From Alligators to All In - Generalized Predator-Prey Problems and Guided Online Research

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with
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Modeling Makes Mathematics More Meaningful (Certainly in Differential Equations Education)

SIAM Conference on Applied Mathematics Education, Cleveland, Ohio, July 10, 2026

https://web.williams.edu/Mathematics/sjmiller/public_html/math/talks/talks.html

Supported in part by NSF Grant number: DMS2341670

OUTLINE

- 1) Polymath Jr. (description + appeal!).
- 2) Know your team.
- 3) Research that makes a difference.
- 4) PP models and how to approach.
- 5) Results, Collaborations, future steps, and lessons learned.

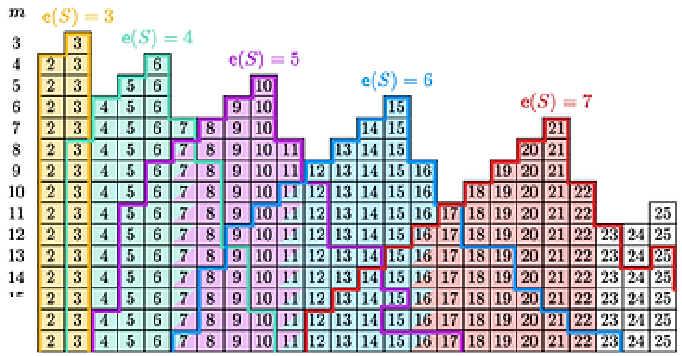
Polymath Jr.

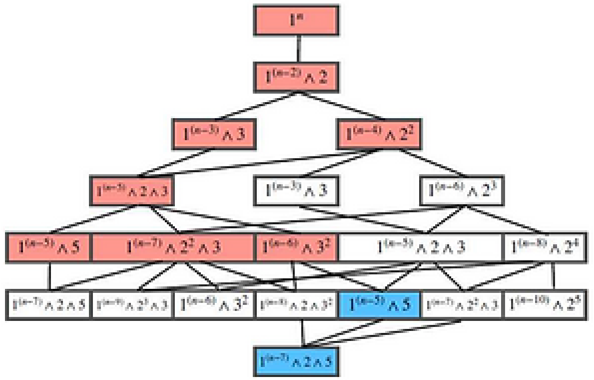
Child of Pandemic Response.

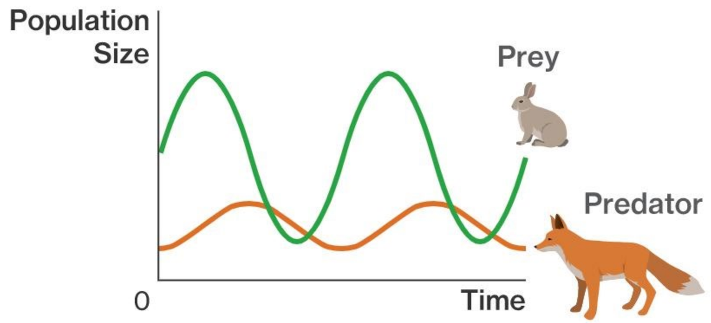
Provide research opportunities to many undergraduates to explore advanced math, train graduate students / young faculty in mentoring.

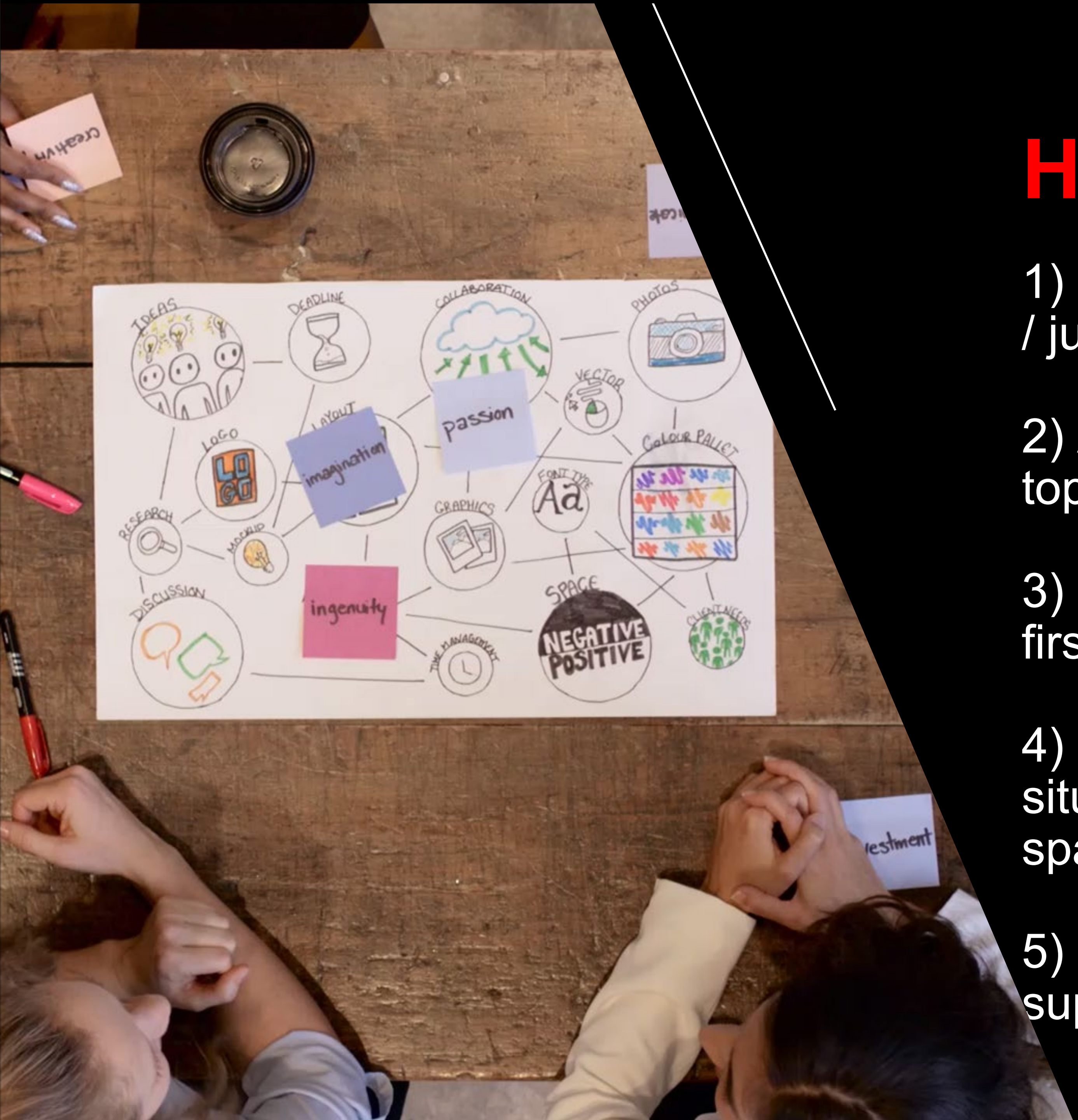
Projects advised by an active researcher with experience in undergrad mentoring, assisted by young faculty / graduate students / advanced undergraduates.

https://geometrynyc.wixsite.com/polymathreu









How Polymath Works

1) Hundreds of students, priority sophomores / juniors (help decide on grad school).

2) All attend online sessions introducing topics.

3) Students list top three projects (usually get first or second).

4) Mentors trained on handling stressful situations, leading effectively, giving students space, motivating them, life advice….

5) Graduate mentors paired with a faculty supervisor who oversees the project.

OUR TEAM

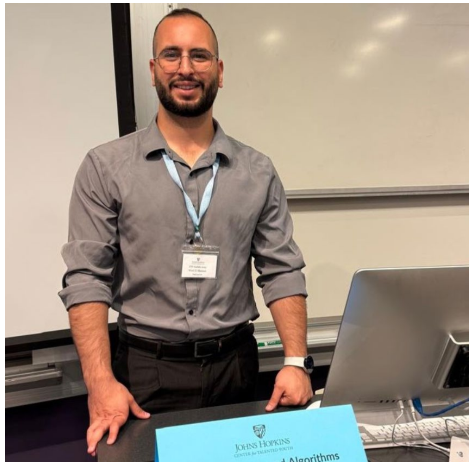
Anh Quynh Bui

Jasmine Pham

Jessie Wang

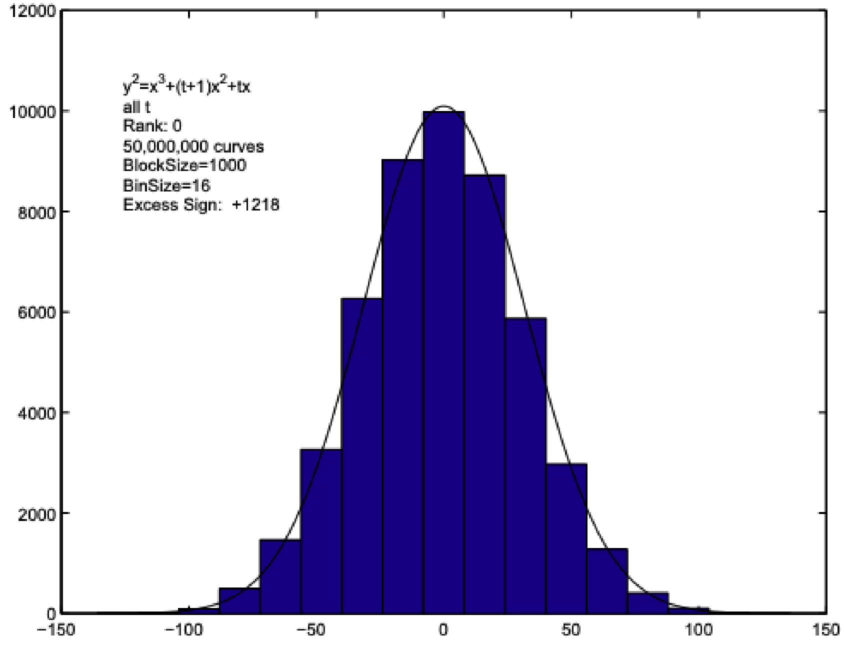
Sangam Dhakal

Steven Miller

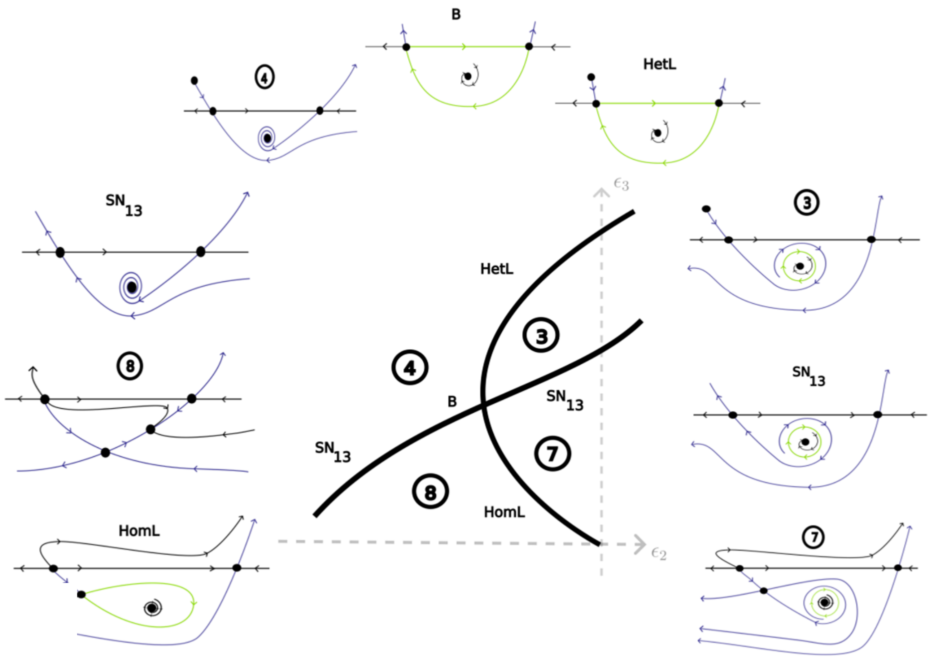


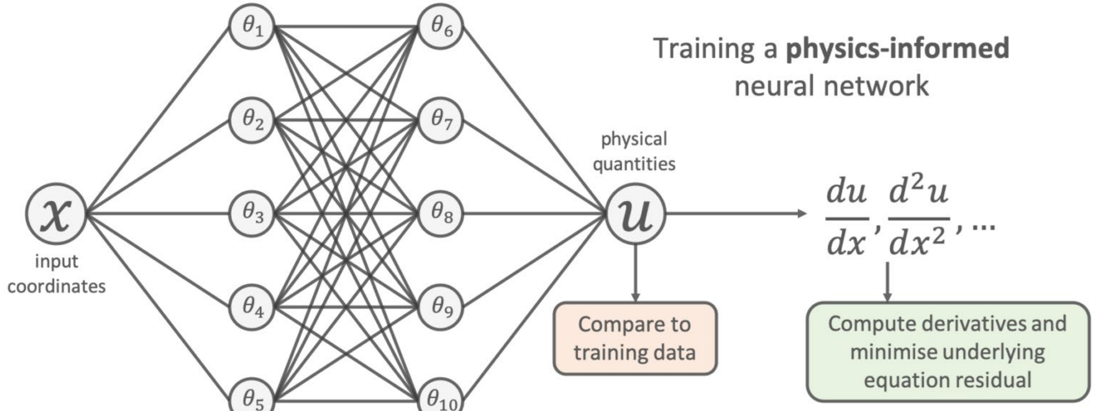
Wael El Khateeb

MY/OUR BACKGROUND



Histogram plot: All $t \in [2, 5 \cdot 10^7]$





PHASE 1: Introduction

How to guide students without prior differential equations experience to produce high-quality research in just six weeks?

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How to guide students without prior differential equations experience to produce high-quality research in just six weeks?

Established a clear framework built on the following.

Consistent and Timely Communication

- The average email response time, amazingly, did not exceed 30 minutes.

Resricted Material

- Restrict to teaching only essential tools (System of ODEs, Jacobian, basic bifurcations,…).
- Give students control over direction of project / readings.

Weekly group meetings

- Complemented by one-on-one office hours.

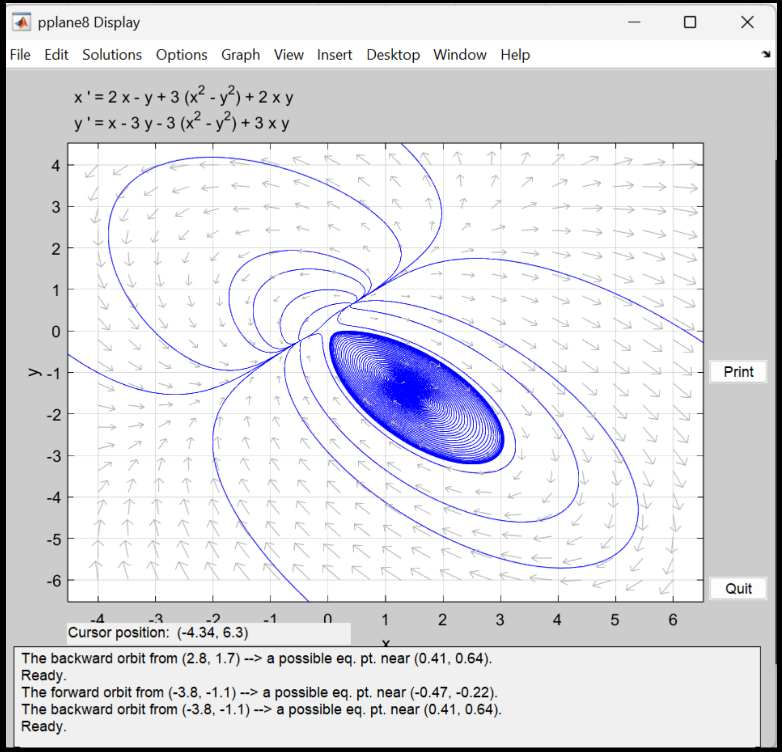
Positive Reinforcement and motivation

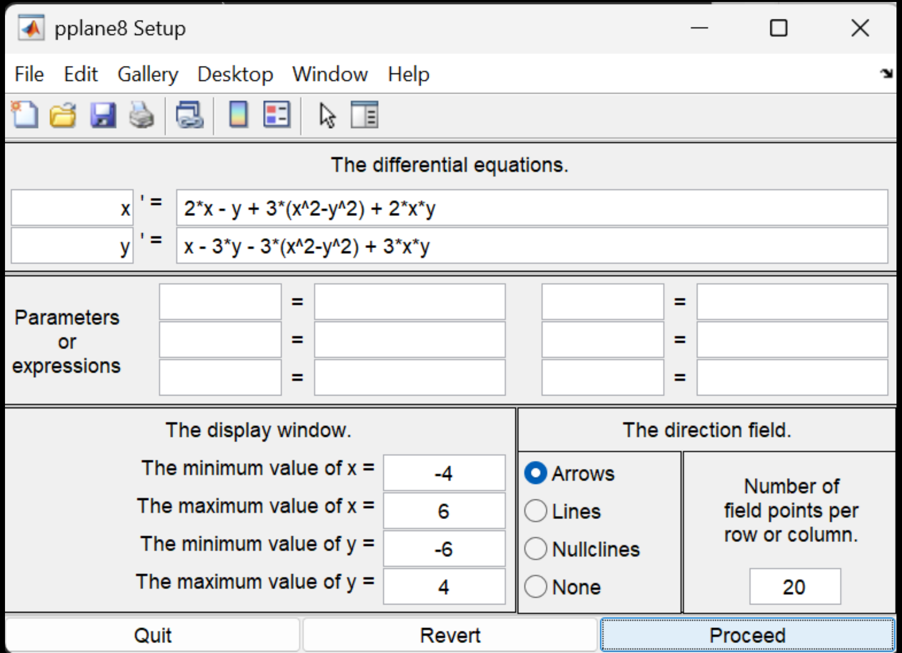
- Maintain high morale to encourage confidence and sustained engagement.
- Motivating goal: All expense paid trip to UT Austin to present at a research conference.

SOFTWARE TOOLS

1) Pplane
2) ODE 45

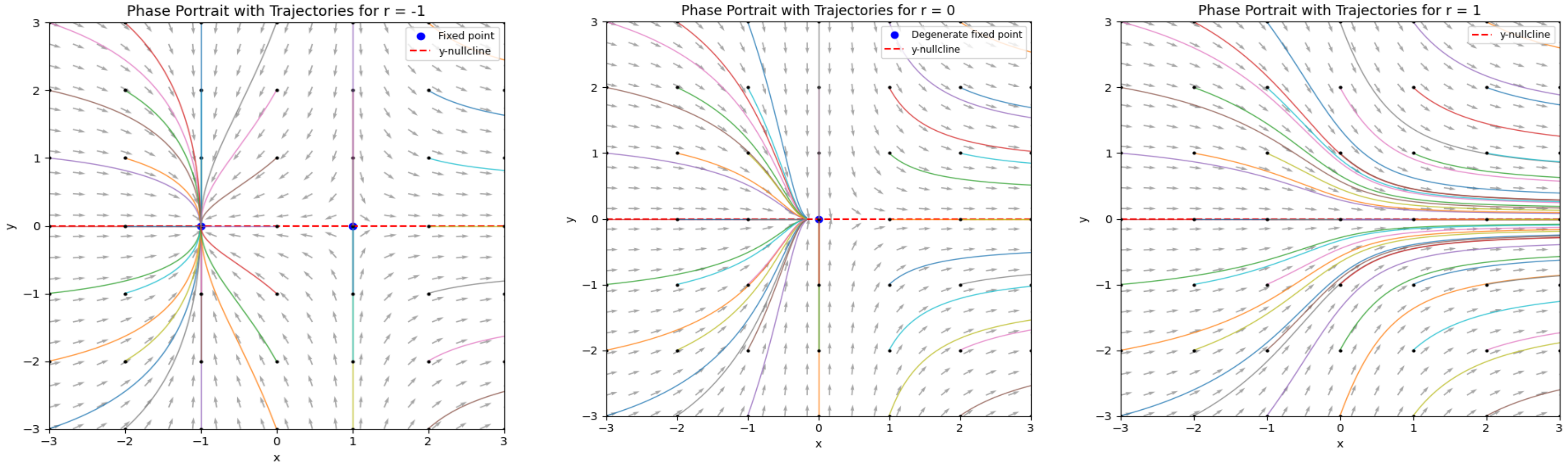
Well written code that students read, analyze, understand, and use instead of writing from scratch. AI in future?





BIFURCATIONS: CHANGE IN BEHAVIOR AS VARY PARAMETER

$$\begin{cases} \dfrac{dx}{dt} = r + x^2, \\ \dfrac{dy}{dt} = -y. \end{cases}$$



r = -1

r = 0

r = 1

HOW TO KEEP STUDENTS ON TRACK?
WHAT IF STUDENTS NOT PREPARED?

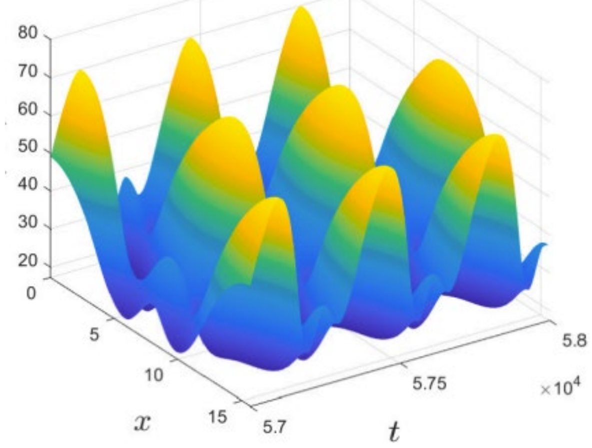
Think-Pair-Share:

- Present on the spot (we go offline for a bit).

$$\begin{cases} \dfrac{dx(t)}{dt} = rx(t)\left(1 - \dfrac{x(t)}{K}\right) - \dfrac{mx(t)y(t)}{a + x(t)}, \\ \dfrac{dy(t)}{dt} = -dy(t) + e^{-d\tau}\dfrac{cmx(t-\tau)y(t-\tau)}{a + x(t-\tau)}, \end{cases}$$

Initial condition is a trajectory!

$$\begin{aligned} S_t - r_1 \Delta S &= A - dS - \beta SI, \\ I_t - r_2 \Delta I &= \beta SI - dI - h(I)I \\ R_t - r_3 \Delta R &= h(I)I - dR, \end{aligned}$$



BACKGROUND (ALLIGATOR BIOLOGY)

1800s to mid-1900s: Intense hunting for leather and meat.

1950s to 1960s: Populations crash, poaching persists, weak enforcement.

1967: Listed as endangered in the United States.

1970s: States close hunting, protect nests, begin science-based management.

BACKGROUND (CONTINUED)

Late 1970s to 1980s: Rebound via protection, anti-poaching, wetland conservation.

1987: Delisted federally, managed as "threatened due to similarity of appearance."

FWC lottery permits: two alligators per permit.

Burmese Pythons

- Intentional releases by owners who could no longer keep large snakes.
- Accidental escapes from pet stores, breeders, and private collections.
- Facility damage during Hurricane Andrew (1992) likely released additional snakes into the Everglades.

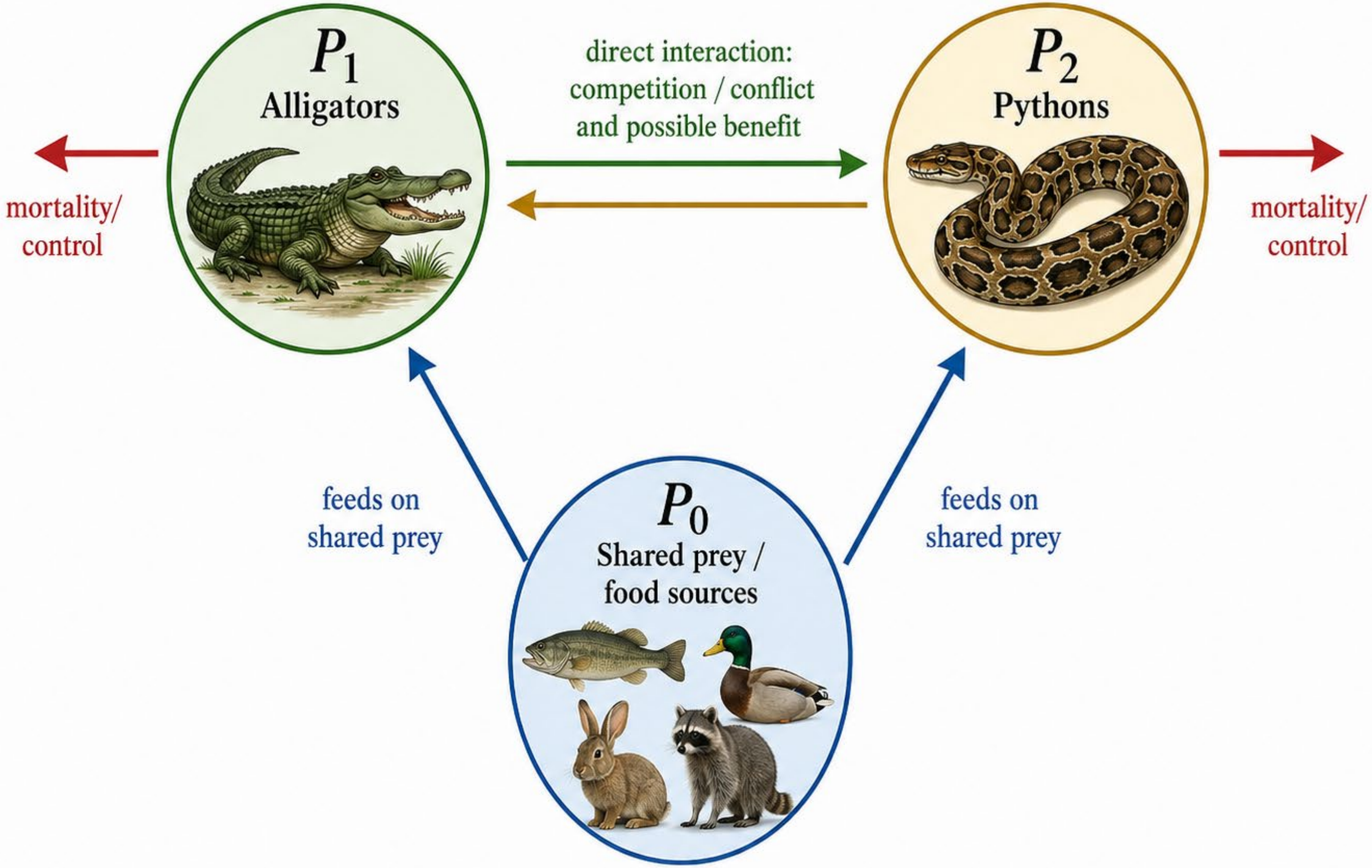
ECOLOGICAL IMPLICATIONS

- Sharp declines of midsized mammals in parts of Everglades (raccoons, opossums, marsh rabbits).

- Competition with native apex predators for prey, altering food webs.

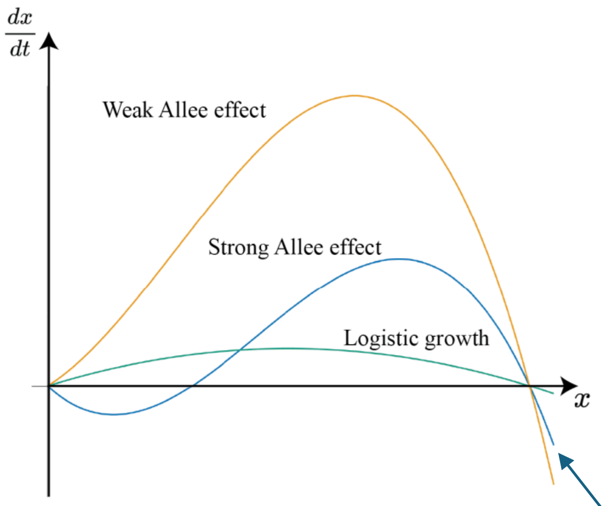
Interaction Diagram for Pythons–Alligators Model

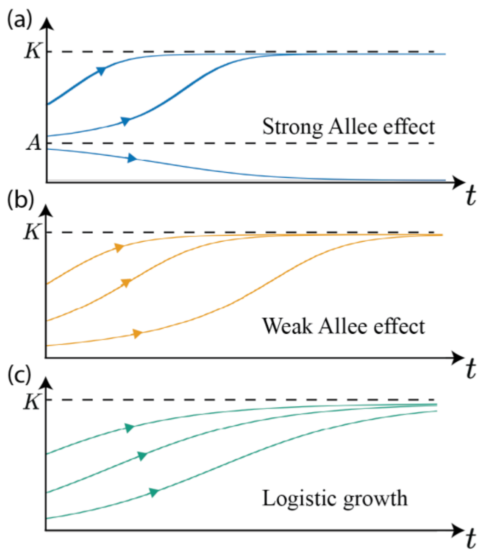
P_0 = *shared prey/food sources*, P_1 = *alligators*, P_2 = *pythons*

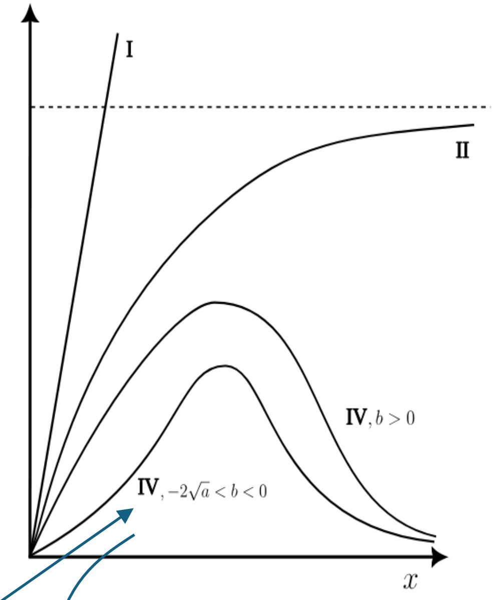


Generated using ChatGPT

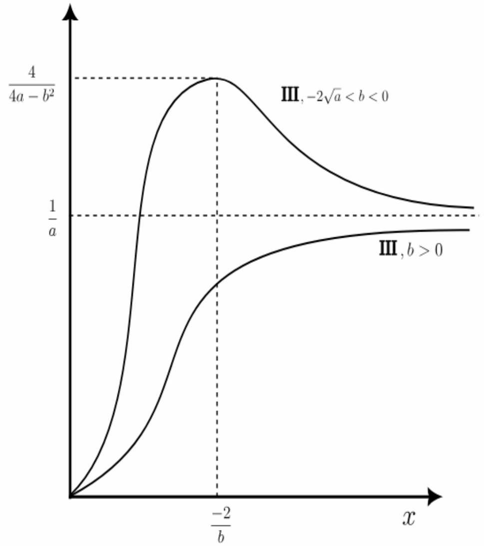
PP MODEL AND EXTENSIONS







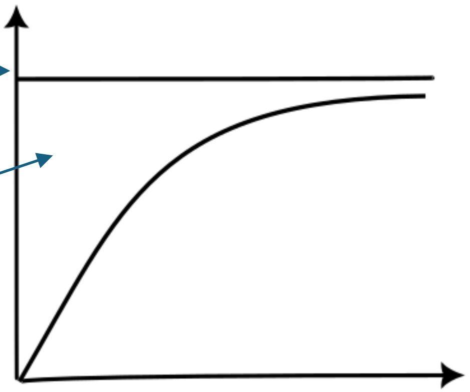
(a) Holling Type I,II, and IV



(b) Holling Type III

$$\frac{dx}{dt} = G_1(x) - \alpha_1 F(x)y - H_1(x)$$

$$\frac{dy}{dt} = -G_2(y) + \alpha_2 F(x)y - H_2(y)$$



FINAL MODEL

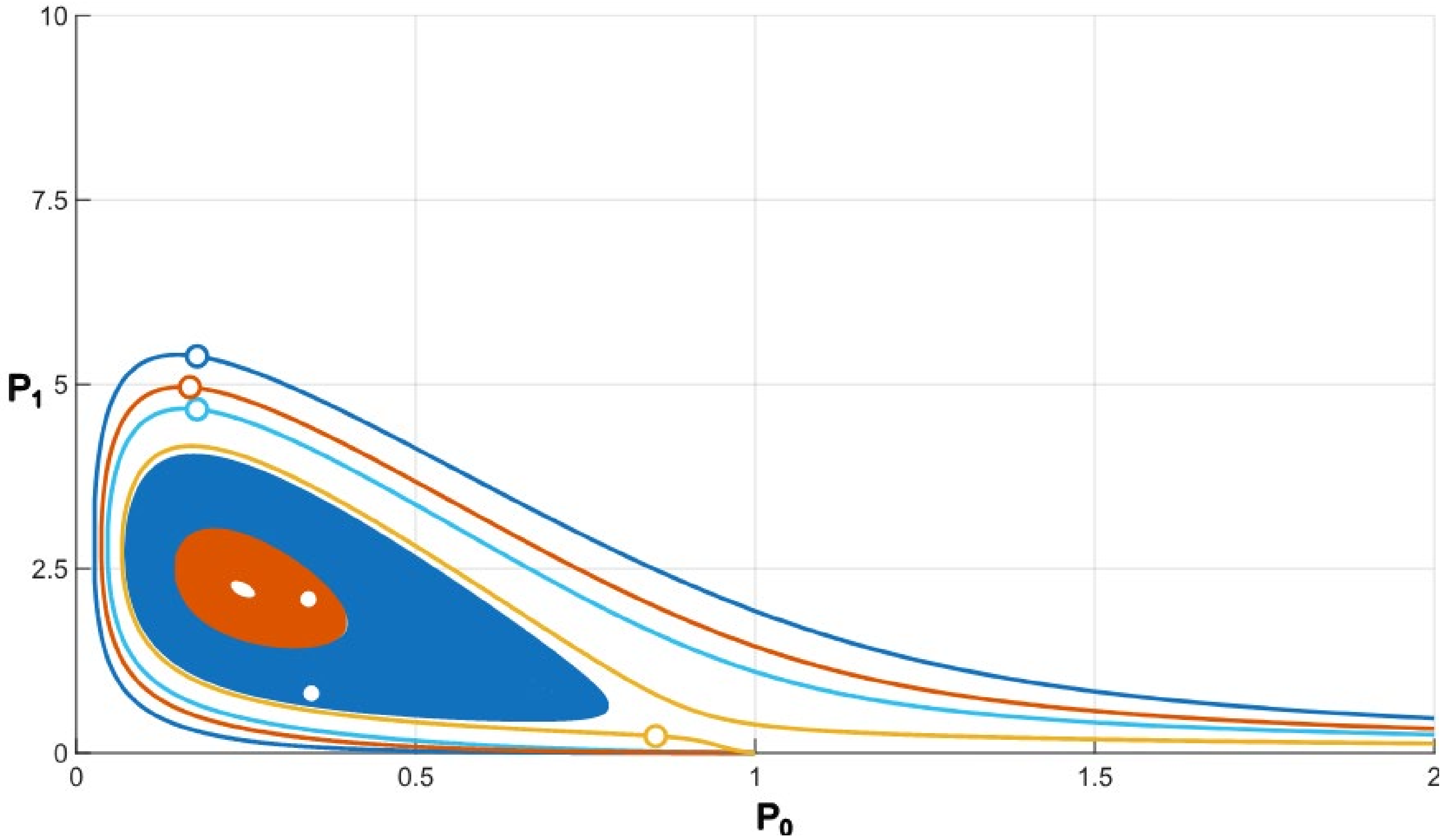
$$\begin{cases} \dfrac{dP_0}{dt} = rP_0\left(1 - \dfrac{P_0}{K}\right) - m_{10}P_0P_1 - m_{20}P_0P_2, \\ \dfrac{dP_1}{dt} = c_{10}P_0P_1 - d_1P_1 + (c_{12} - m_{21})P_1P_2 - \dfrac{h_1P_1}{a + P_1}, \\ \dfrac{dP_2}{dt} = c_{20}P_0P_2 + (c_{21} - m_{12})P_1P_2 - d_2P_2 - h_2P_2. \end{cases}$$

Equilibria: 7 in total

- 2 Equilibria on the P_0 axis.
- 2 equilibria on the $P_0 - P_1$ plane suggesting that the prey can coexist with the alligators.
- 2 non-zero equilibria where all 3 species coexist.
- 1 equilibrium in the $P_0 - P_2$ plane suggesting that python can coexist with the general prey.

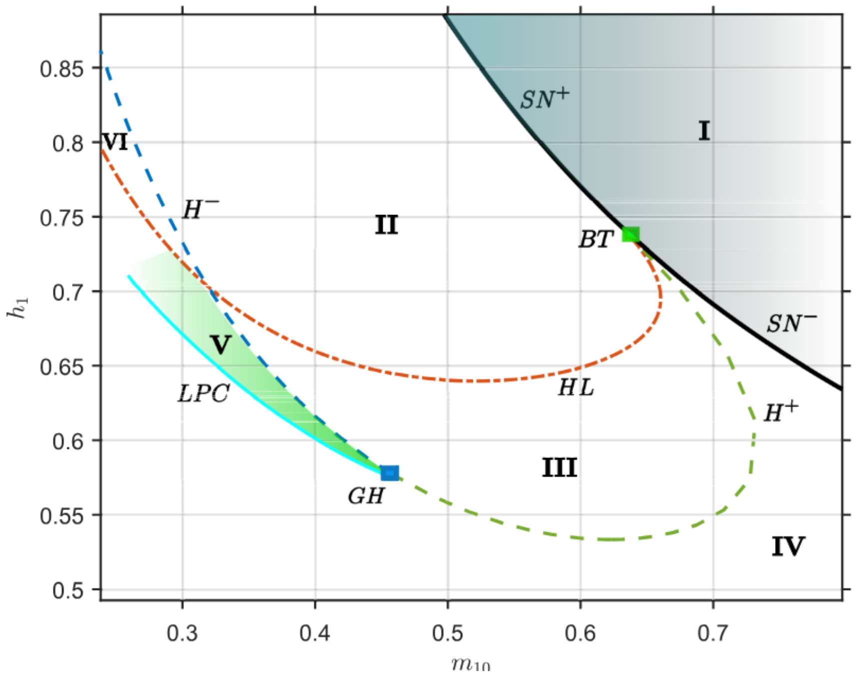
THE CASE WITH NO BURMESE PYTHONS

- The system provides grounds to modeling the relation between Alligators and general prey in the absence of pythons.



What happens if both Hopf and Saddle node occur simultaneously?

Students were able to do so using Bogdanov takens bifurcations, which explained to formation of limit cycles



1. **Shift the equilibrium:** Find the equilibrium $P^* = (P_0^*, P_1^*, P_2^*)$ and shift variables:

$$u = P_0 - P_0^*, \quad v = P_1 - P_1^*, \quad w = P_2 - P_2^*$$

so that the system becomes $\dot{z} = F(z)$ with $F(0) = 0$.

2. **Linearize and identify eigen-directions:** Compute the Jacobian $J = DF(0)$. At the BT point: $\mathrm{spec}(J) = \{0, 0, \lambda_s\}$ with $\lambda_s < 0$. The eigenvectors corresponding to zero eigenvalues define the center subspace.

3. **Center manifold reduction (3D $\rightarrow$ 2D):** There exists a smooth invariant manifold $w = h(u, v)$ with $h(0) = 0$, $Dh(0) = 0$. Substituting $w = h(u, v)$ reduces the system to:

$$\dot{u} = \tilde{F}_1(u, v), \quad \dot{v} = \tilde{F}_2(u, v)$$

4. **Reduced dynamics:** Expand $\tilde{F}_1, \tilde{F}_2$ in a Taylor series near the origin and retain quadratic terms:

$$\dot{u} = v + \mathcal{O}(2), \quad \dot{v} = au^2 + buv + \cdots$$

5. **Transform to BT normal form:** Apply smooth coordinate and parameter changes to obtain:

$$\boxed{\dot{x} = y, \quad \dot{y} = \beta_1 + \beta_2 x + ax^2 + bxy + \mathcal{O}(3)}$$

ULTIMATE GOAL: MAKING POLICY

- **QUESTION:** With harvesting restricted to 2 alligators per hunter per season, should the hunting policies be altered, especially that the common prey population is decreasing (racoons drop by 95%...)?
- **PROBLEM:** No good data is available, even with the Everglades administration. (only hunting records, sightings,…)
- **PLAN:**

Analyze without Burmese pythons, what does bifurcation theory say?

Introduce pythons and theoretical work on harvesting efforts.

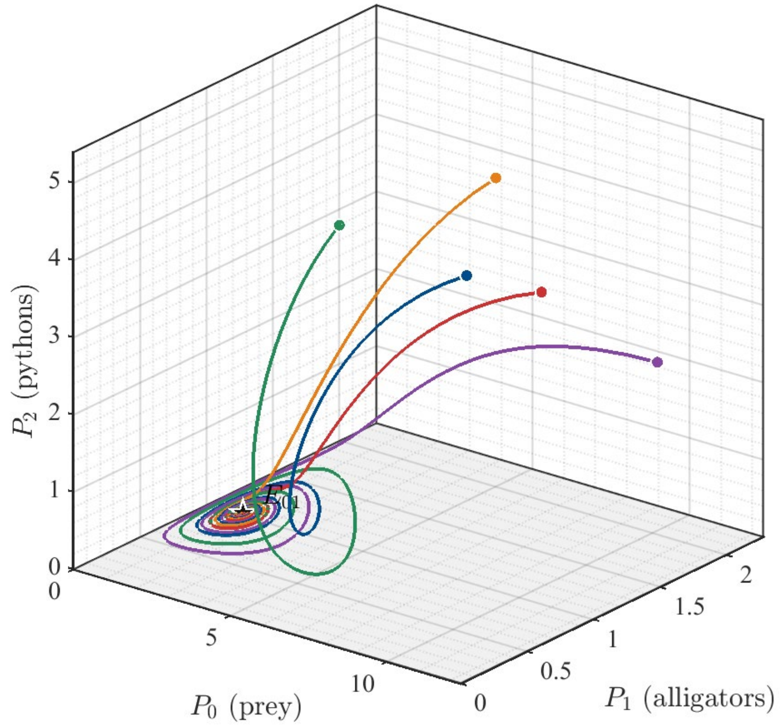
Analyze the formed thresholds.

DEDUCTION 1:

For

$$h_2 > h_2^{\mathrm{crit}}(h_1) = \frac{Kc_{20}[m_{20}(ad_1+h_1)+a(m_{21}-c_{12})r]-ad_2(Kc_{10}m_{20}+(m_{21}-c_{12})r)}{a(Kc_{10}m_{20}+(m_{21}-c_{12})r)},$$

Python population goes extinct.

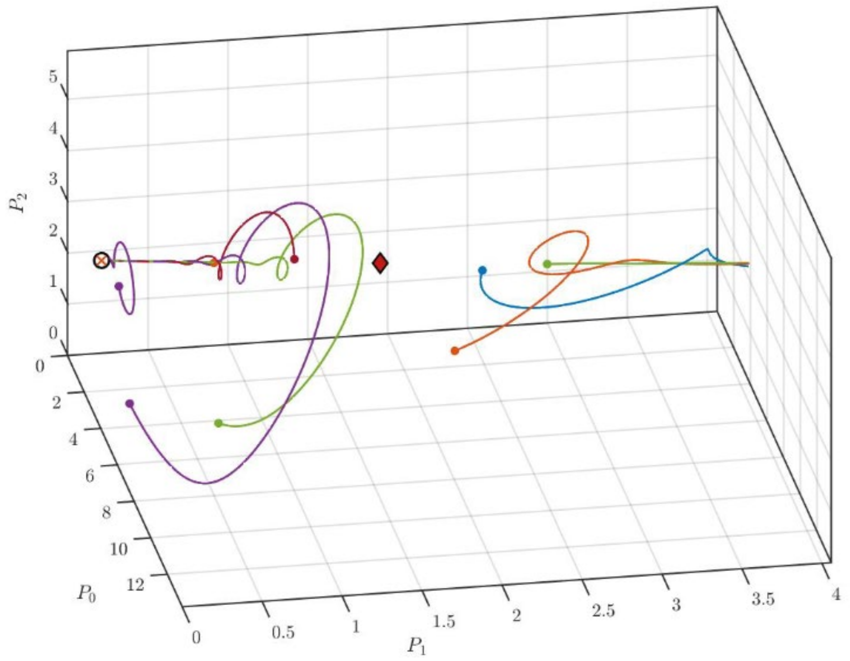


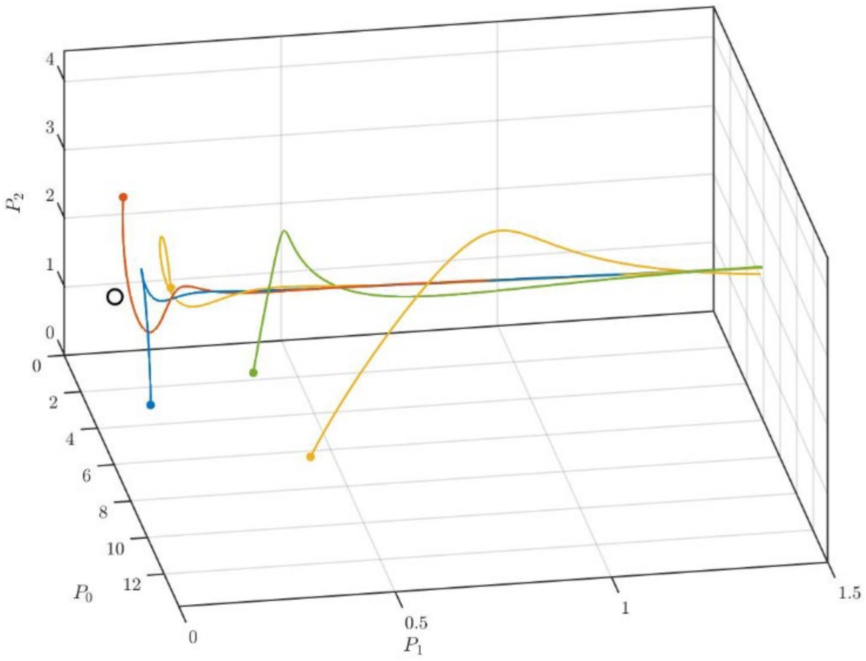
DEDUCTION 2: INTERACTION MODES

Let

$$A_{012} = Kc_{20}m_{10}(m_{21} - c_{12}) - Kc_{10}(c_{21} - m_{12})m_{20} - (c_{21} - m_{12})(m_{21} - c_{12})r,$$

If $A_{012} > 0$, the interaction between pythons and alligators is competitive:

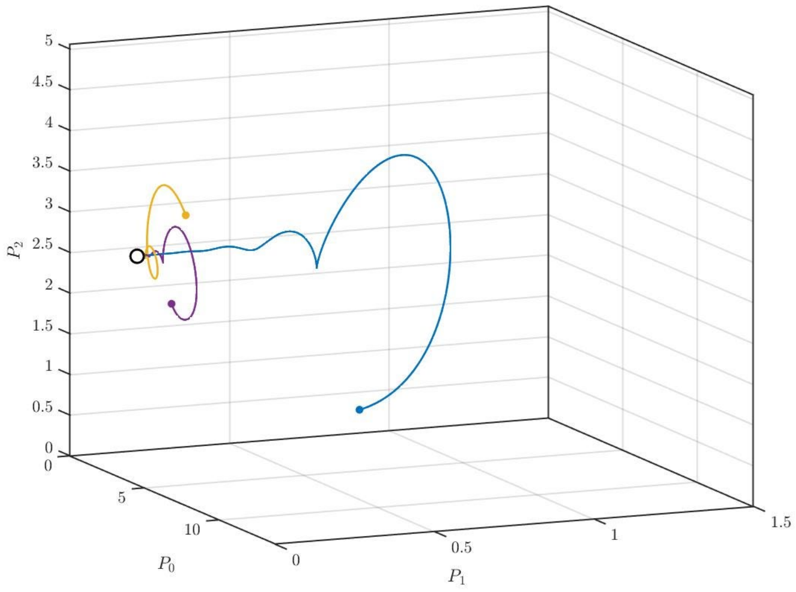




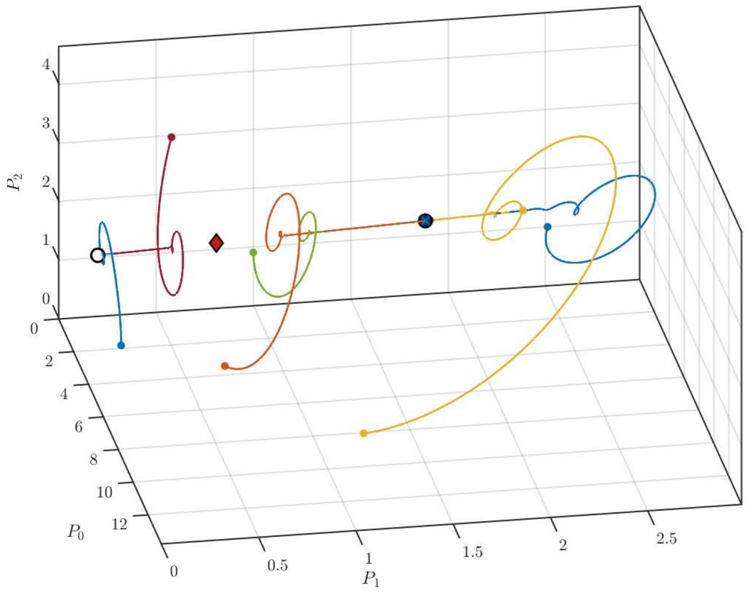
No Coexistance! One of the predators goes extinct!

If $A_{012} < 0$, the interaction sustains coexistence: alligators and pythons support one another

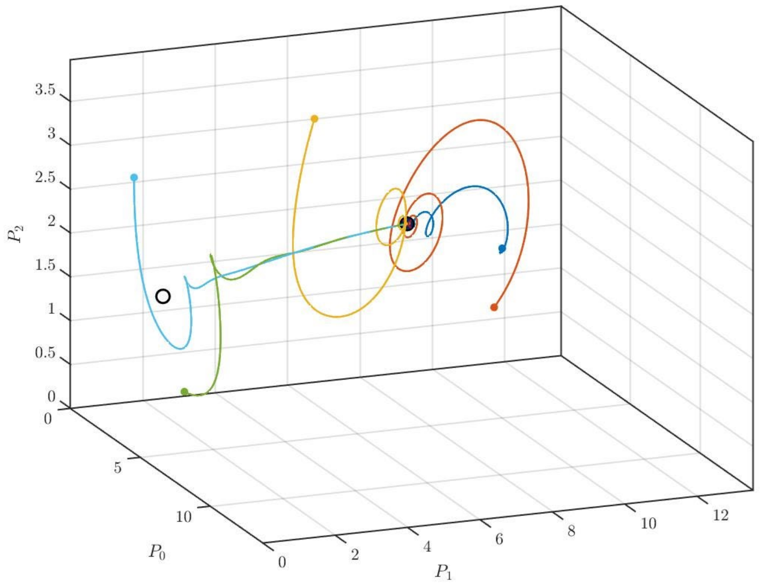
$h_2 < h_2^2 < h_2^3$



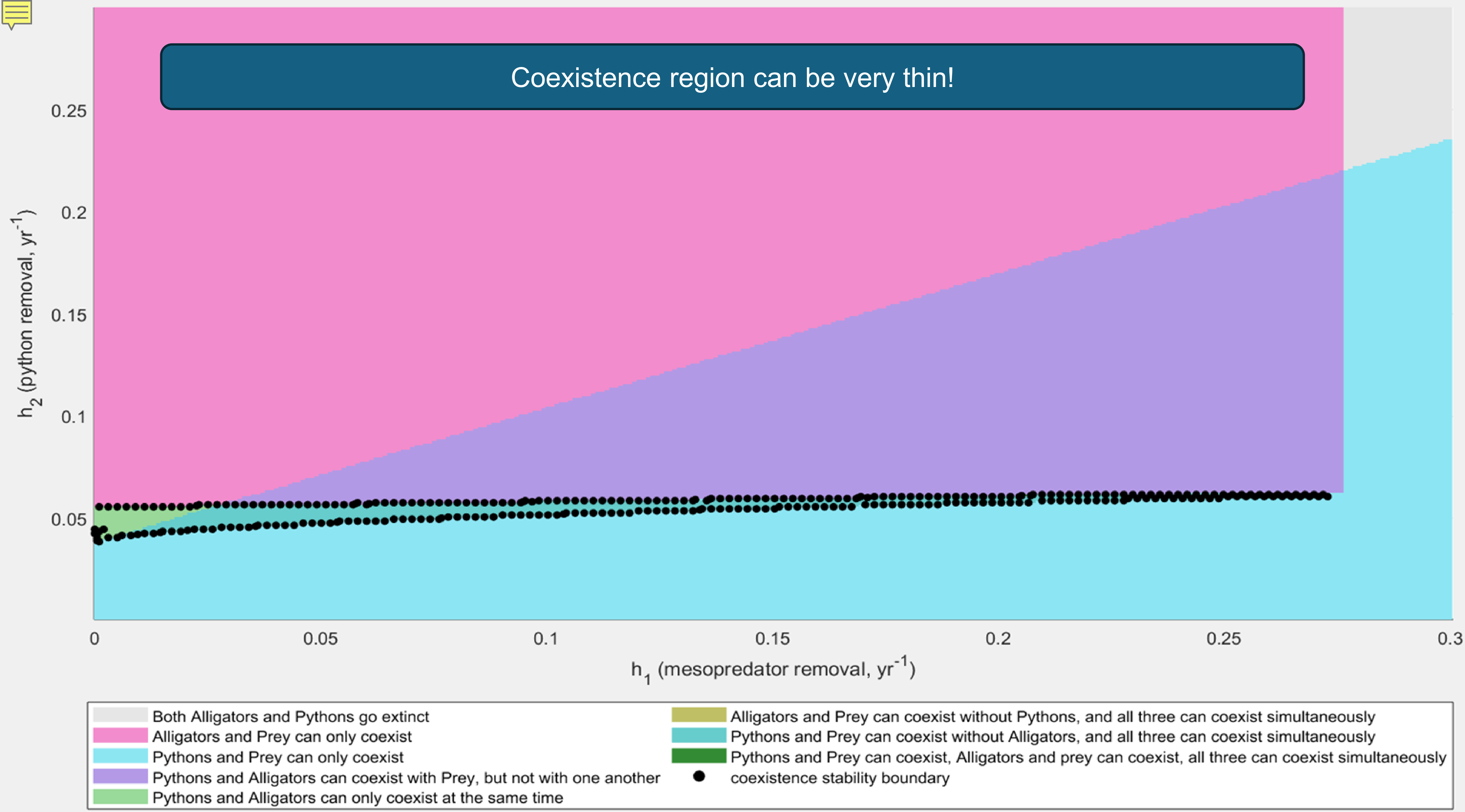
$h_2^2 < h_2 < h_2^3$



$h_2^2 < h_2^3 < h_2 < h_2^{critical}$



Coexistance is achievable. Low alligator population cannot persist if h_2 is relatively low.



Coexistence region can be very thin!
0.25
0.2
0.15
0.1
0.05
h2 (python removal, yr-1)
0
0.05
0.1
0.15
0.2
0.25
0.3
h1 (mesopredator removal, yr-1)
Both Alligators and Pythons go extinct
Alligators and Prey can only coexist
Pythons and Prey can only coexist
Pythons and Alligators can coexist with Prey, but not with one another
Pythons and Alligators can only coexist at the same time
Alligators and Prey can coexist without Pythons, and all three can coexist simultaneously
Pythons and Prey can coexist without Alligators, and all three can coexist simultaneously
Pythons and Prey can coexist, Alligators and prey can coexist, all three can coexist simultaneously
coexistence stability boundary

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1872

SIAM TX-LA Conference and JMM 2026

USEFUL TECHNIQUES / STRATEGIES.

- Ownership.
- Know strong / weak points (me vs Wael, experts).
- Reach out to experts.
- Always share something.
- Provide opportunities, but students must seize.

THANK YOU!

Contact Steven Miller at sjm1@williams.edu if interested in participating in the Polymath Jr Program (as student, TA, mentor, …)

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