

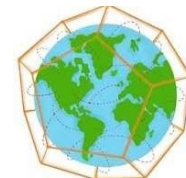
From Alligators to All In - Generalized Predator-Prey Problems and Guided Online Research

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Modeling Makes Mathematics More Meaningful (M - Certainly in Differential Equations Education)

SIAM Conference on Applied Mathematics Education (ED26)-Cleveland Ohio



https://web.williams.edu/Mathematics/sjmillier/public_html/math/talks/talks.html

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Sections

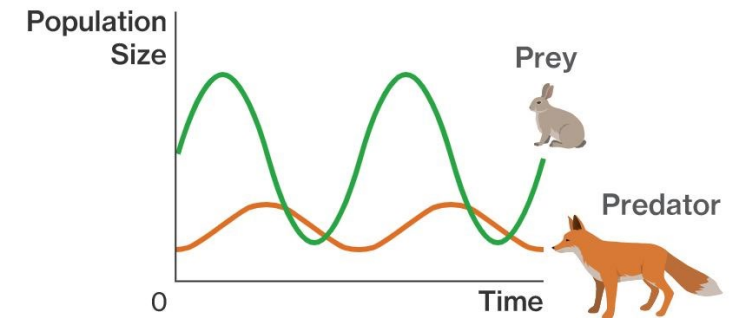
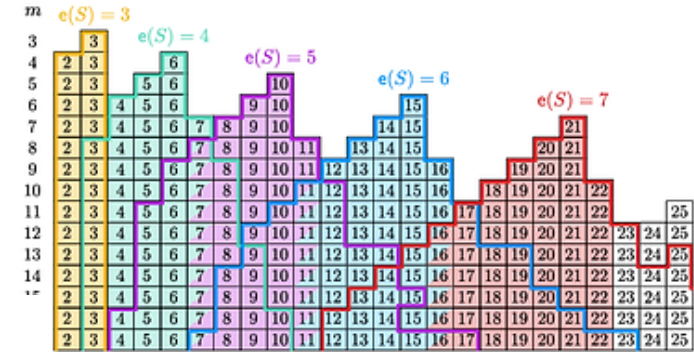
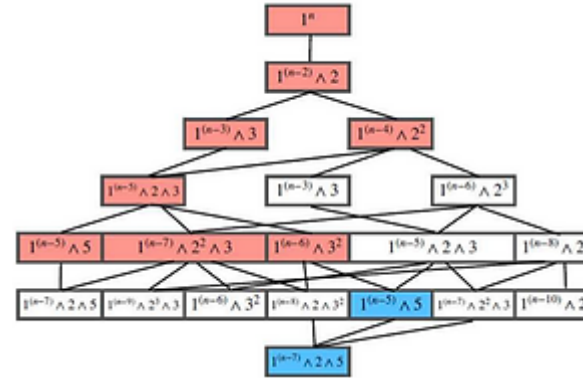
- 1) Polymath Jr.
- 2) Know your team
- 3) Research that makes a difference
- 4) PP models and how to approach
- 5) Results, Collaborations, future steps, and lessons learned

Polymath Jr.

Provide research opportunities to many undergraduates who wish to explore advanced mathematics, train graduate students / young faculty in mentoring.

Projects advised by an active researcher with experience in undergraduate mentoring, assisted by young faculty / graduate students / advanced undergraduates.

<https://geometrynyc.wixsite.com/polymathreu>



How Polymath Works

Hundreds of students every year, primarily to sophomores / juniors (so time help decide if want to go to grad school).

All students attend a series of online sessions introducing a wide range of topics and provide insight into potential projects.

After these sessions, students list their top three projects (usually get one).

Mentors receive training on handling stressful situations, leading effectively, giving students space, motivating them....

Graduate mentors paired with a faculty supervisor who oversees the project.



Meet the Team



Anh Quynh Bui



Jasmine Pham



Jessie Wang



Sangam Dhakal

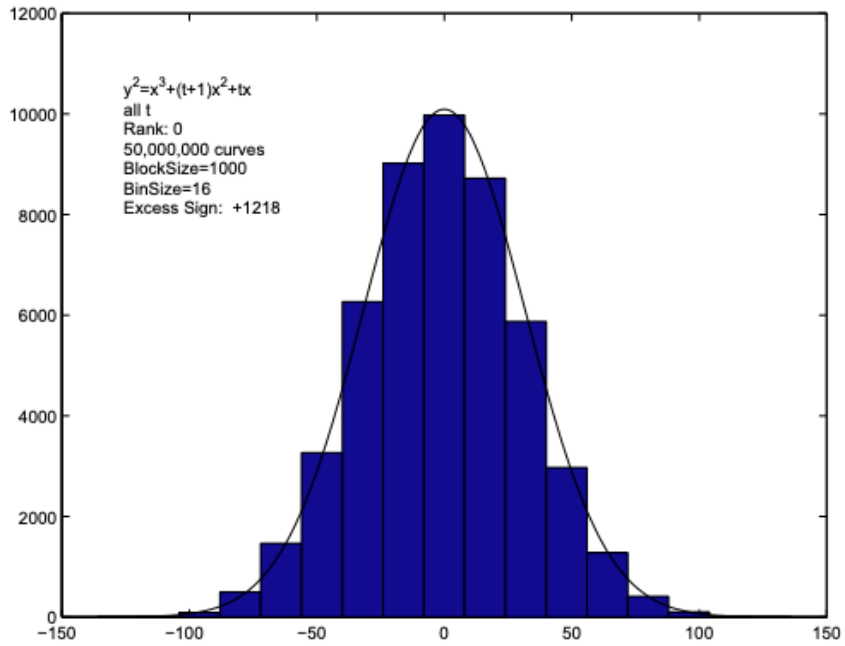
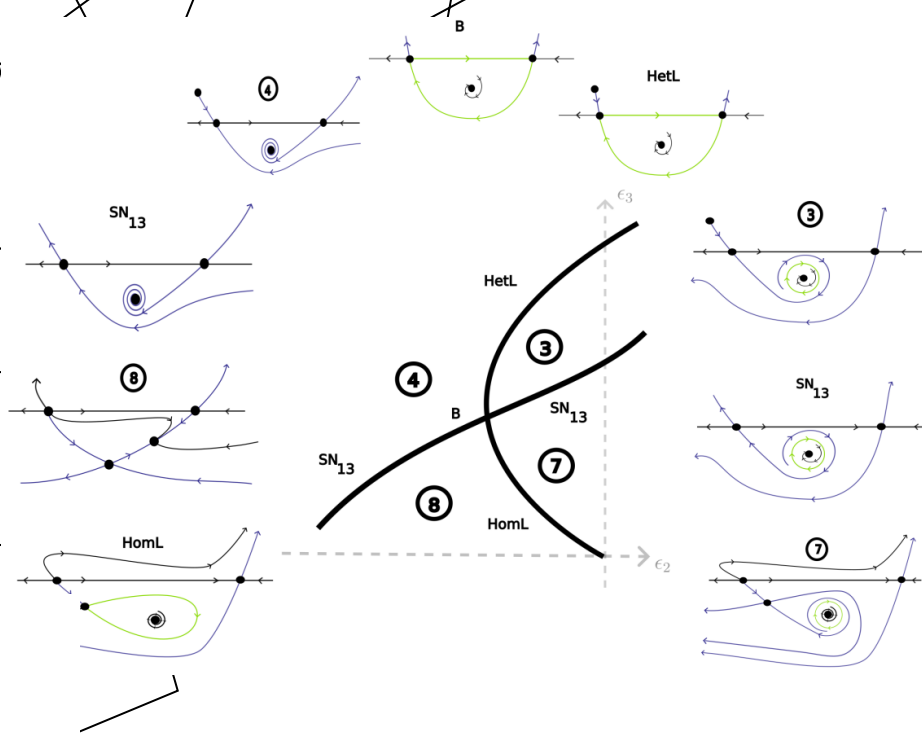
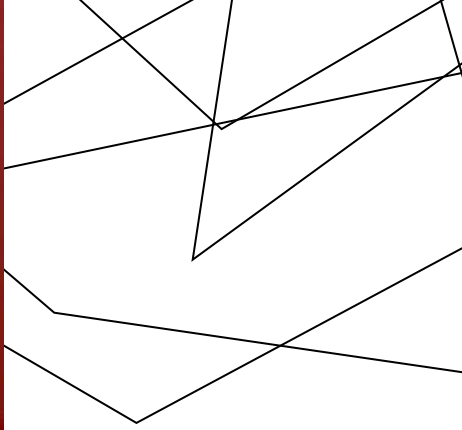


Steven Miller

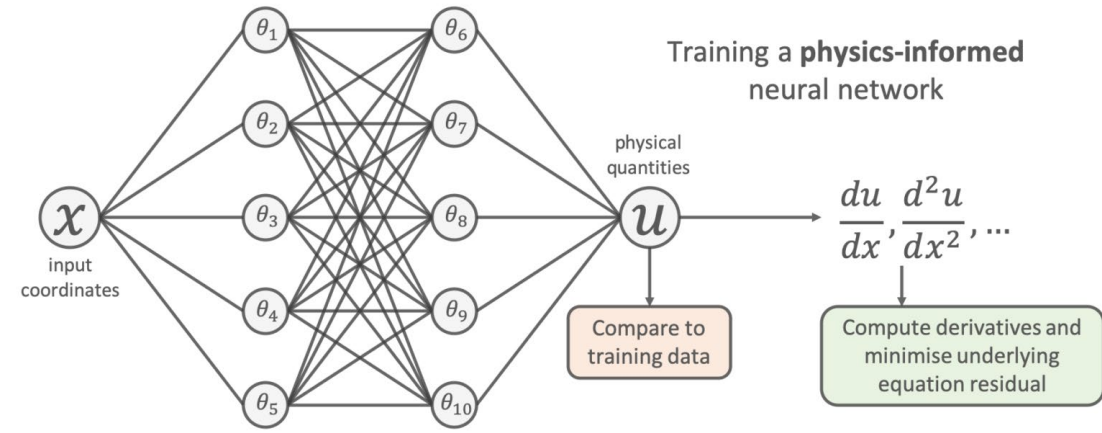


Wael El Khateeb

Our Background



Histogram plot: All $t \in [2, 5 \cdot 10^7]$



Training a physics-informed neural network

PHASE 1: Introduction

How to guide students without prior differential equations experience to produce high-quality research in just six weeks?

We established a clear framework built on the following principles.

Consistent and Timely Communication

- The average email response time, amazingly, did not exceed 30 minutes.

Restricted Material

- Restricting our sharing of knowledge to only the essential tools (System of ODEs, Jacobian, basic bifurcations,...).

Weekly group meetings

- Complemented by one-on-one office hours.

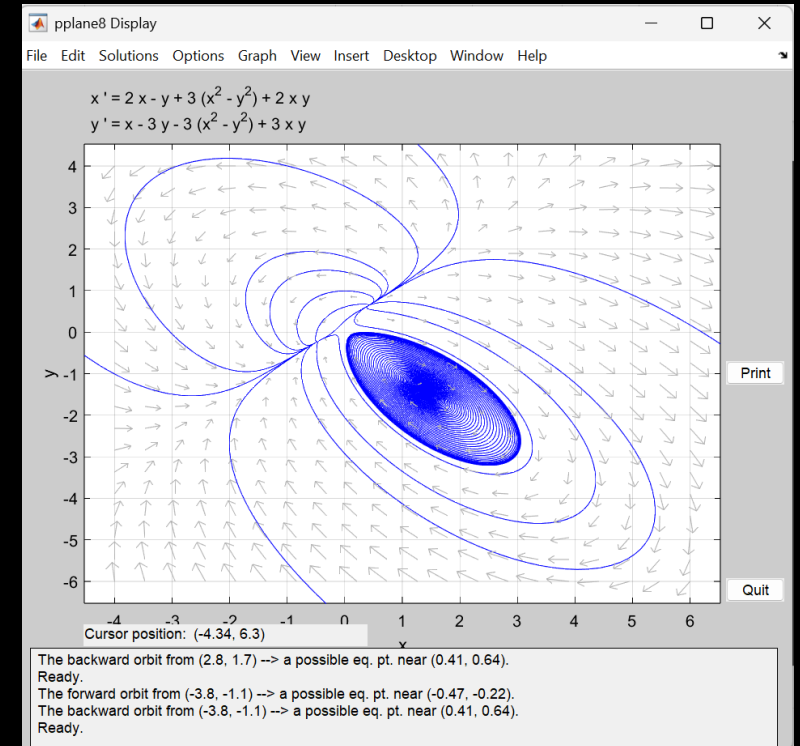
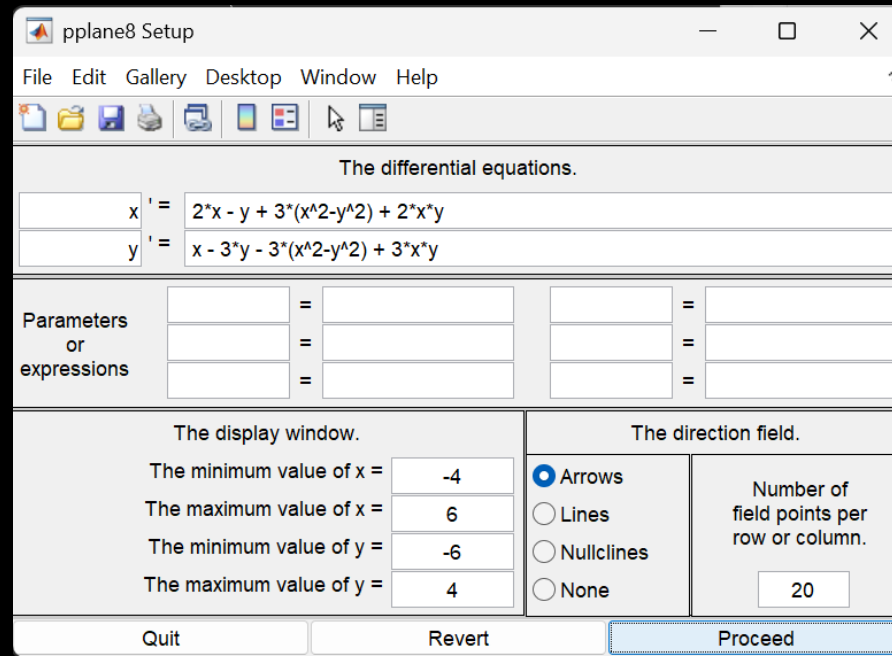
Positive Reinforcement and motivation

- Maintain high morale throughout the program to encourage confidence and sustained engagement.
- Motivating goal: All expense paid trip to UT Austin to present at a research conference.

Software tools

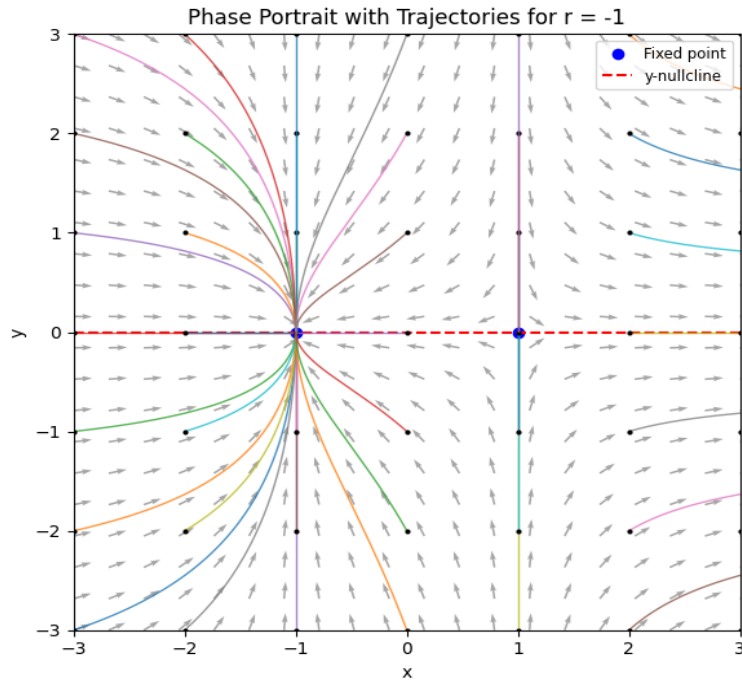
- 1) Pplane
- 2) ODE 45

Well written code that students read, analyze, understand, and use instead of writing from scratch.

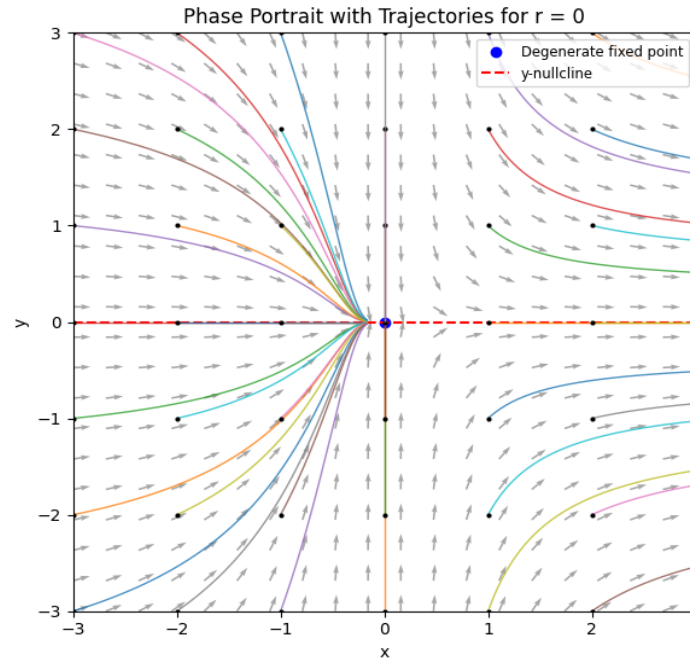


Bifurcations: qualitative change in the behavior of a system as a parameter is varied.

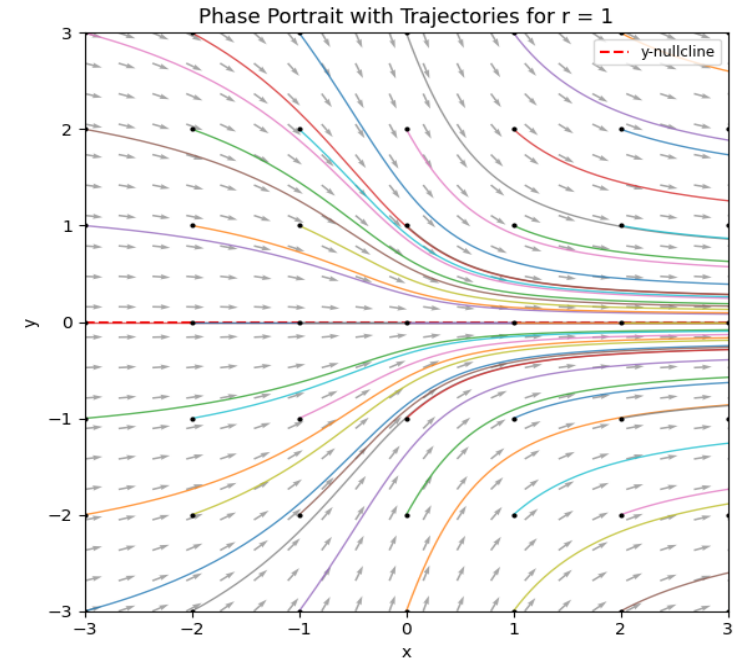
$$\begin{cases} \frac{dx}{dt} = r + x^2, \\ \frac{dy}{dt} = -y. \end{cases}$$



$r = -1$



$r = 0$

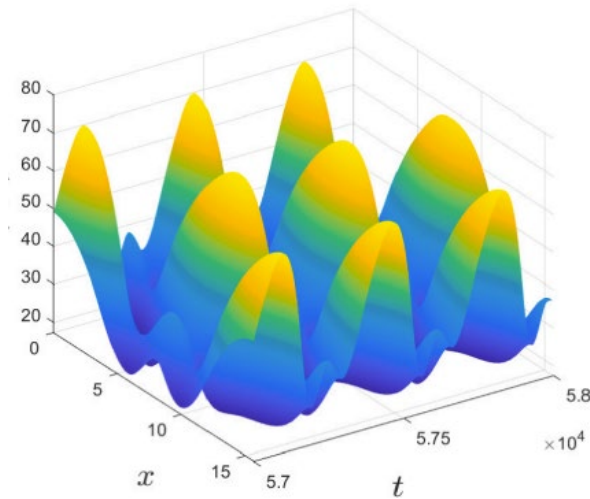


$r = 1$

HOW TO KEEP STUDENTS ON TRACK? WHAT IF STUDENTS WERE NOT PREPARED?

Think-Pair-Share
Slide preparation on the spot

$$\begin{aligned} S_t - r_1 \Delta S &= A - dS - \beta SI, \\ I_t - r_2 \Delta I &= \beta SI - dI - h(I)I, \\ R_t - r_3 \Delta R &= h(I)I - dR, \end{aligned}$$



$$\begin{cases} \frac{dx(t)}{dt} = rx(t) \left(1 - \frac{x(t)}{K} \right) - \frac{mx(t)y(t)}{a + x(t)}, \\ \frac{dy(t)}{dt} = -dy(t) + e^{-d\tau} \frac{cmx(t-\tau)y(t-\tau)}{a + x(t-\tau)}, \end{cases}$$

Initial condition is a trajectory!



BACKGROUND (BIOLOGY)

1800s to mid-1900s: Intense hunting for leather and meat.

1950s to 1960s: Populations crash; poaching persists; weak enforcement.

1967: Listed as endangered in the United States.

1970s: States close hunting, protect nests, begin science-based management.



BACKGROUND (BIOLOGY)

Late 1970s to 1980s: Rebound via protection, anti-poaching, wetland conservation.

1987: Delisted federally; managed as “threatened due to similarity of appearance.”

FWC lottery permits; two alligators per permit.



Burmese Pythons



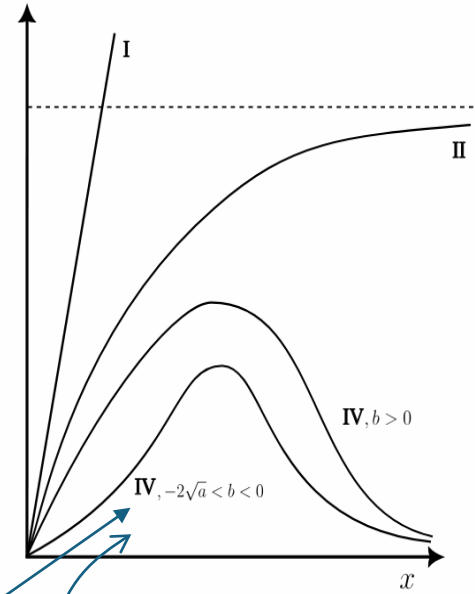
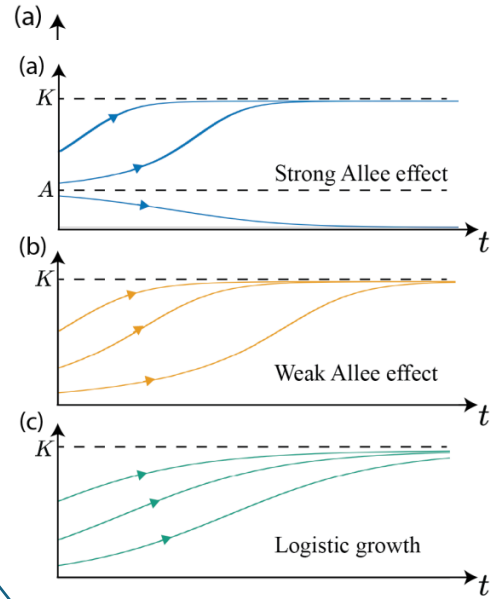
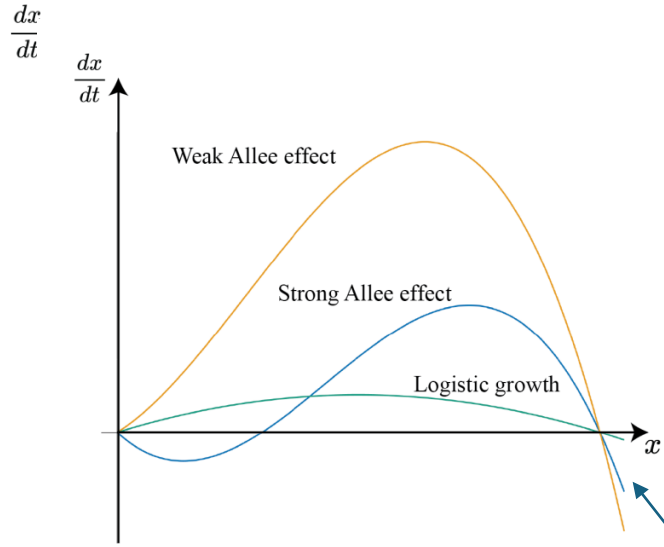
- Intentional releases by owners who could no longer keep large snakes.
- Accidental escapes from pet stores, breeders, and private collections.
- Facility damage during Hurricane Andrew (1992) likely released additional snakes into the Everglades.

ECOLOGICAL IMPLICATIONS

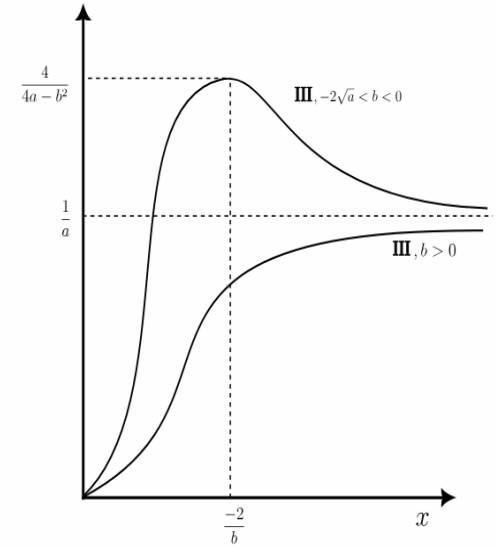
- Sharp declines of midsized mammals in parts of the Everglades (raccoons, opossums, marsh rabbits)
- Competition with native apex predators for prey, altering food webs



PP model and extensions



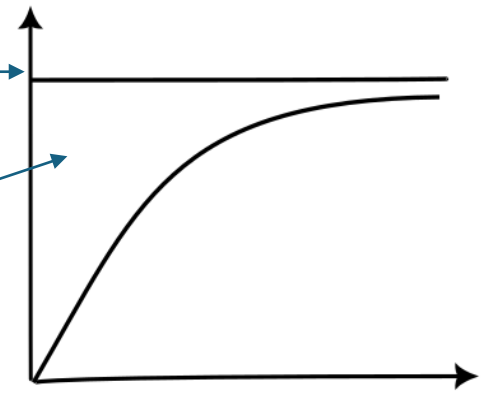
(a) Holling Type I, II, and IV



(b) Holling Type III

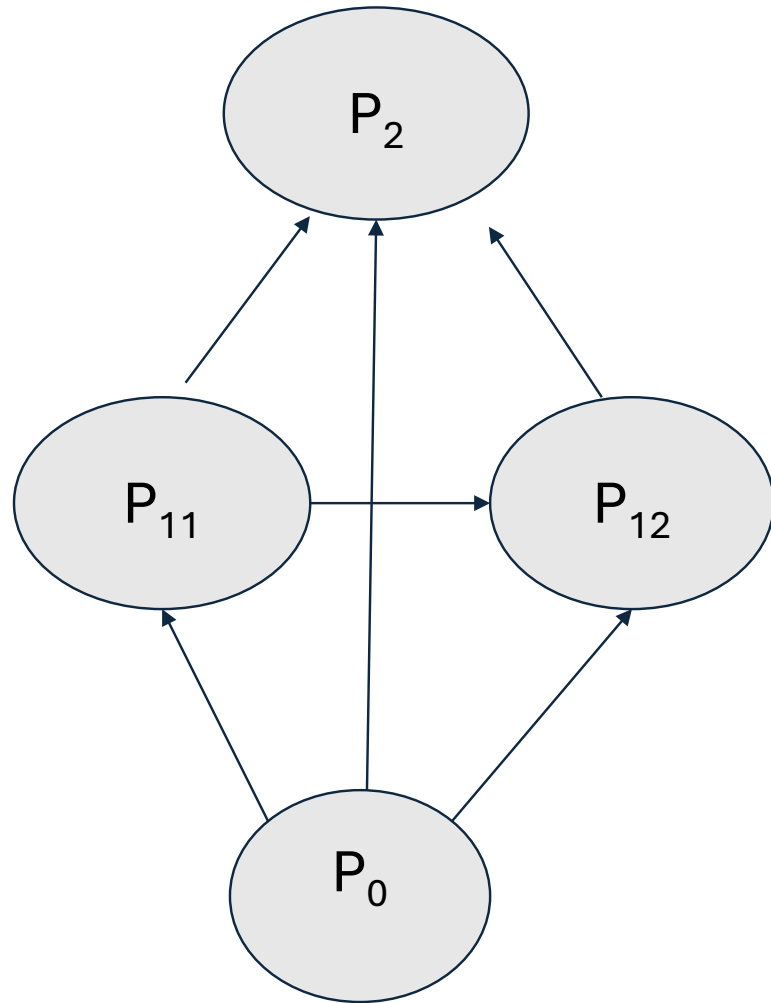
$$\frac{dx}{dt} = G_1(x) - \alpha_1 F(x)y - H_1(x)$$

$$\frac{dy}{dt} = -G_2(y) + \alpha_2 F(x)y - H_2(y)$$





Model Illustrated:



P_0 - Raccoon/prey population

P_{11} - Juvenile Alligator population

P_{12} - Adult Alligator population

P_2 - Burmese Python population



Initial Model

$$\begin{cases} \frac{dP_0}{dt} = rP_0 - m_{20}P_0P_2 - m_{30}P_0P_2 \\ \frac{dP_1}{dt} = c_{20}P_0P_2 - m_{31}P_1P_3 - d_{11}P_{11} \\ \frac{dP_2}{dt} = d_{12}P_1 - d_2P_2 - h_2 \\ \frac{dP_2}{dt} = c_{30}P_0P_3 + c_{31}P_3P_1 - d_3P_2 - h_3P_2 \end{cases}$$



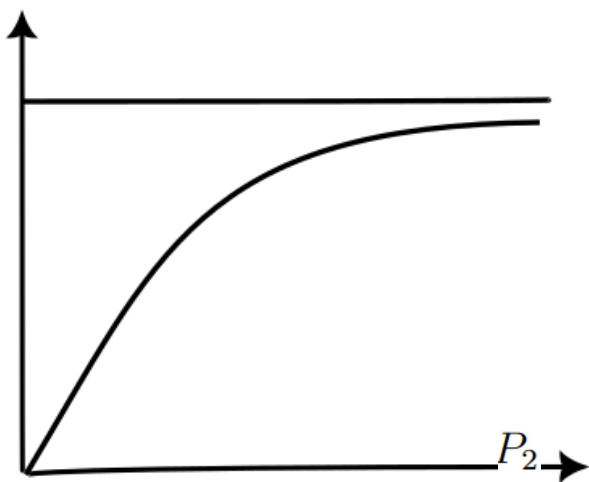
$h_2 \Rightarrow$ Saturated

$$\begin{cases} \frac{dP_0}{dt} = rP_0 - m_{20}P_0P_{12} - m_{30}P_0P_2 \\ \frac{dP_{11}}{dt} = c_{20}P_0P_{12} - m_{31}P_{11}P_3 - d_{11}P_{11} \\ \frac{dP_{12}}{dt} = d_{12}P_{11} - d_2P_{12} - h_2 \frac{P_{12}}{a + P_{12}} \\ \frac{dP_2}{dt} = c_{30}P_0P_2 + c_{31}P_2P_{11} - d_3P_2 - h_3P_2 \end{cases}$$

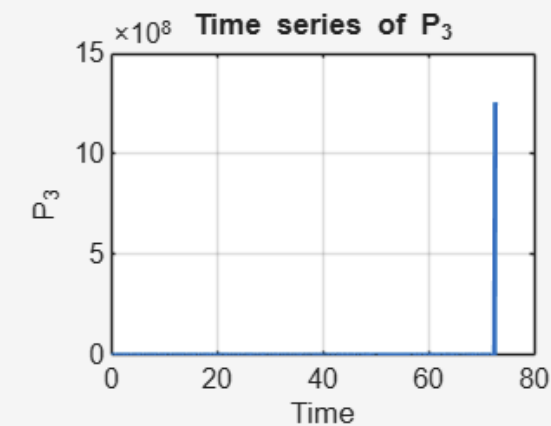
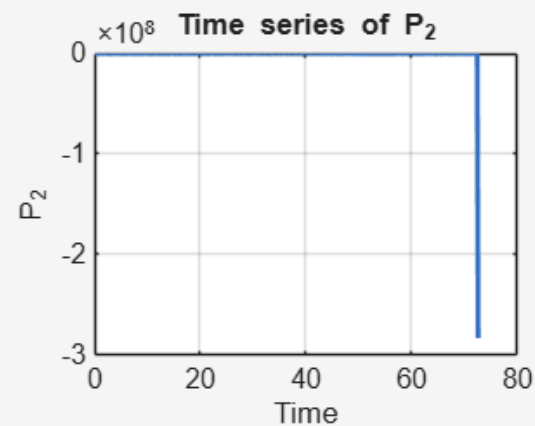
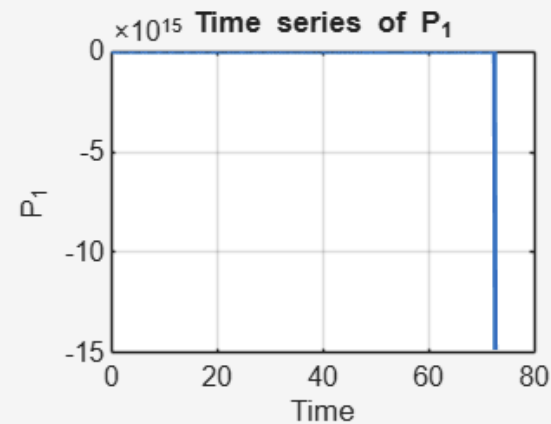
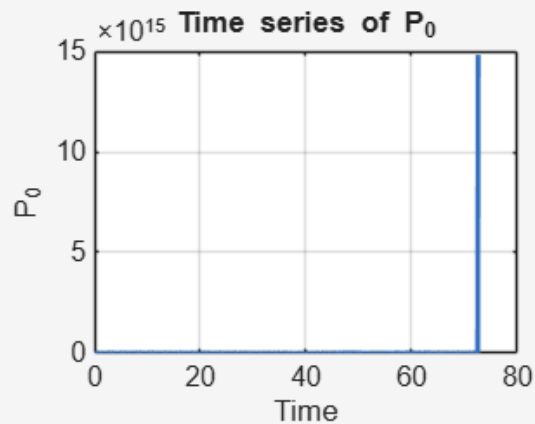


Why constant harvesting doesn't work

Applying constant harvesting shows that P_1 and P_2 eventually become **negative**, which is biologically unrealistic and highlights why **constant harvesting does not work**.



Simulation of 4-ODE system (P2 harvesting: constant)



Final Model

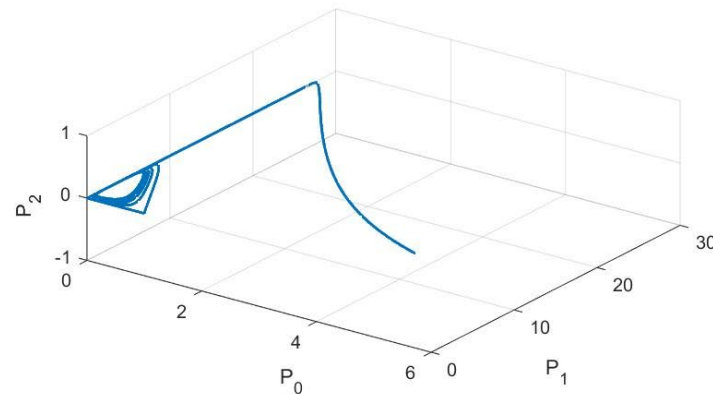
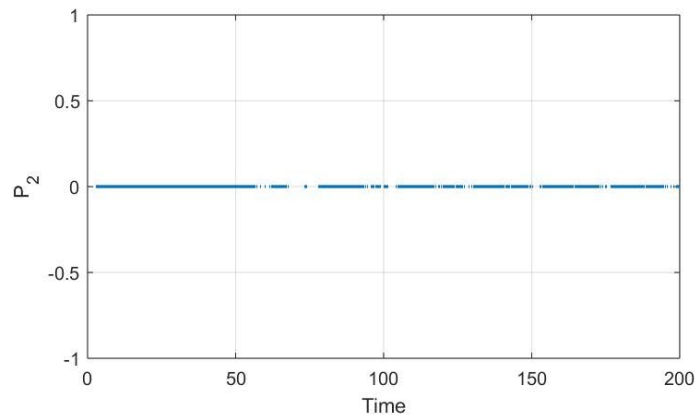
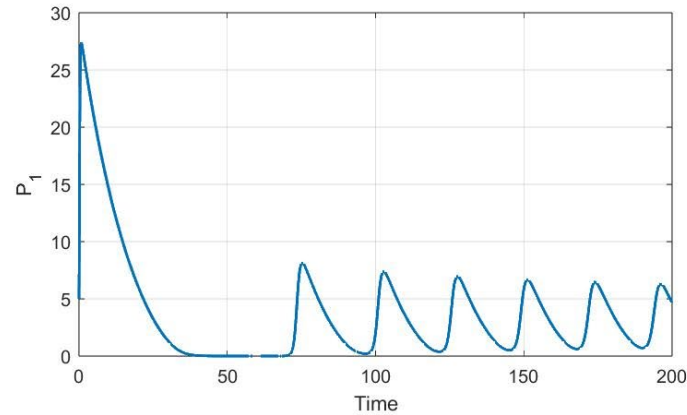
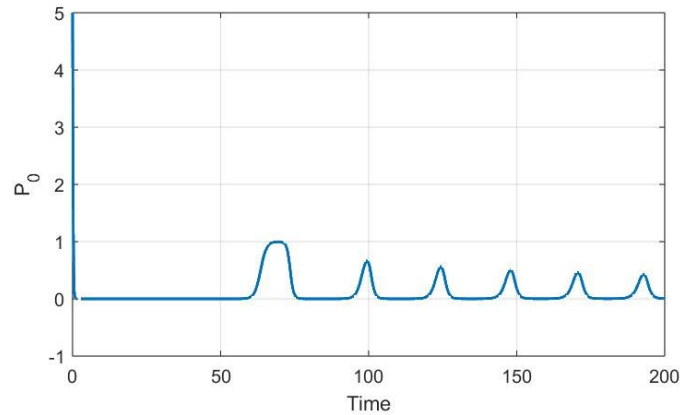
$$\begin{cases} \frac{dP_0}{dt} = rP_0 \left(1 - \frac{P_0}{K}\right) - m_{10}P_0P_1 - m_{20}P_0P_2, \\ \frac{dP_1}{dt} = c_{10}P_0P_1 - d_1P_1 - \frac{h_1P_1}{a + P_1} - m_{31}P_1P_2, \\ \frac{dP_2}{dt} = c_{20}P_0P_2 + c_{21}P_1P_2 - d_2P_2 - h_2P_2. \end{cases}$$

Equilibria: 7 in total

- 2 Equilibria on the P_0 axis.
- 2 equilibria on the $P_0 - P_1$ plane suggesting that the prey can coexist with the alligators.
- 2 non-zero equilibria where all 3 species coexist.
- 1 equilibrium in the $P_0 - P_2$ plane suggesting that python can coexist with the general prey.

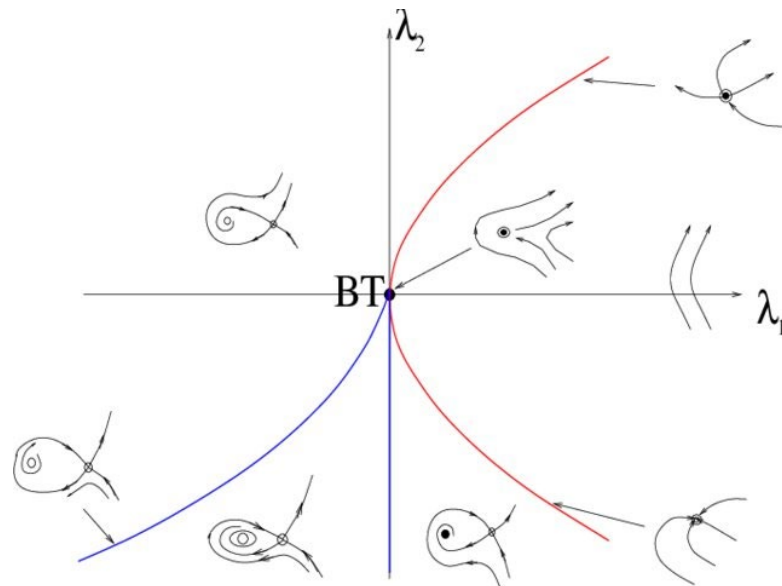
THE CASE WITH NO BURMESE PYTHONS

- The system provides grounds to modeling the relation between Alligators and general prey in the absence of pythons.



What happens if both Hopf and Saddle node occur simultaneously?

Students were able to do so using Bogdanov takes bifurcations, which explained to formation of limit cycles



1. **Shift the equilibrium:** Find the equilibrium $P^* = (P_0^*, P_1^*, P_2^*)$ and shift variables:

$$u = P_0 - P_0^*, \quad v = P_1 - P_1^*, \quad w = P_2 - P_2^*$$

so that the system becomes $\dot{z} = F(z)$ with $F(0) = 0$.

2. **Linearize and identify eigen-directions:** Compute the Jacobian $J = DF(0)$. At the BT point: $\text{spec}(J) = \{0, 0, \lambda_s\}$ with $\lambda_s < 0$. The eigenvectors corresponding to zero eigenvalues define the center subspace.

3. **Center manifold reduction (3D \rightarrow 2D):** There exists a smooth invariant manifold $w = h(u, v)$ with $h(0) = 0, Dh(0) = 0$. Substituting $w = h(u, v)$ reduces the system to:

$$\dot{u} = \tilde{F}_1(u, v), \quad \dot{v} = \tilde{F}_2(u, v)$$

4. **Reduced dynamics:** Expand \tilde{F}_1, \tilde{F}_2 in a Taylor series near the origin and retain quadratic terms:

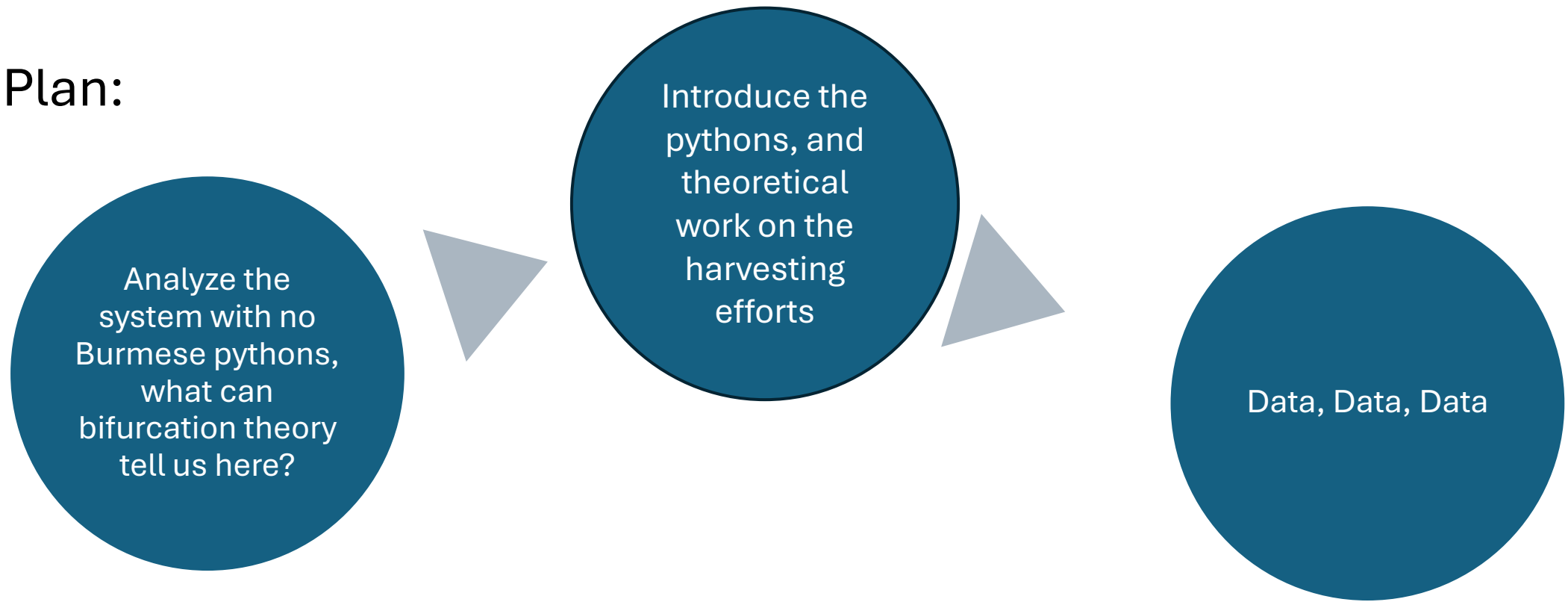
$$\dot{u} = v + \mathcal{O}(2), \quad \dot{v} = au^2 + buv + \dots$$

5. **Transform to BT normal form:** Apply smooth coordinate and parameter changes to obtain:

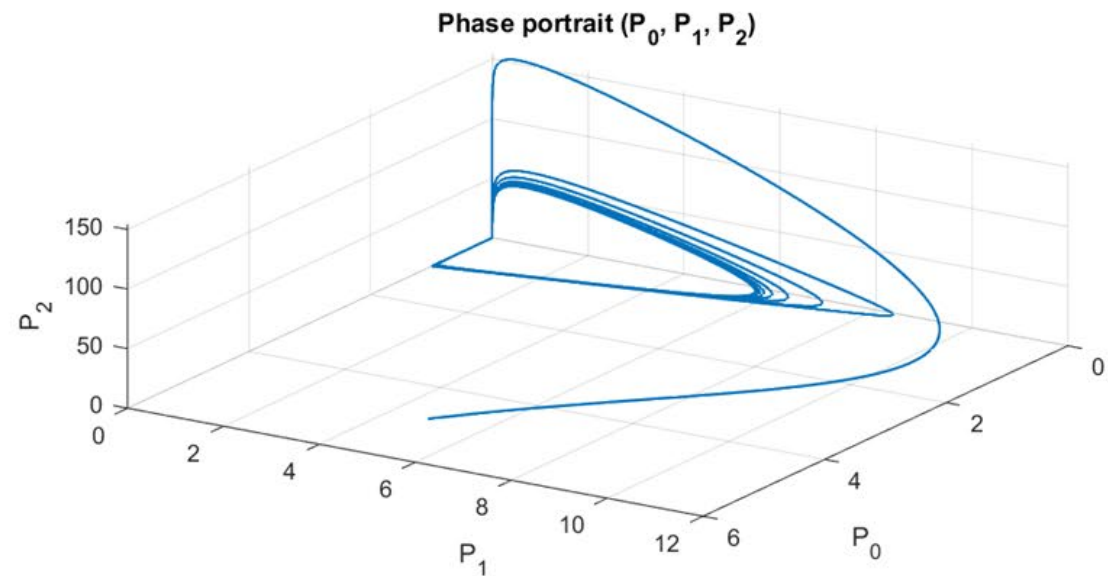
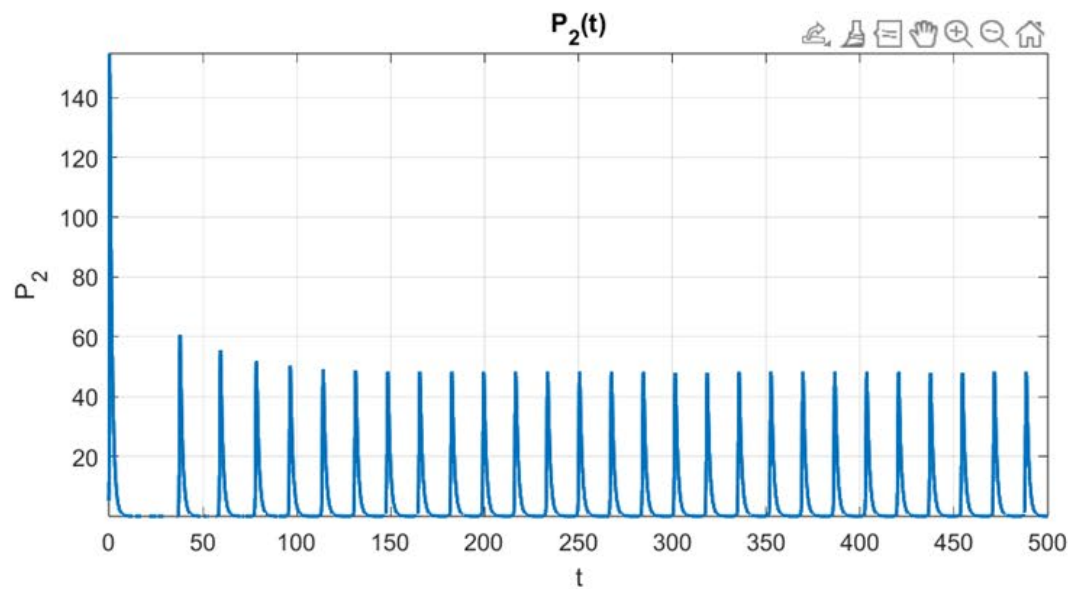
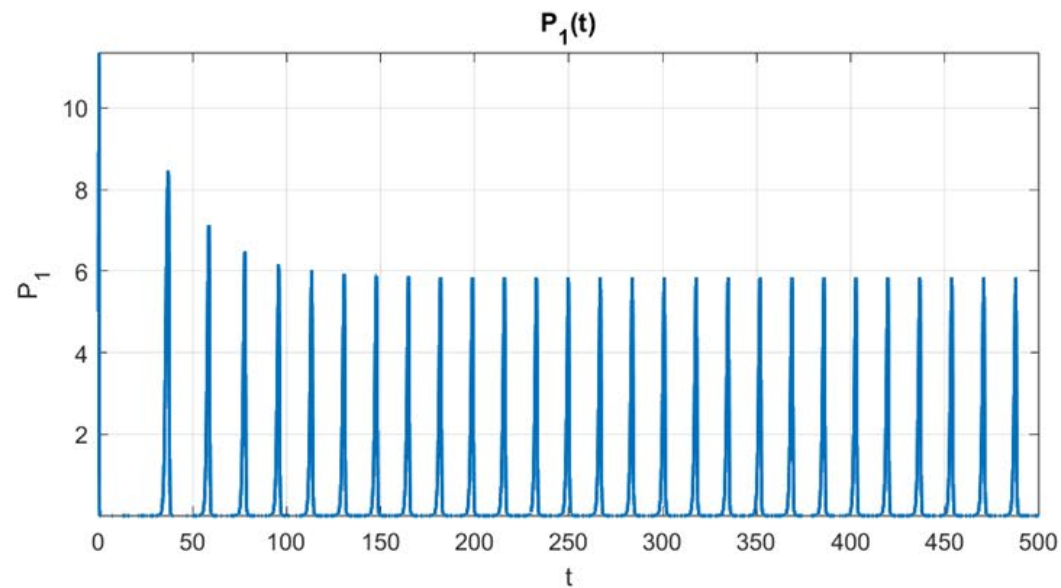
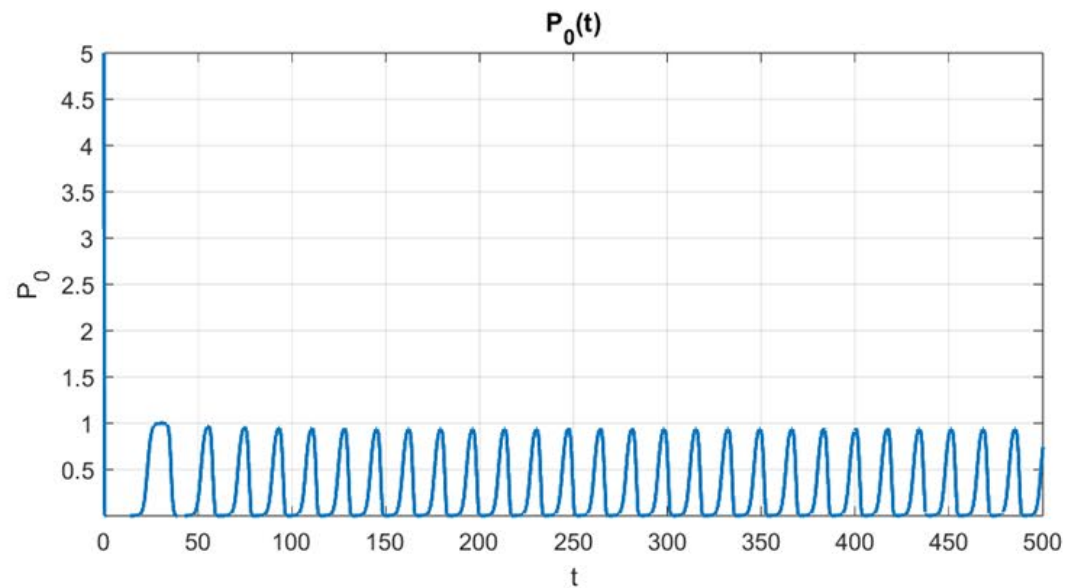
$$\dot{x} = y, \quad \dot{y} = \beta_1 + \beta_2 x + ax^2 + bxy + \mathcal{O}(3)$$

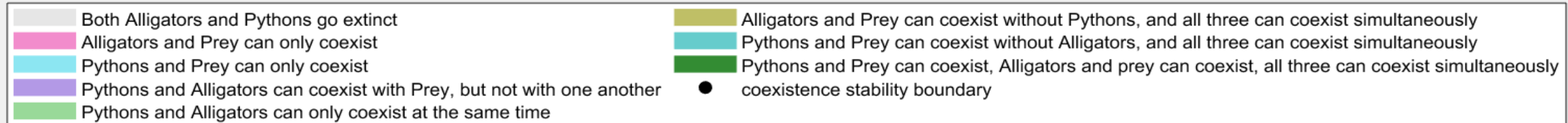
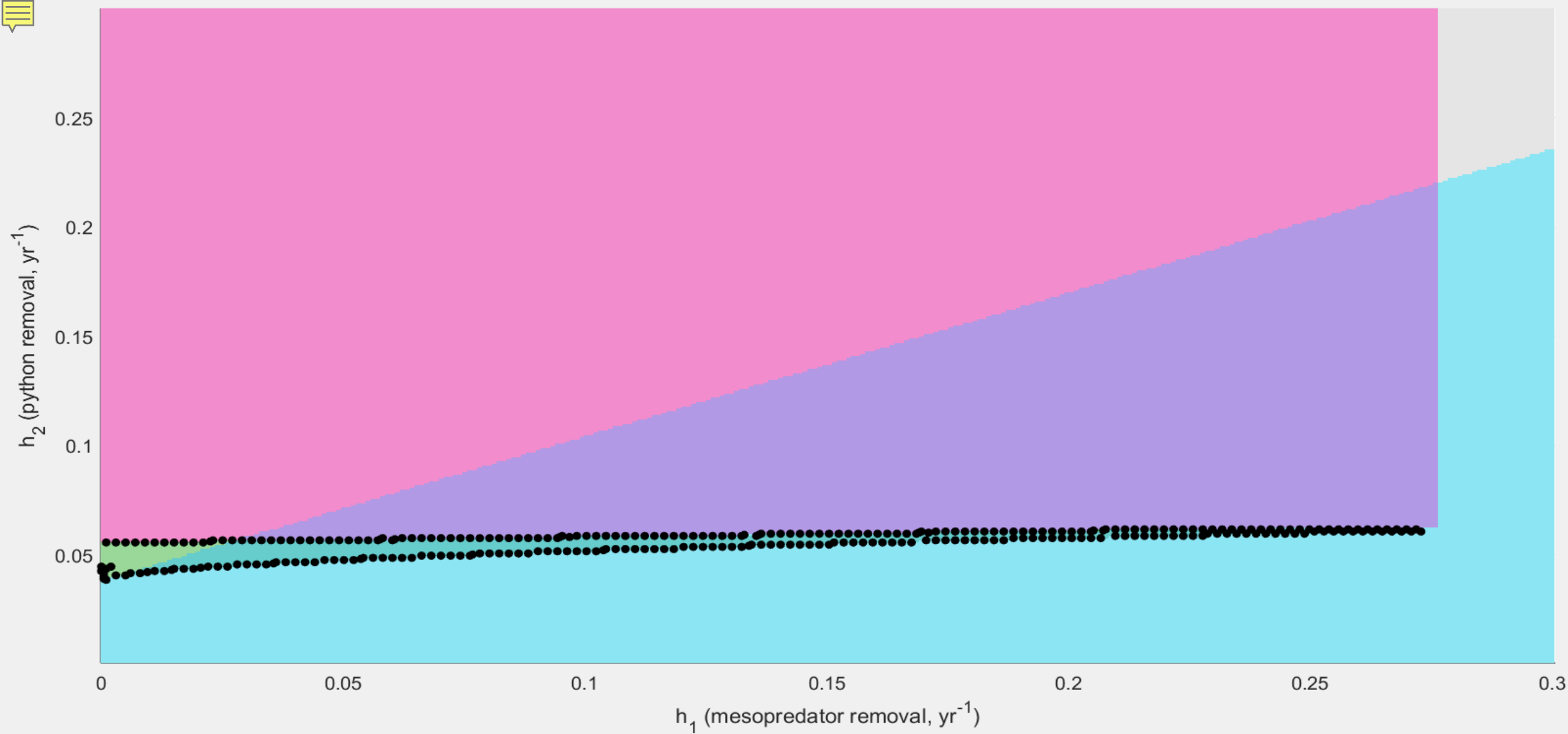
Ultimate goal: Policy making

- With current harvesting restricted to 2 alligators per hunter, per season, should the hunting policies be altered, especially that the common prey population is decreasing (Racoons drop by 95%...)
- Plan:



The presence of Burmese Pythons







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SIAM TX-LA Conference and JMM 2026



Useful techniques/ Strategies implemented

- **Ownership**
- **Reaching out to contacts**
- **Know your strong points, know your weak points**
- **Always have something to share**
- **Provide opportunities, but students must seize**



Thank you!

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