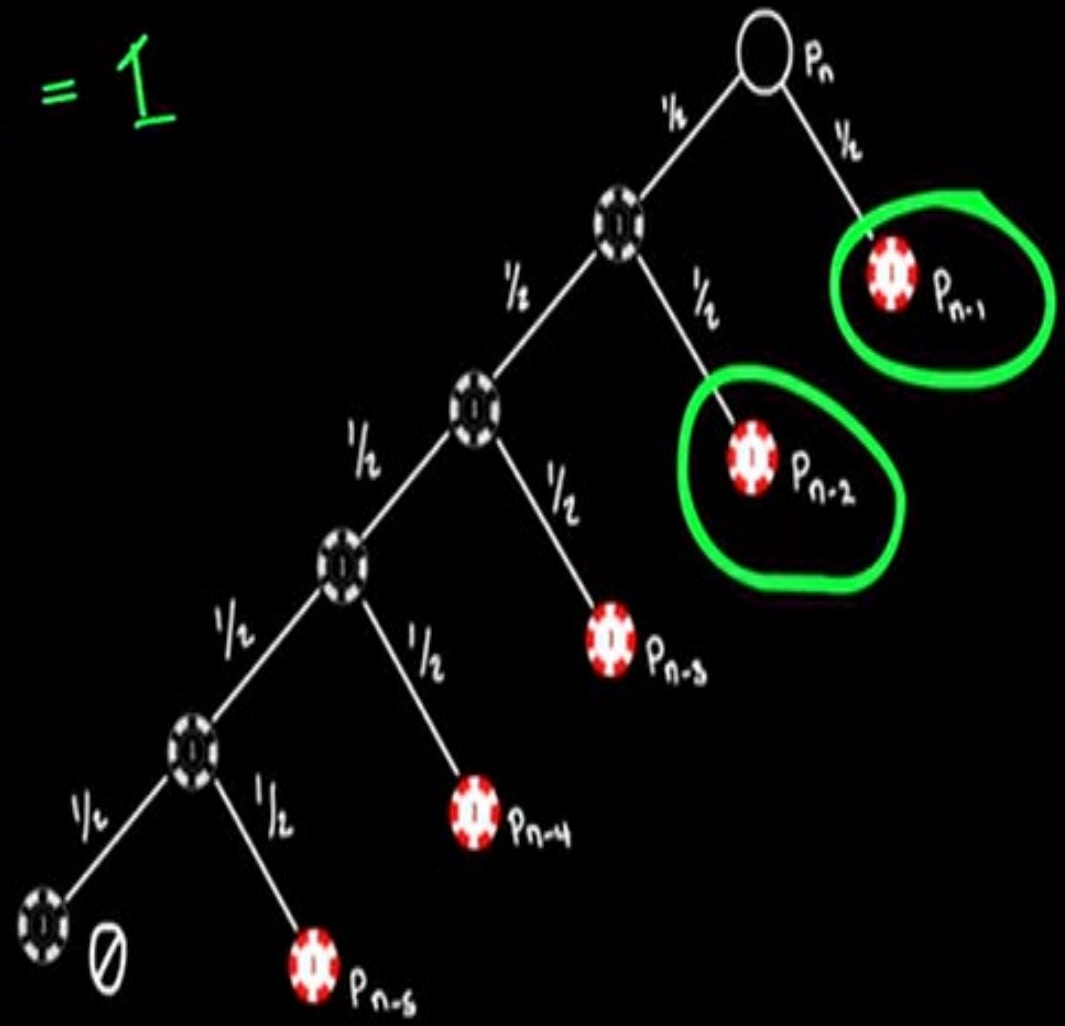


$$P_0 = P_1 = P_2 = P_3 = P_4 = 1$$



$$P_n = \frac{1}{2}(P_{n-1}) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(P_{n-2}) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(P_{n-3}) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(P_{n-4}) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(P_{n-5})$$

Iterate to Solution (target = 5)

Advantage: can do with young students (rational).

n	P(n)	Q(n)	n	P(n)	Q(n)	n	P(n)	Q(n)
0	100.00	0.00	100	18.99	81.01	200	3.41	96.59
1	100.00	0.00	101	18.67	81.33	201	3.35	96.65
2	100.00	0.00	102	18.35	81.65	202	3.29	96.71
3	100.00	0.00	103	18.04	81.96	203	3.24	96.76
4	100.00	0.00	104	17.73	82.27	204	3.18	96.82
5	96.88	3.13	105	17.43	82.57	205	3.13	96.87
6	95.31	4.69	106	17.13	82.87	206	3.08	96.92
7	93.75	6.25	107	16.84	83.16	207	3.02	96.98
8	92.19	7.81	108	16.55	83.45	208	2.97	97.03
9	90.63	9.38	109	16.27	83.73	209	2.92	97.08
10	89.06	10.94	110	15.99	84.01	210	2.87	97.13
11	87.55	12.45	111	15.72	84.28	211	2.82	97.18
12	86.06	13.94	112	15.45	84.55	212	2.77	97.23
13	84.59	15.41	113	15.19	84.81	213	2.73	97.27
14	83.15	16.85	114	14.93	85.07	214	2.68	97.32
15	81.74	18.26	115	14.68	85.32	215	2.64	97.36
16	80.35	19.65	116	14.43	85.57	216	2.59	97.41
17	78.98	21.02	117	14.18	85.82	217	2.55	97.45
18	77.63	22.37	118	13.94	86.06	218	2.50	97.50
19	76.31	23.69	119	13.70	86.30	219	2.46	97.54
20	75.01	24.99	120	13.47	86.53	220	2.42	97.58

THEORY

- Closed form solution (Generalized Binet Formula).
- Yields large #spins behavior (cannot see from recurrence).
- Lots of ways to solve: linear algebra, characteristic polynomials.
- Generalizing Fibonaccis – what else can model with recurrences? (Predator-Prey, ….)

CONCLUSIONS

- Code to get data.
- Use data to extrapolate.
- View data right way.
- Power of modeling with recurrences.

Thank you!

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