From Alligators to All In - Generalized Predator-Prey Problems and Guided Online Research

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https://web.williams.edu/Mathematics/sjmiller/public_html/math/talks/talks.html

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Sections

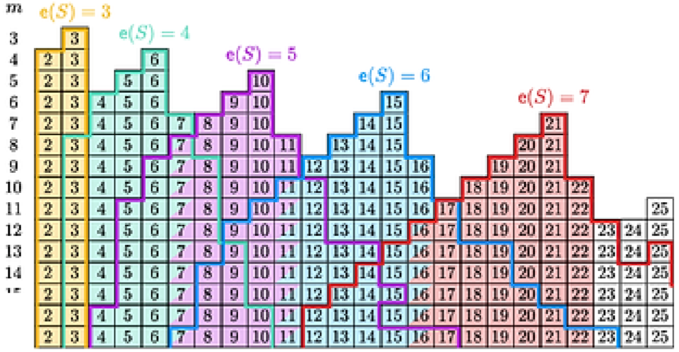
- 1) Polymath Jr.
- 2) Know your team
- 3) Research that makes a difference
- 4) PP models and how to approach
- 5) Results, Collaborations, future steps, and lessons learned

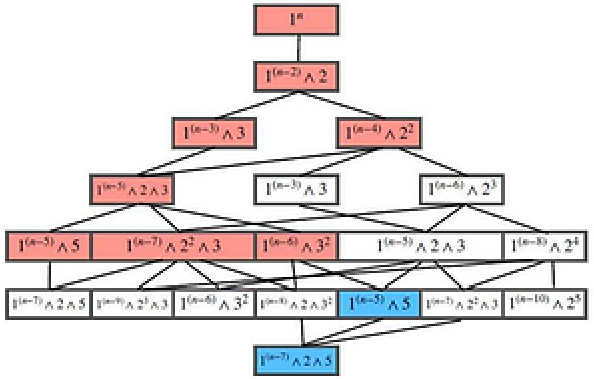
Polymath Jr.

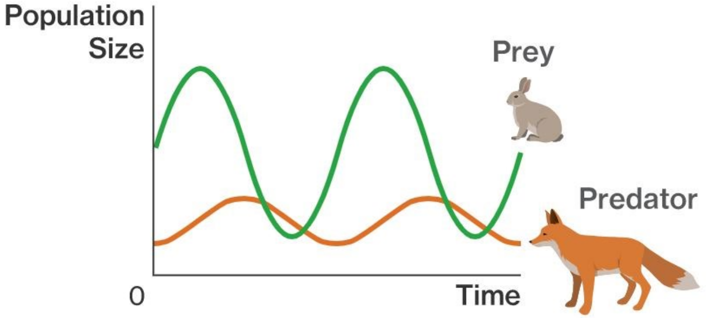
Our goal is to provide research opportunities to every undergraduate who wishes to explore advanced mathematics.

Each project is mentored by an active researcher with experience in undergraduate mentoring.

https://geometrynyc.wixsite.com/polymathreu







How Polymath Works

1) Polymath accepts hundreds of applications every year, with priority given to seniors.

2) All students attend a series of online sessions that introduce a wide range of topics and provide insight into potential projects.

3) Mentors receive training on handling stressful situations, leading effectively, giving students space, motivating them, and more.

4) Each graduate mentor is paired with a faculty supervisor who guides and oversees the project.

5) After these sessions, students choose their top three projects. Students are then matched with a mentor and a supervisor.

Meet the Team

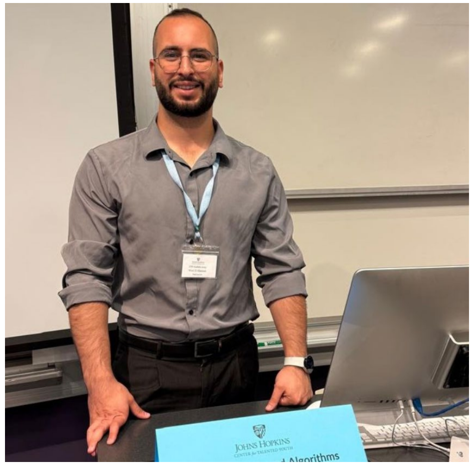
Anh Quynh Bui

Jasmine Pham

Jessie Wang

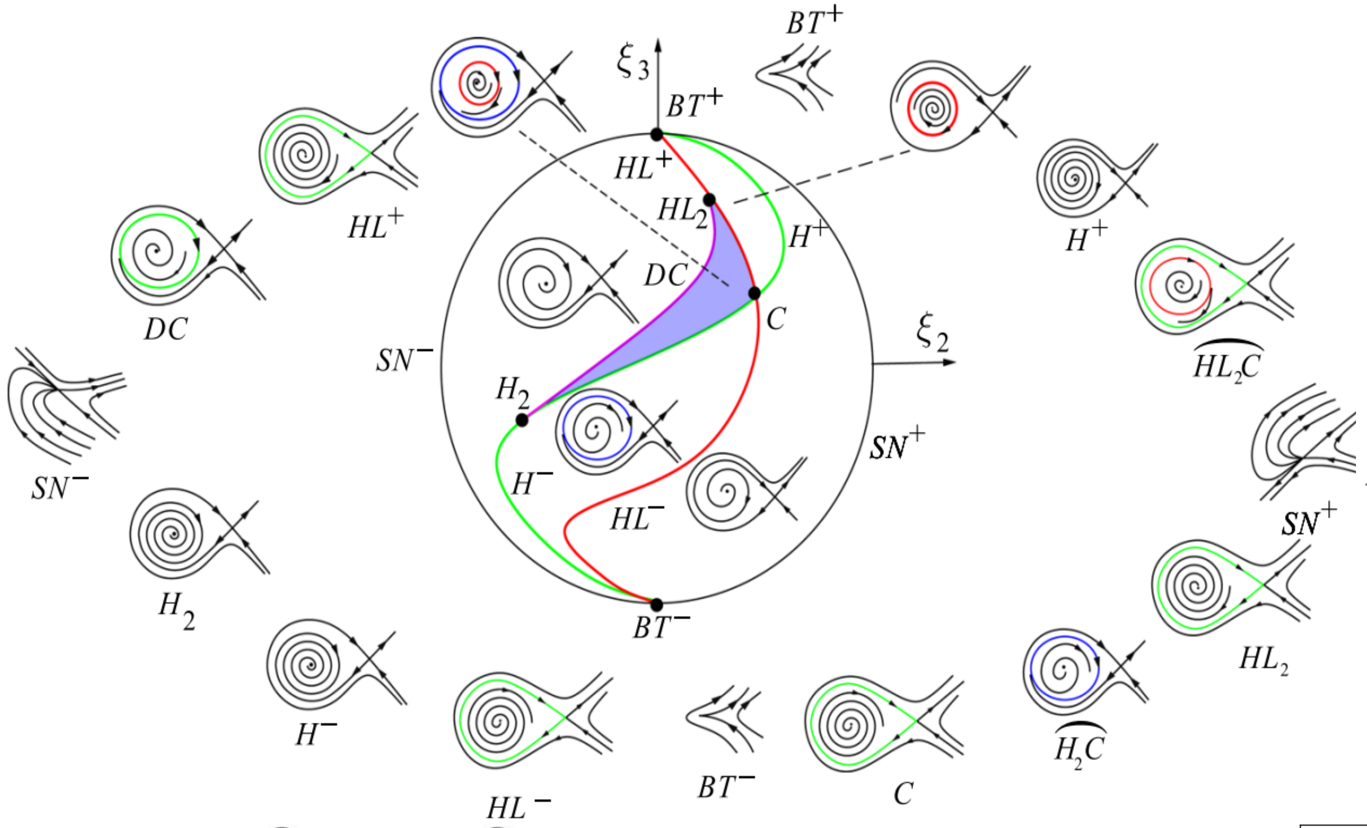
Sangam Dhakal

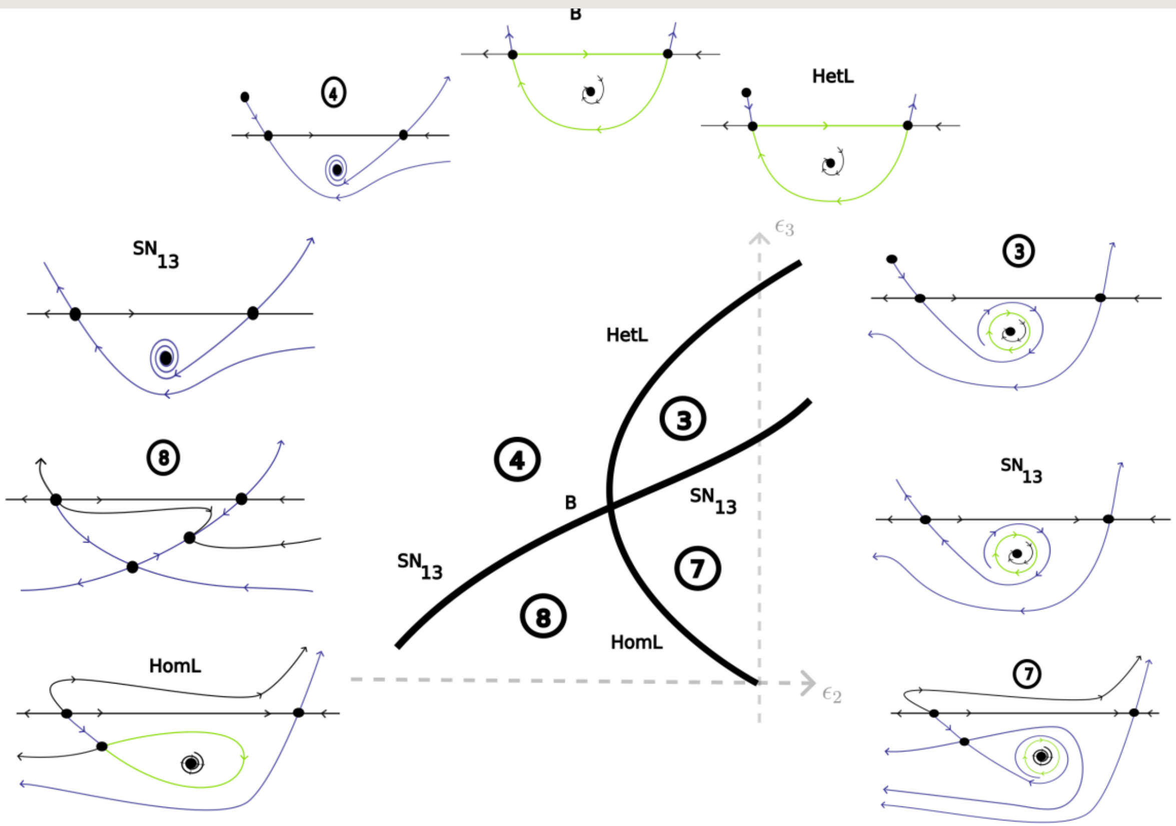
Steven Miller

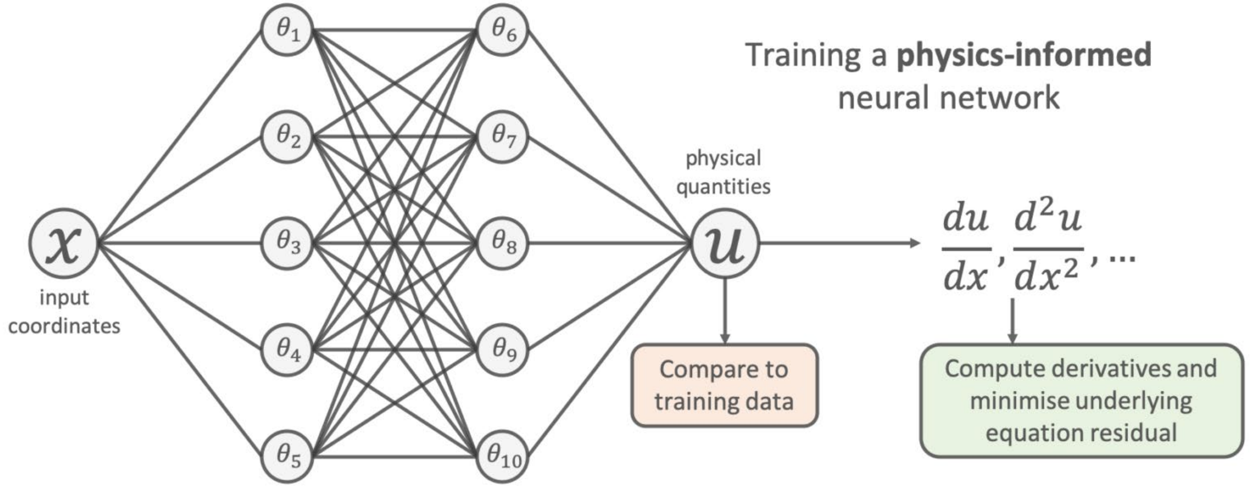


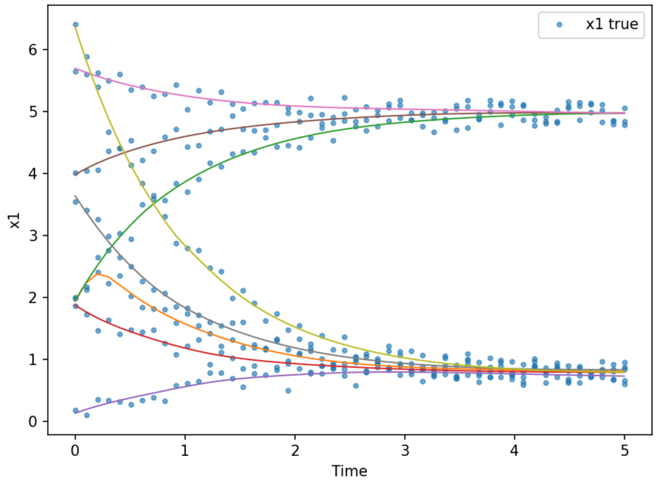
Wael El Khateeb

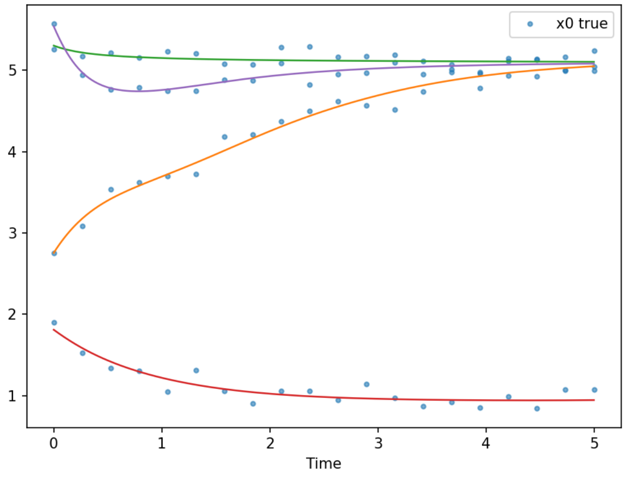
My Background











PHASE 1: Introduction

How to guide students without prior differential equations experience to produce high-quality research in just six weeks?

We established a clear framework built on the following principles.

Consistent and Timely Communication

- The average email response time, amazingly, did not exceed 30 minutes.

Resricted Material

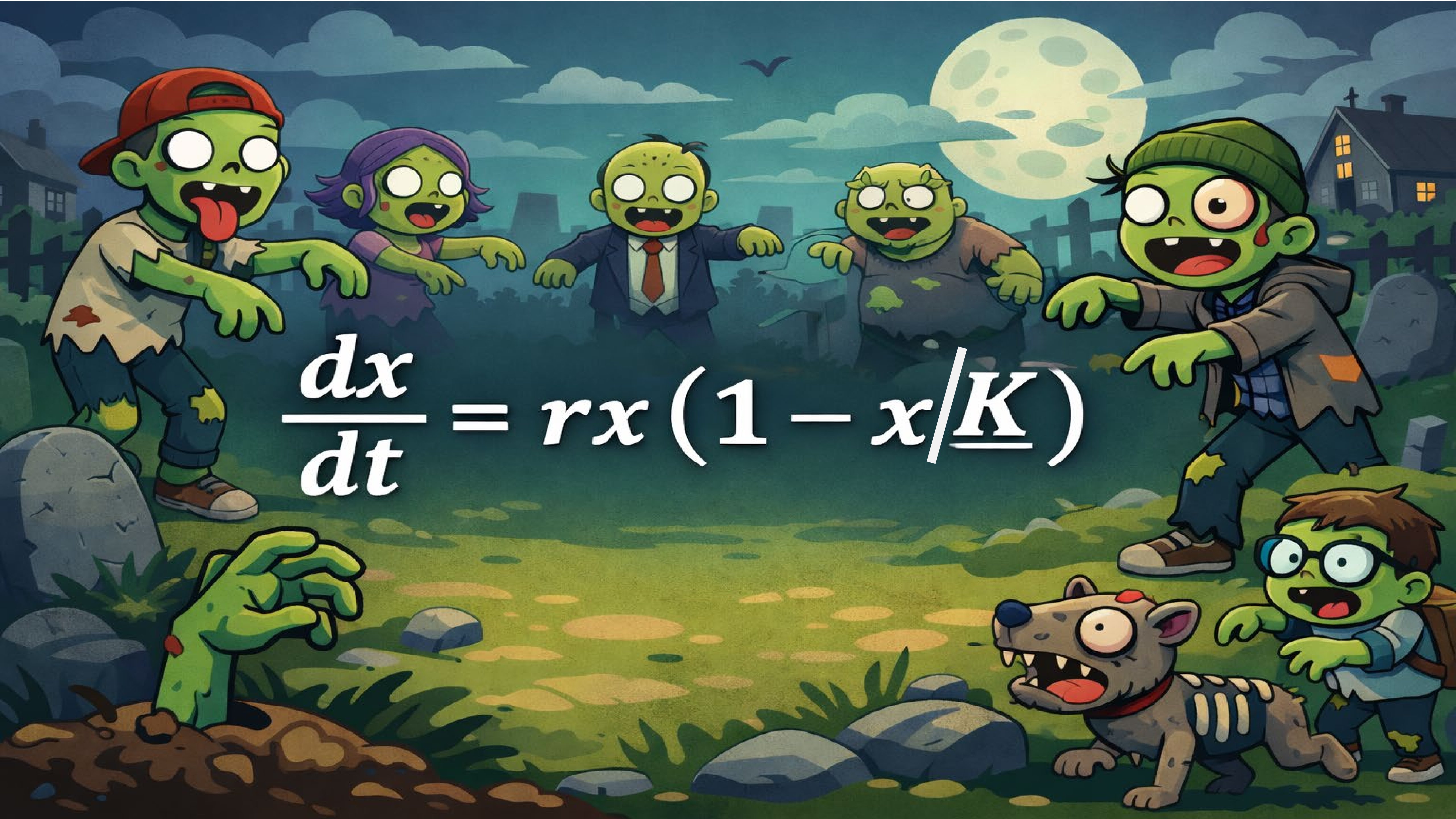
- Restriciting our sharing of knowledge to only the essential tools (System of ODEs, Jacobian, basic bifurcations,...).

Weekly group meetings

- Complemented by one-on-one office hours.

Positive Reinforcement and motivation

- Maintain high morale throughout the program to encourage confidence and sustained engagement.
- Motivating goal: All expense paid trip to UT Austin to present at a research conference.



$$\frac{dx}{dt} = rx\left(1 - x/K\right)$$

Find the values of the functions

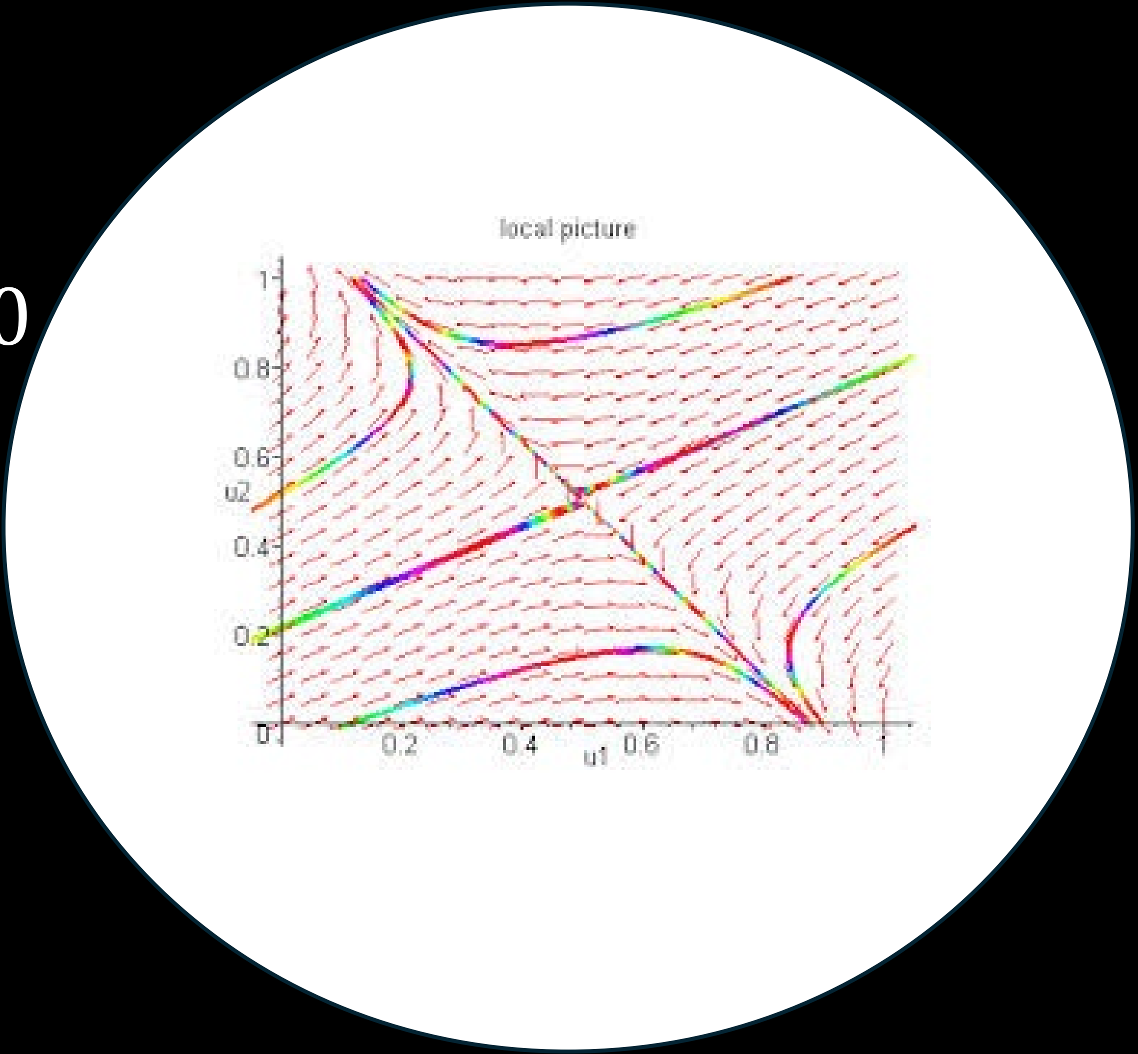
$$f_1(x) = x, f_2(x) = x^2, f_3(x) = x^3$$

near $x = 100, x = 1000, x = 10000$

Now do the same for

$$x = 0.01, x = 0.001, x = 0.0001$$

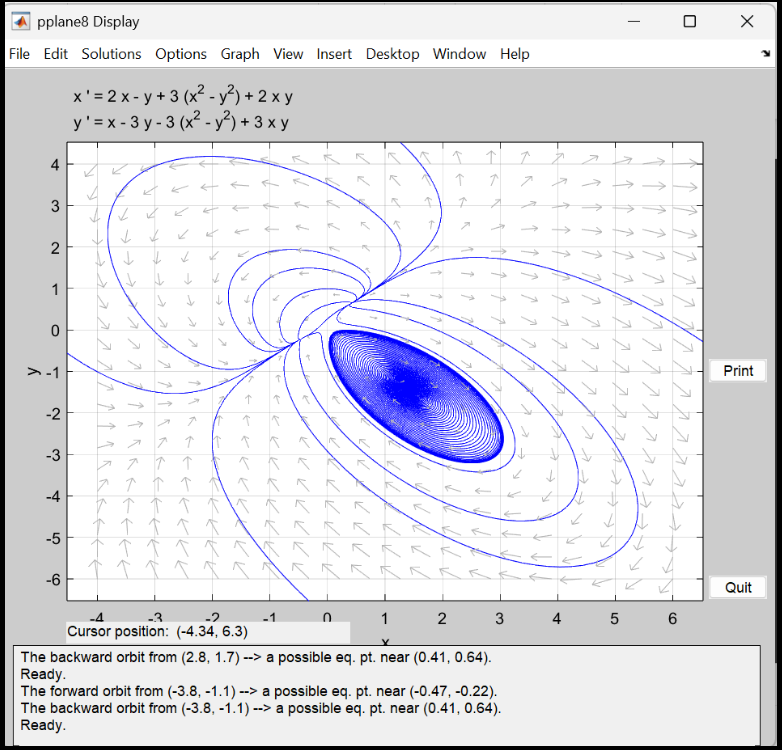
What can you conclude?

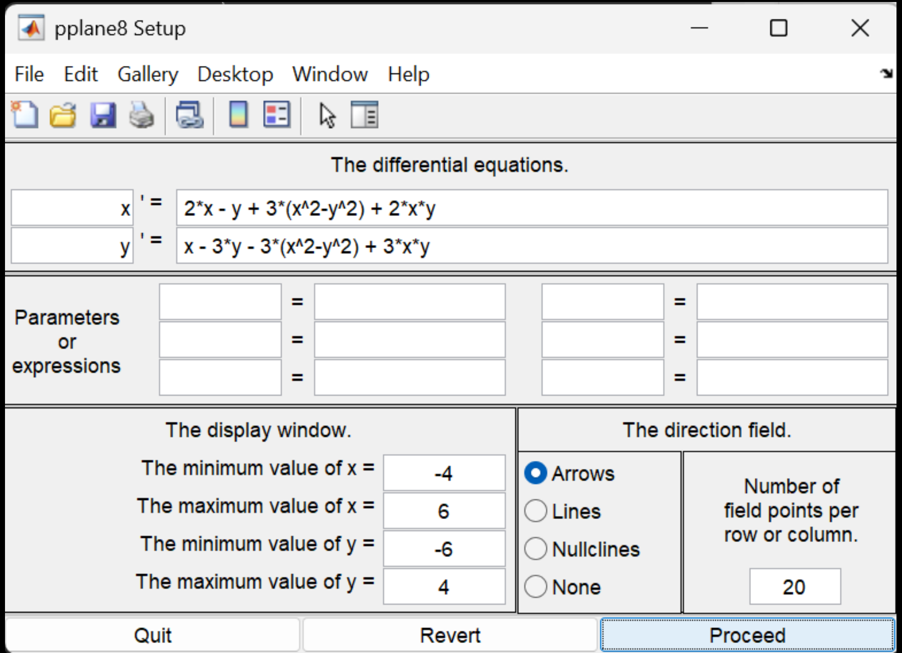


Software tools

1) pplane
2) ODE 45

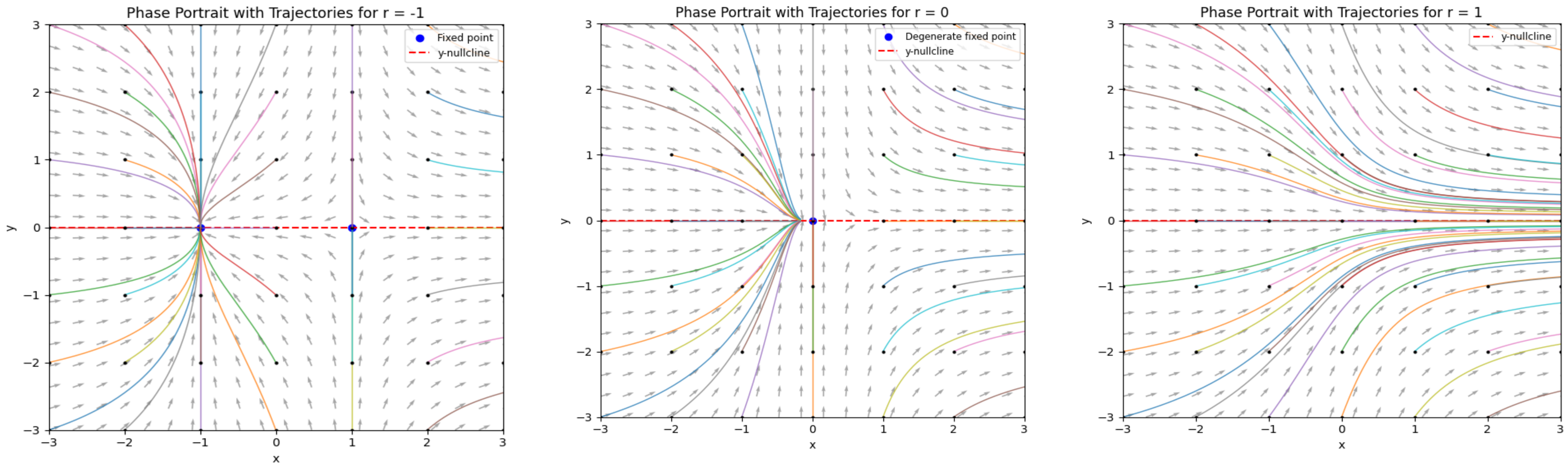
Well written code that students read, analyze, understand, and use instead of writing from scratch.





Bifurcations: qualitative change in the behavior of a system as a parameter is varied.

$$\begin{cases} \frac{dx}{dt} = r + x^2, \\ \frac{dy}{dt} = -y. \end{cases}$$



HOW TO KEEP STUDENTS ON TRACK?
WHAT IF STUDENTS WERE NOT PREPARED?

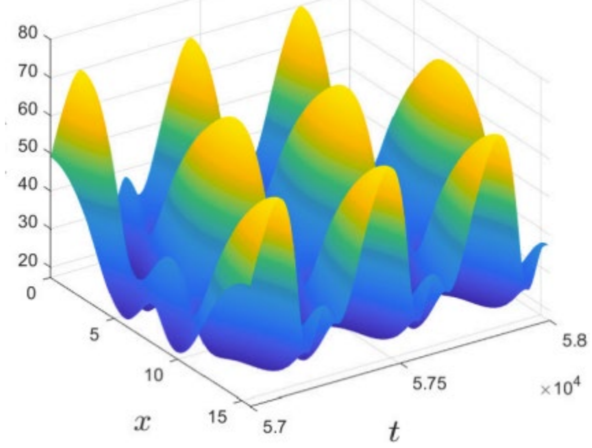
$$\begin{cases} \dfrac{dx(t)}{dt} = rx(t)\left(1 - \dfrac{x(t)}{K}\right) - \dfrac{mx(t)y(t)}{a + x(t)}, \\ \dfrac{dy(t)}{dt} = -dy(t) + e^{-d\tau}\dfrac{cmx(t-\tau)y(t-\tau)}{a + x(t-\tau)}, \end{cases}$$

Initial condition is a trajectory!

Think-Pair-Share

Slide preparation on the spot

$$S_t - r_1 \Delta S = A - dS - \beta SI,$$
$$I_t - r_2 \Delta I = \beta SI - dI - h(I)I$$
$$R_t - r_3 \Delta R = h(I)I - dR,$$



BACKGROUND (BIOLOGY)

1800s to mid-1900s: Intense hunting for leather and meat.

1950s to 1960s: Populations crash; poaching persists; weak enforcement.

1967: Listed as endangered in the United States.

1970s: States close hunting, protect nests, begin science-based management.

BACKGROUND (BIOLOGY)

Late 1970s to 1980s: Rebound via protection, anti-poaching, wetland conservation.

1987: Delisted federally; managed as "threatened due to similarity of appearance."

FWC lottery permits; two alligators per permit.

Burmese Pythons

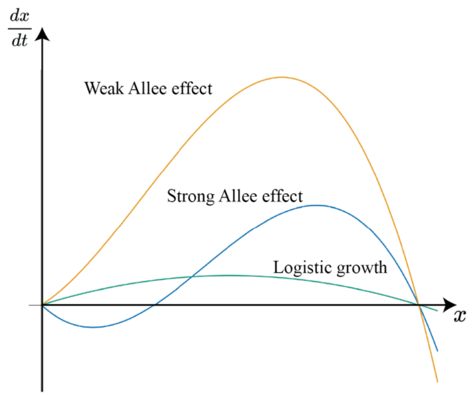
- Intentional releases by owners who could no longer keep large snakes.
- Accidental escapes from pet stores, breeders, and private collections.
- Facility damage during Hurricane Andrew (1992) likely released additional snakes into the Everglades.

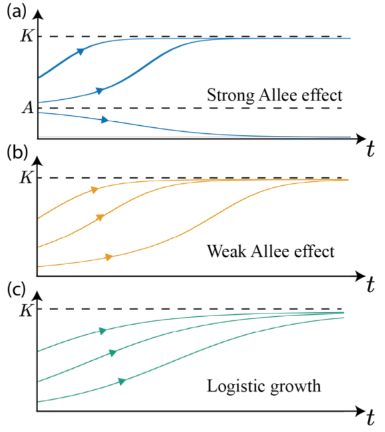
ECOLOGICAL IMPLICATIONS

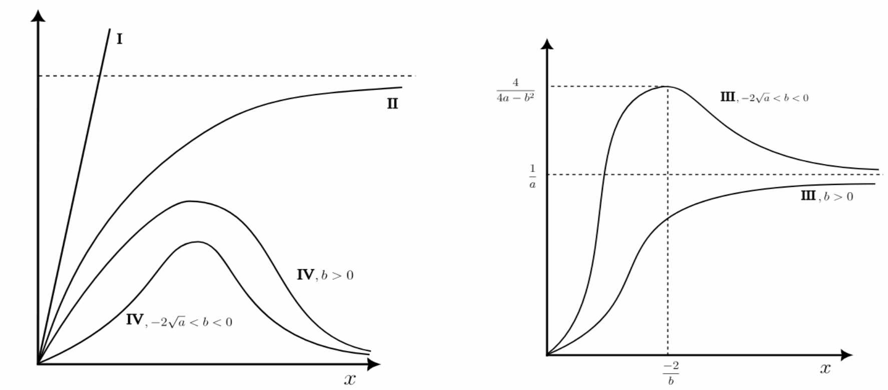
- Sharp declines of midsized mammals in parts of the Everglades (raccoons, opossums, marsh rabbits)

- Competition with native apex predators for prey, altering food webs

PP model and extensions



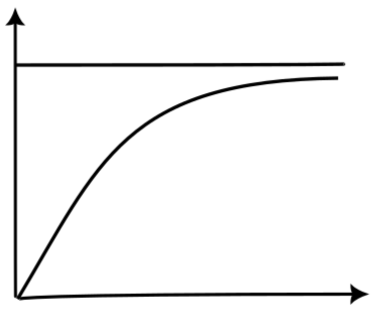




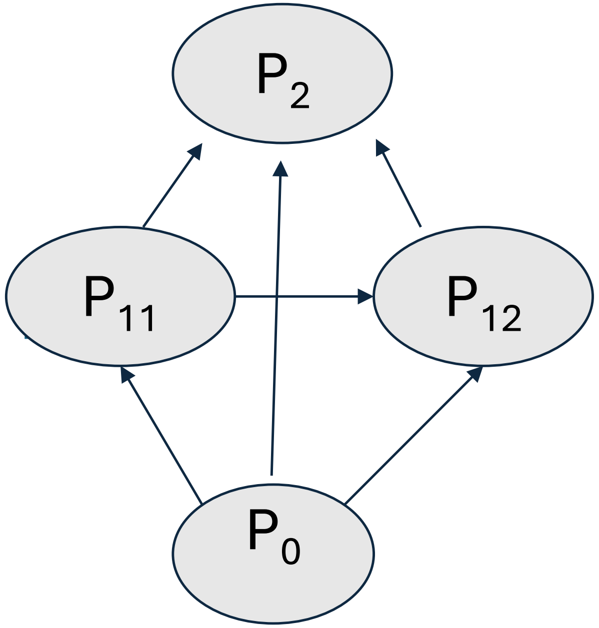
(a) Holling Type I,II, and IV (b) Holling Type III

$$\frac{dx}{dt} = G_1(x) - \alpha_1 F(x)y - H_1(x)$$

$$\frac{dy}{dt} = -G_2(y) + \alpha_2 F(x)y - H_2(y)$$



Model Illustrated:



P_0 - Raccoon/prey population

P_{11} - Juvenile Alligator population

P_{12} - Adult Alligator population

P_2 - Burmese Python population

Initial Model

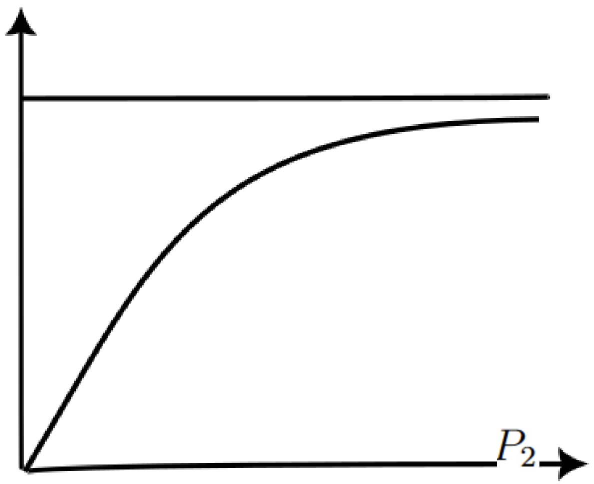
$$\begin{cases} \frac{dP_0}{dt} &= rP_0 - m_{20}P_0P_2 - m_{30}P_0P_2 \\ \frac{dP_1}{dt} &= c_{20}P_0P_2 - m_{31}P_1P_3 - d_{11}P_{11} \\ \frac{dP_2}{dt} &= d_{12}P_1 - d_2P_2 - h_2 \\ \frac{dP_2}{dt} &= c_{30}P_0P_3 + c_{31}P_3P_1 - d_3P_2 - h_3P_2 \end{cases}$$

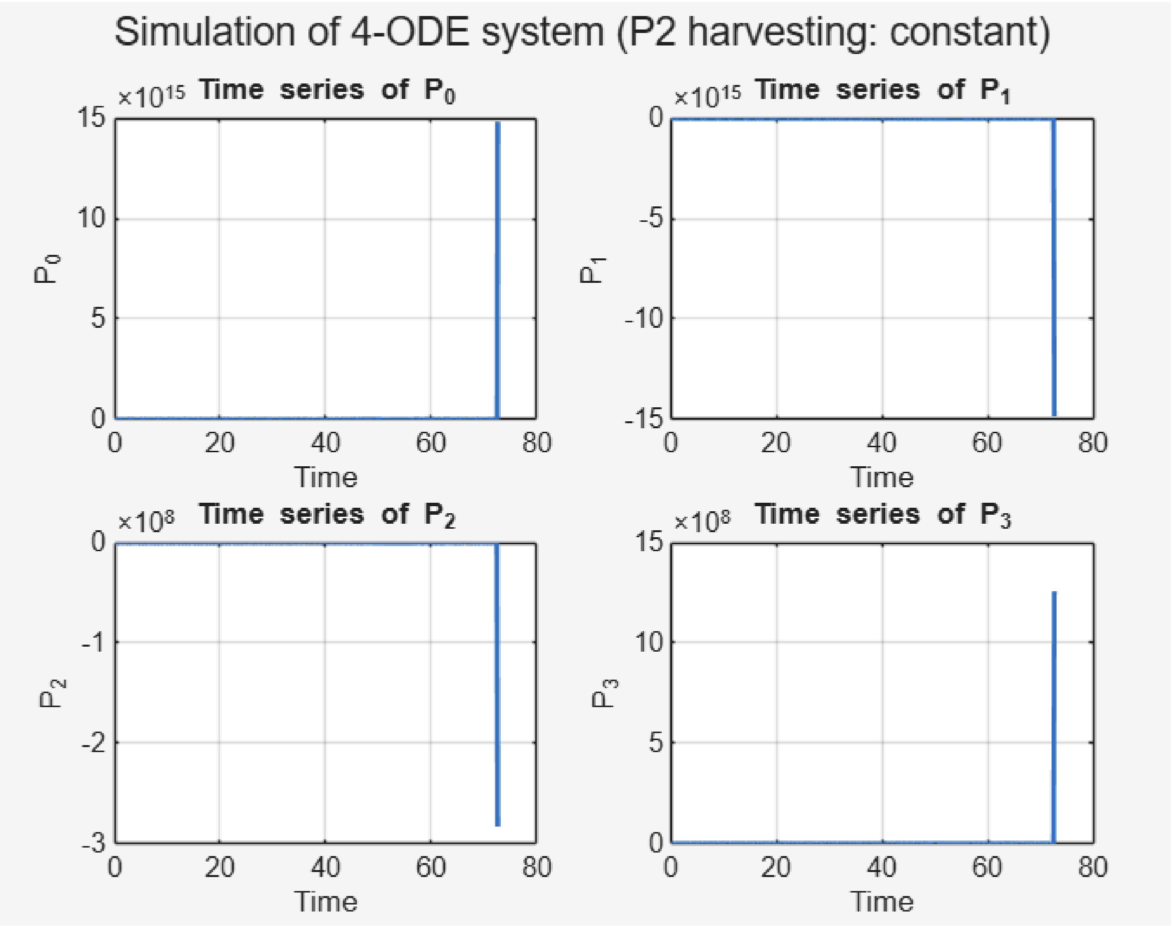
$h_2 \Rightarrow$ ***Saturated***

$$\begin{cases} \frac{dP_0}{dt} &= rP_0 - m_{20}P_0P_{12} - m_{30}P_0P_2 \\ \frac{dP_{11}}{dt} &= c_{20}P_0P_{12} - m_{31}P_{11}P_3 - d_{11}P_{11} \\ \frac{dP_{12}}{dt} &= d_{12}P_{11} - d_2P_{12} - h_2\frac{P_{12}}{a+P_{12}} \\ \frac{dP_2}{dt} &= c_{30}P_0P_2 + c_{31}P_2P_{11} - d_3P_2 - h_3P_2 \end{cases}$$

Why constant harvesting doesn't work

Applying constant harvesting shows that P_1 and P_2 eventually become **negative**, which is biologically unrealistic and highlights why **constant harvesting does not work.**





Final Model

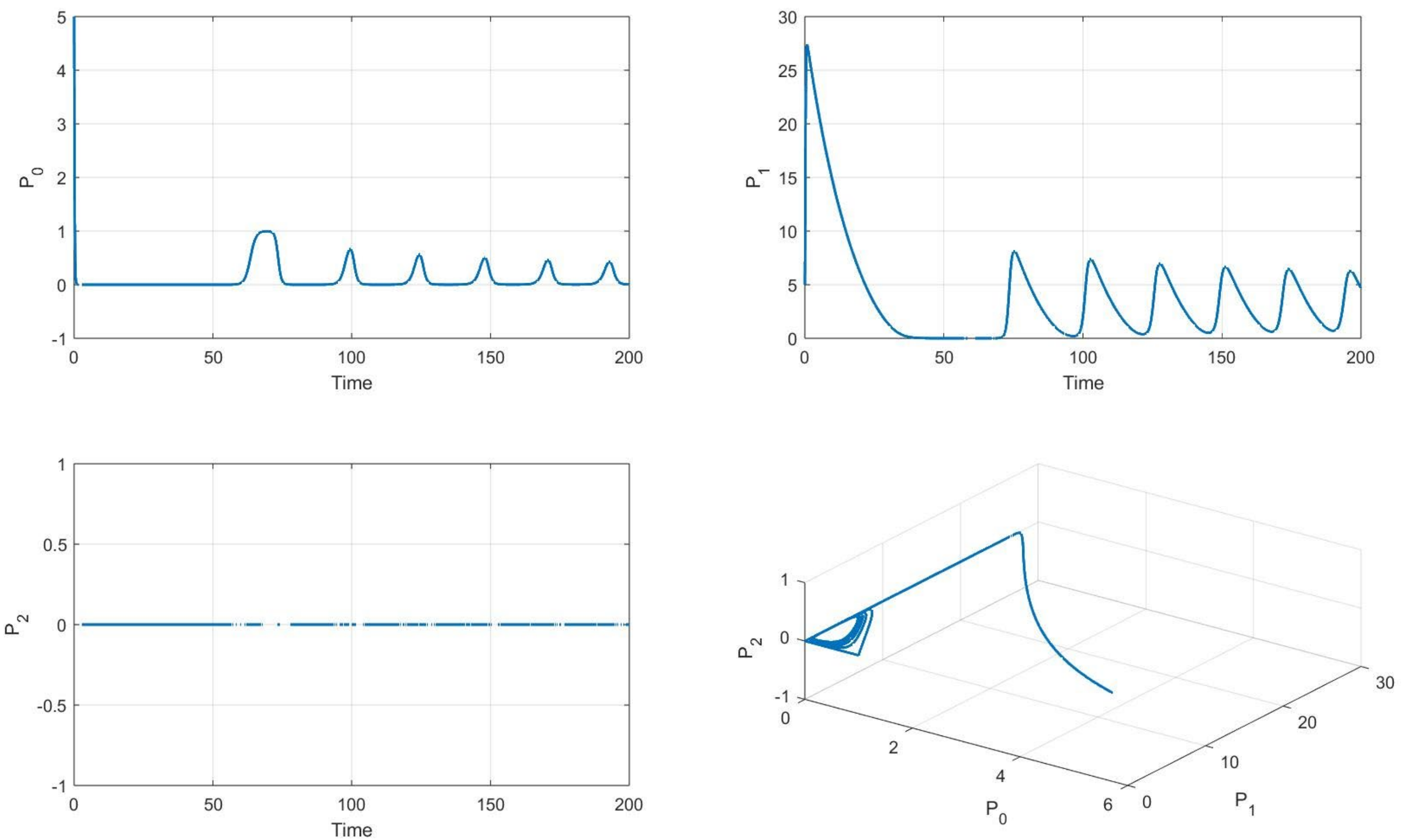
$$\begin{cases} \dfrac{dP_0}{dt} = rP_0\left(1 - \dfrac{P_0}{K}\right) - m_{10}P_0P_1 - m_{20}P_0P_2, \\ \dfrac{dP_1}{dt} = c_{10}P_0P_1 - d_1P_1 - \dfrac{h_1P_1}{a + P_1} - m_{31}P_1P_2, \\ \dfrac{dP_2}{dt} = c_{20}P_0P_2 + c_{21}P_1P_2 - d_2P_2 - h_2P_2. \end{cases}$$

Equilibria: 7 in total

- 2 Equilibria on the P_0 axis.
- 2 equilibria on the $P_0 - P_1$ plane suggesting that the prey can coexist with the alligators.
- 2 non-zero equilibria where all 3 species coexist.
- 1 equilibrium in the $P_0 - P_2$ plane suggesting that python can coexist with the general prey.

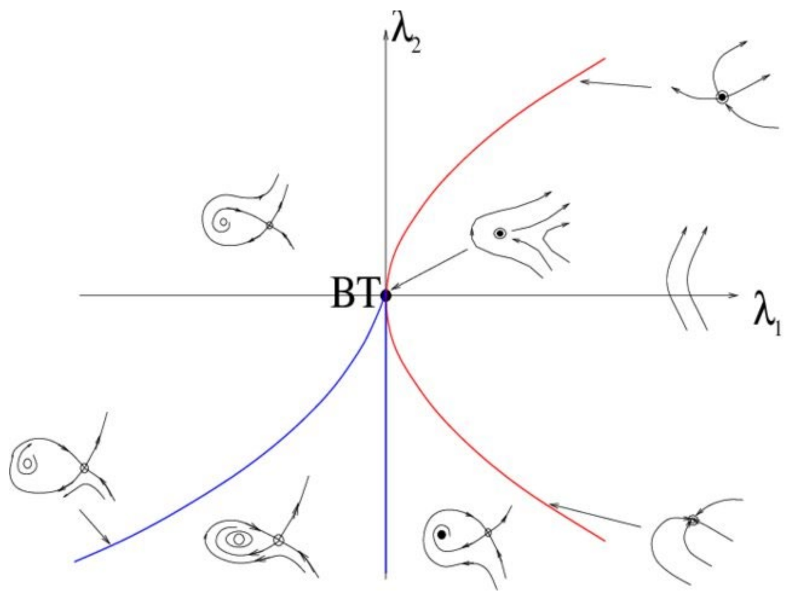
THE CASE WITH NO BURMESE PYTHONS

- The system provides grounds to modeling the relation between Alligators and general prey in the absence of pythons.



What happens if both Hopf and Saddle node occur simultaneously?

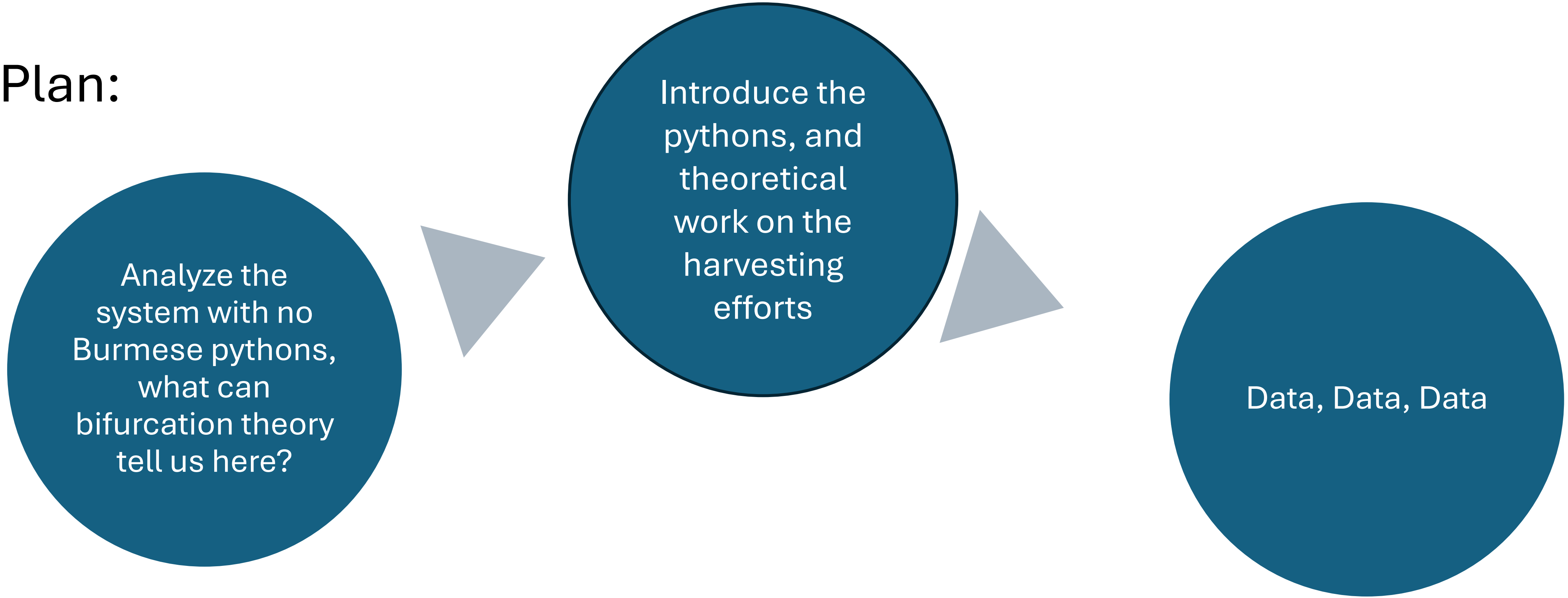
Students were able to do so using Bogdanov takens bifurcations, which explained to formation of limit cycles



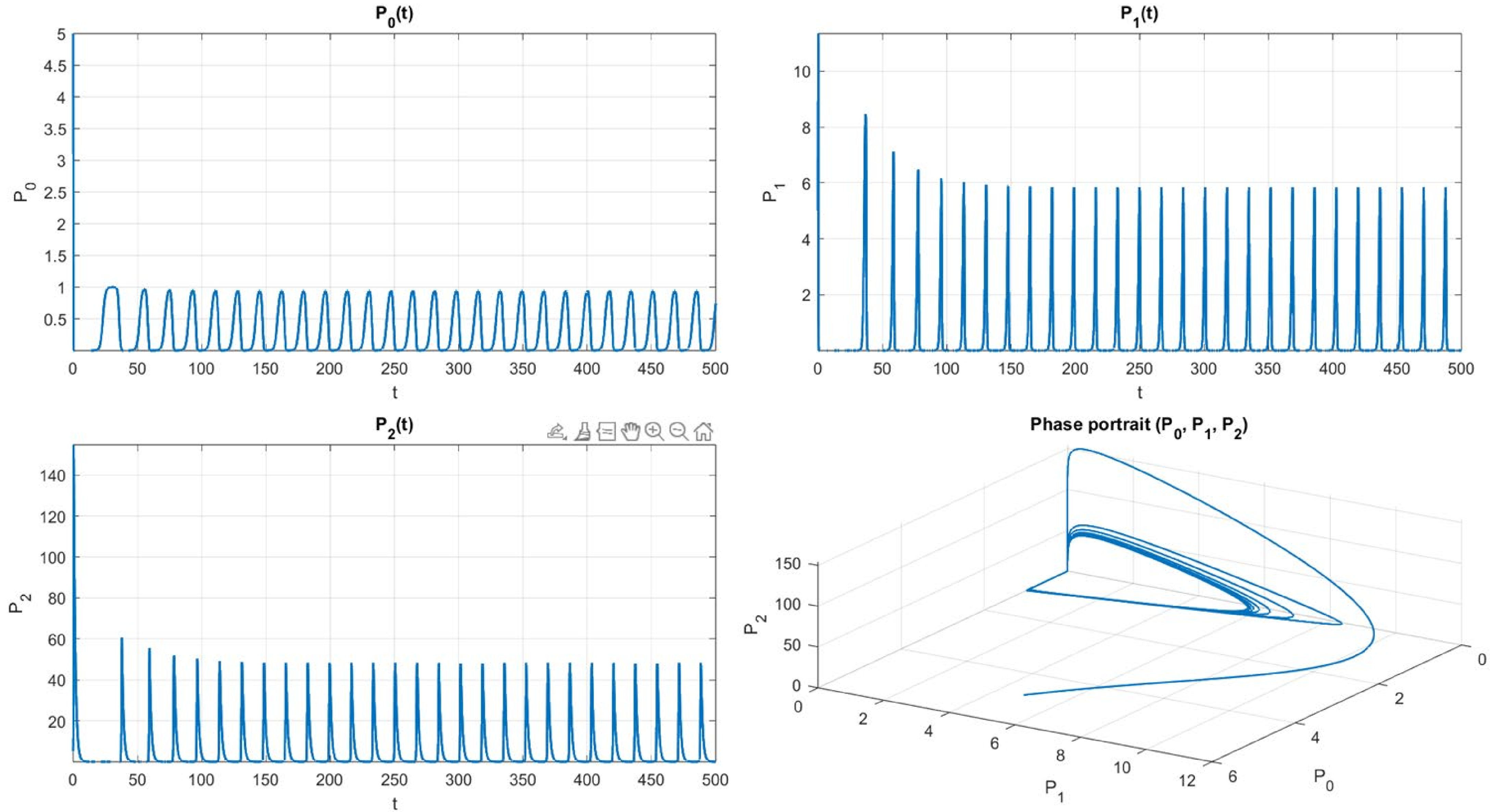
1. **Shift the equilibrium:** Find the equilibrium $P^* = (P_0^*, P_1^*, P_2^*)$ and shift variables:
$$u = P_0 - P_0^*, \quad v = P_1 - P_1^*, \quad w = P_2 - P_2^*$$
so that the system becomes $\dot{z} = F(z)$ with $F(0) = 0$.
2. **Linearize and identify eigen-directions:** Compute the Jacobian $J = DF(0)$. At the BT point: $\mathrm{spec}(J) = \{0, 0, \lambda_s\}$ with $\lambda_s < 0$. The eigenvectors corresponding to zero eigenvalues define the center subspace.
3. **Center manifold reduction (3D → 2D):** There exists a smooth invariant manifold $w = h(u, v)$ with $h(0) = 0$, $Dh(0) = 0$. Substituting $w = h(u, v)$ reduces the system to:
$$\dot{u} = \tilde{F}_1(u, v), \quad \dot{v} = \tilde{F}_2(u, v)$$
4. **Reduced dynamics:** Expand $\tilde{F}_1, \tilde{F}_2$ in a Taylor series near the origin and retain quadratic terms:
$$\dot{u} = v + \mathcal{O}(2), \quad \dot{v} = au^2 + buv + \cdots$$
5. **Transform to BT normal form:** Apply smooth coordinate and parameter changes to obtain:
$$\boxed{\dot{x} = y, \quad \dot{y} = \beta_1 + \beta_2 x + ax^2 + bxy + \mathcal{O}(3)}$$

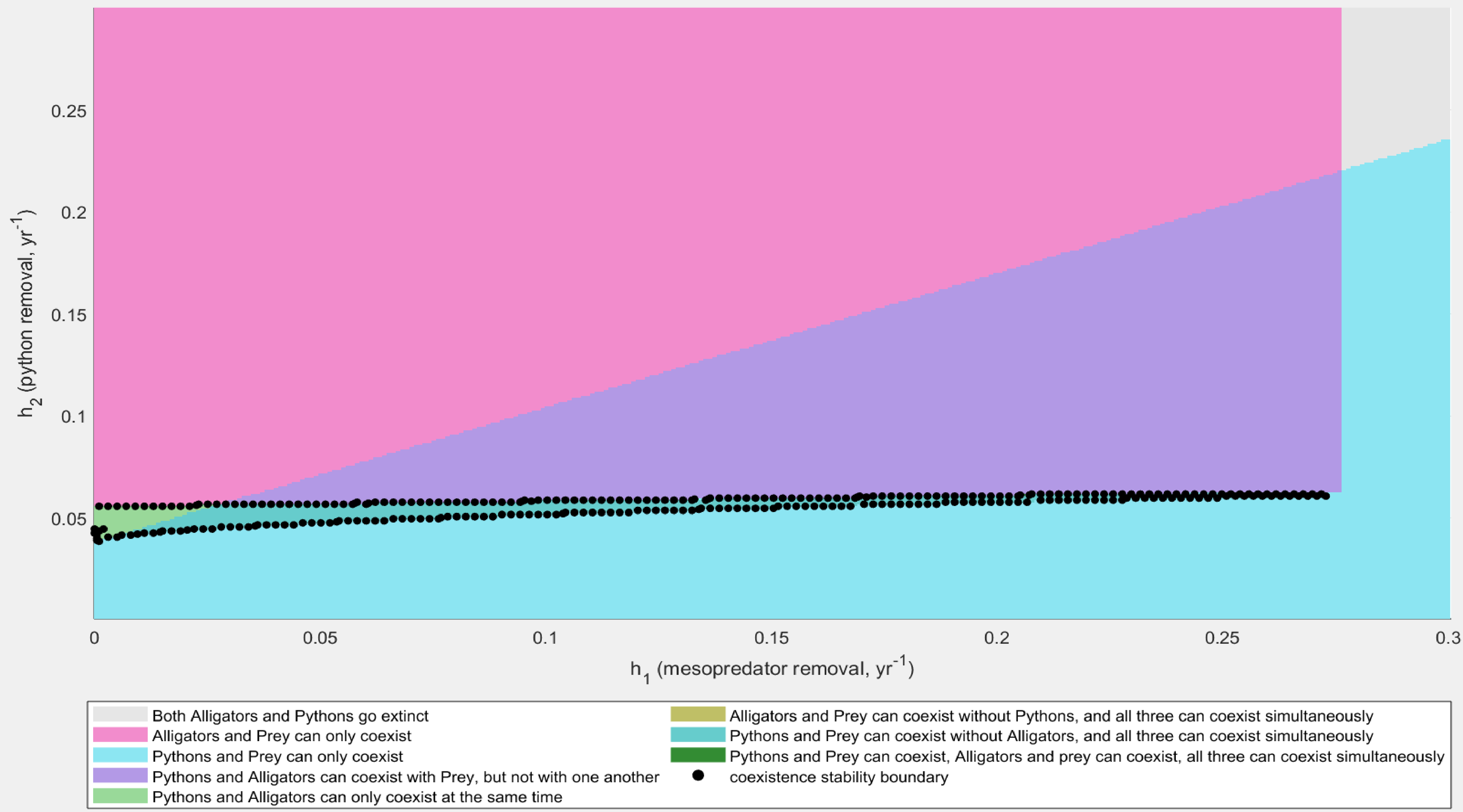
Ultimate goal: Policy making

- With current harvesting restricted to 2 alligators per hunter, per season, should the hunting policies be altered, especially that the common prey population is decreasing (Racoons drop by 95%...)
- Plan:



The presence of Burmese Pythons




h$_2$ (python removal, yr^{-1})
0.25
0.2
0.15
0.1
0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
h$_1$ (mesopredator removal, yr^{-1})
Both Alligators and Pythons go extinct
Alligators and Prey can only coexist
Pythons and Prey can only coexist
Pythons and Alligators can coexist with Prey, but not with one another
Pythons and Alligators can only coexist at the same time
Alligators and Prey can coexist without Pythons, and all three can coexist simultaneously
Pythons and Prey can coexist without Alligators, and all three can coexist simultaneously
Pythons and Prey can coexist, Alligators and prey can coexist, all three can coexist simultaneously
coexistence stability boundary

The Croc Docs
Wildlife Research Team

UF | IFAS Extension
UNIVERSITY *of* FLORIDA

Fort Lauderdale
Research and
Education Center

UT
THE UNIVERSITY OF
TOLEDO
1872

SIAM TX-LA Conference and JMM 2026

Useful techniques/ Strategies implemented

- **Ownership**
- **Reaching out to contacts**
- **Know your strong points, know your weak points**
- **Always have something to share**
- **Provide opportunities, but students must seize**

Thank you!

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