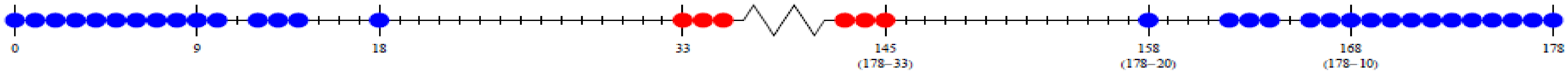
CUR - Goldwater Scholars Faculty Mentor Award



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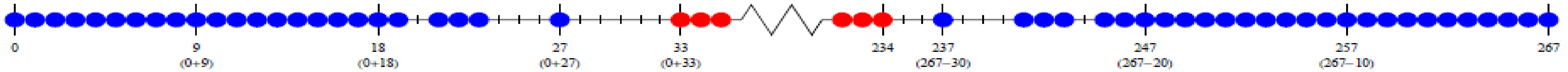
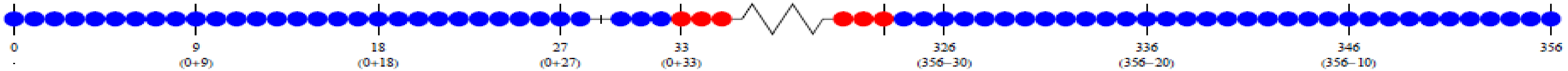
Department of Mathematics and Statistics, Williams College

https://web.williams.edu/Mathematics/sjmiller/public_html/





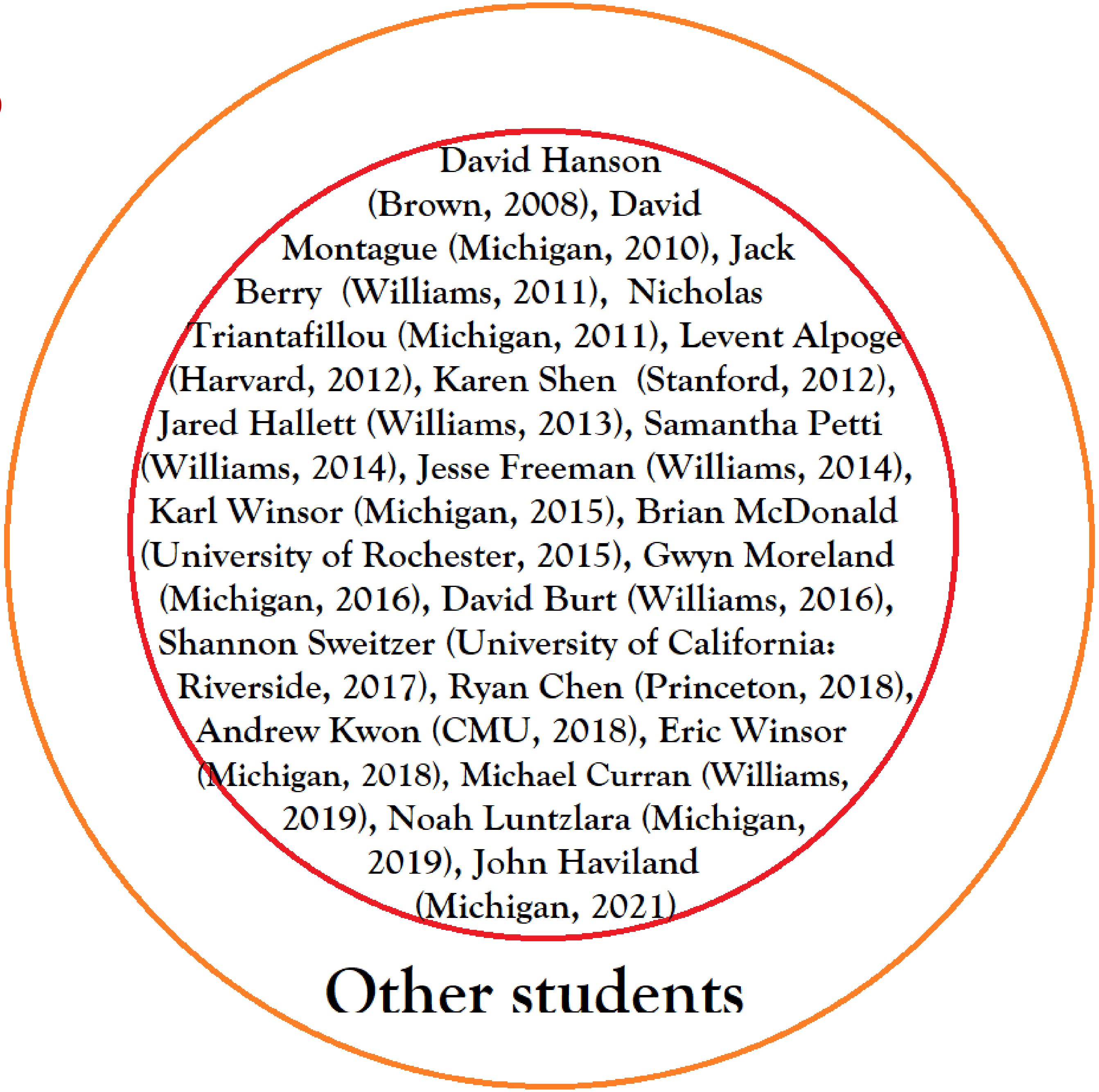
Expanding Circles

- **Goldwater Scholars**

David Hanson
(Brown, 2008), David
Montague (Michigan, 2010), Jack
Berry (Williams, 2011), Nicholas
Triantafillou (Michigan, 2011), Levent Alpoge
(Harvard, 2012), Karen Shen (Stanford, 2012),
Jared Hallett (Williams, 2013), Samantha Petti
(Williams, 2014), Jesse Freeman (Williams, 2014),
Karl Winsor (Michigan, 2015), Brian McDonald
(University of Rochester, 2015), Gwyn Moreland
(Michigan, 2016), David Burt (Williams, 2016),
Shannon Sweitzer (University of California:
Riverside, 2017), Ryan Chen (Princeton, 2018),
Andrew Kwon (CMU, 2018), Eric Winsor
(Michigan, 2018), Michael Curran (Williams,
2019), Noah Luntzlara (Michigan,
2019), John Haviland
(Michigan, 2021)

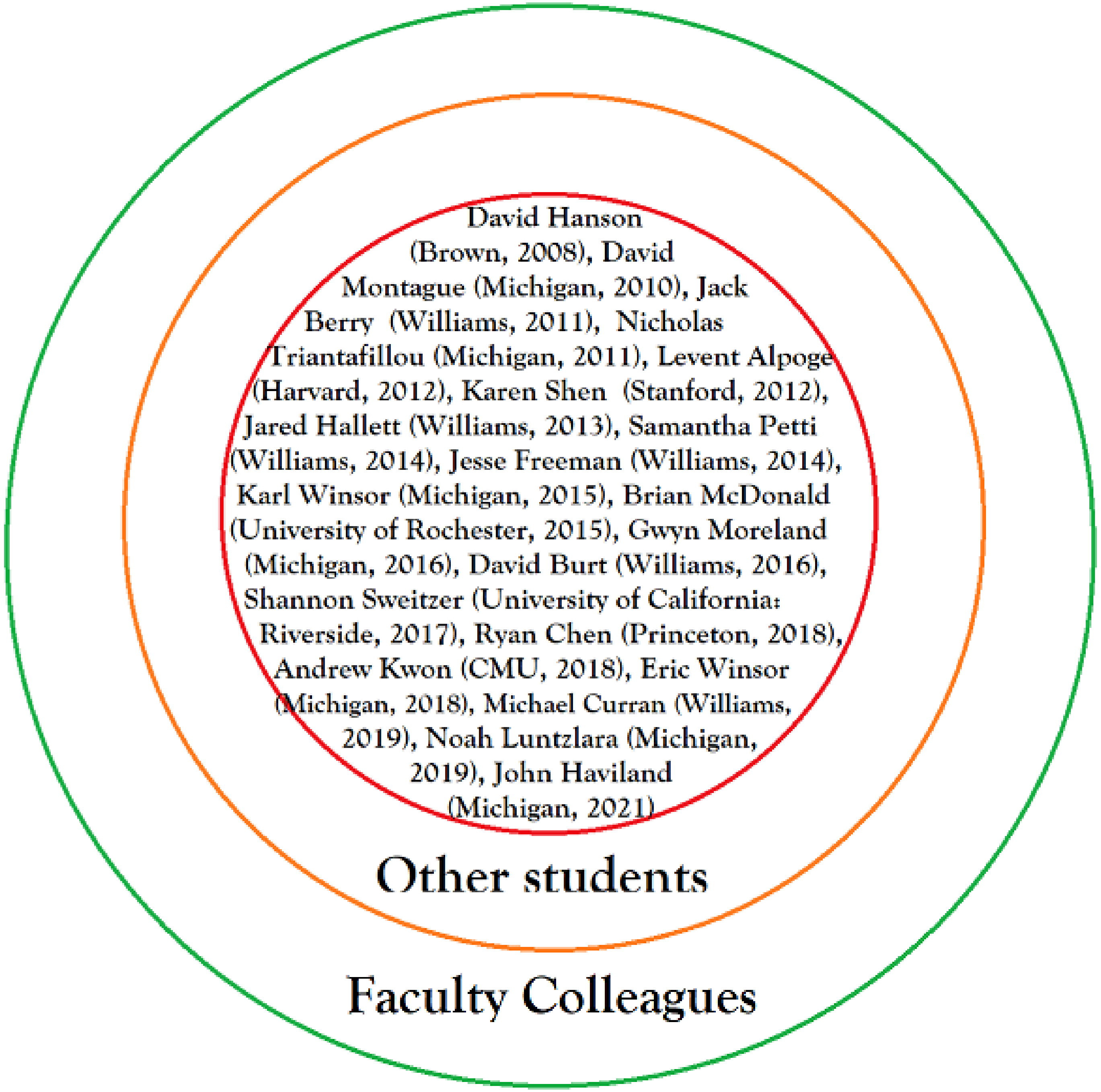
Expanding Circles

- **Goldwater Scholars**
- **Other students: 400+ high school and college students**



Expanding Circles

- **Goldwater Scholars**
- **Other students: 400+ high school and college students**
- **Faculty Colleagues: Especially those at Williams and Michigan**



Expanding Circles

- Goldwater Scholars
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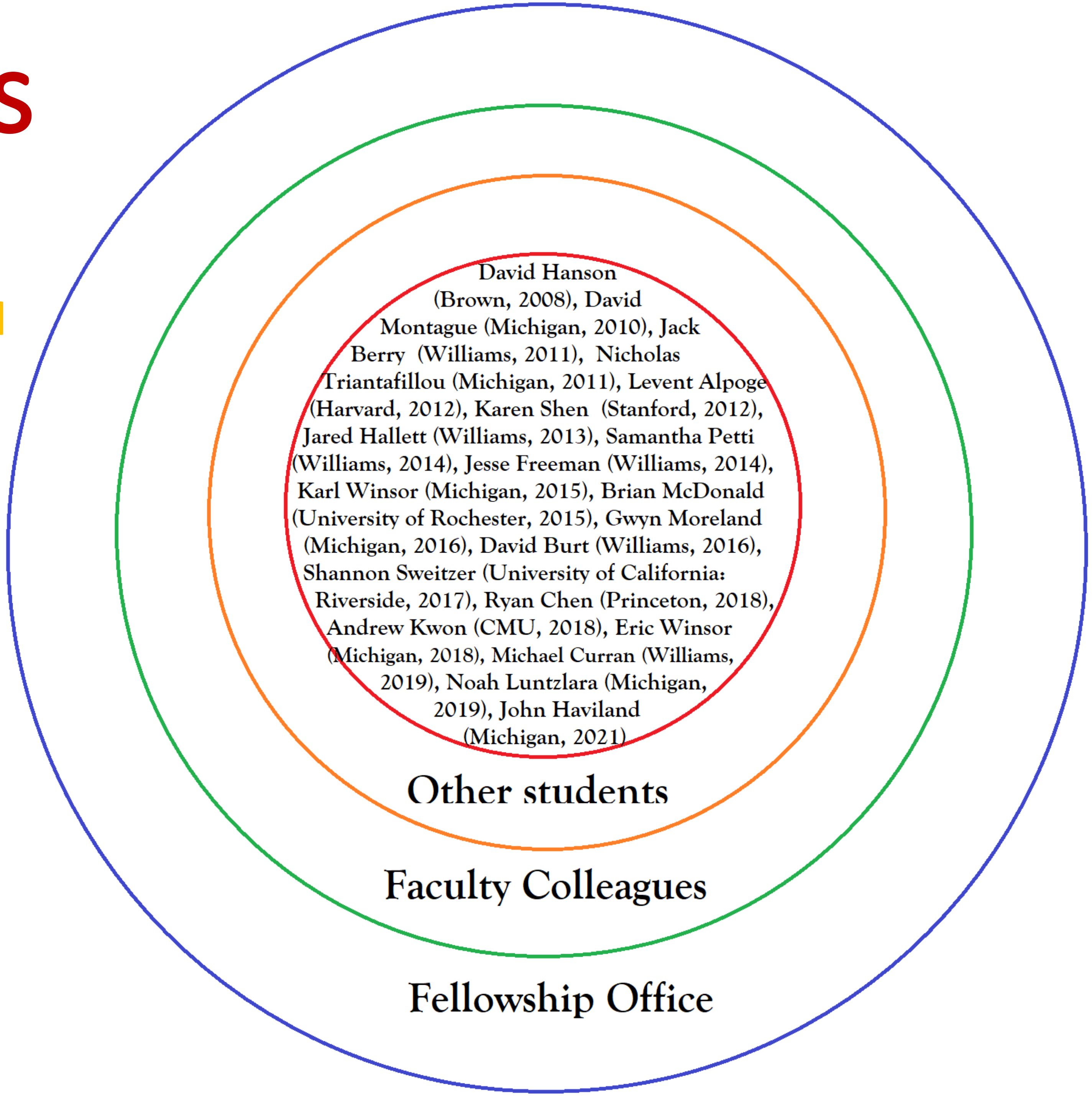
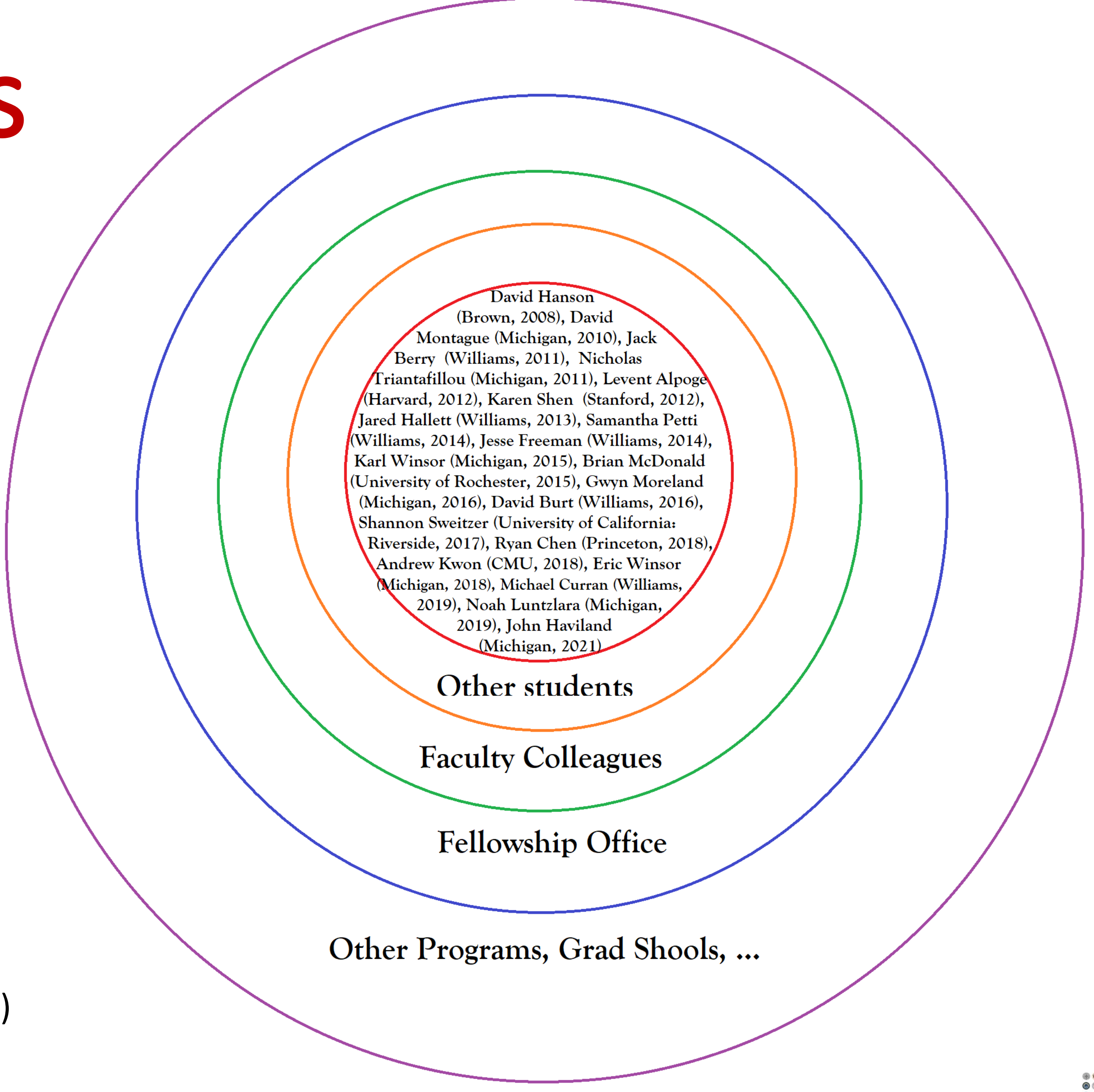
- Fellowship Office: Especially Lynn Chick, Katya King



Expanding Circles

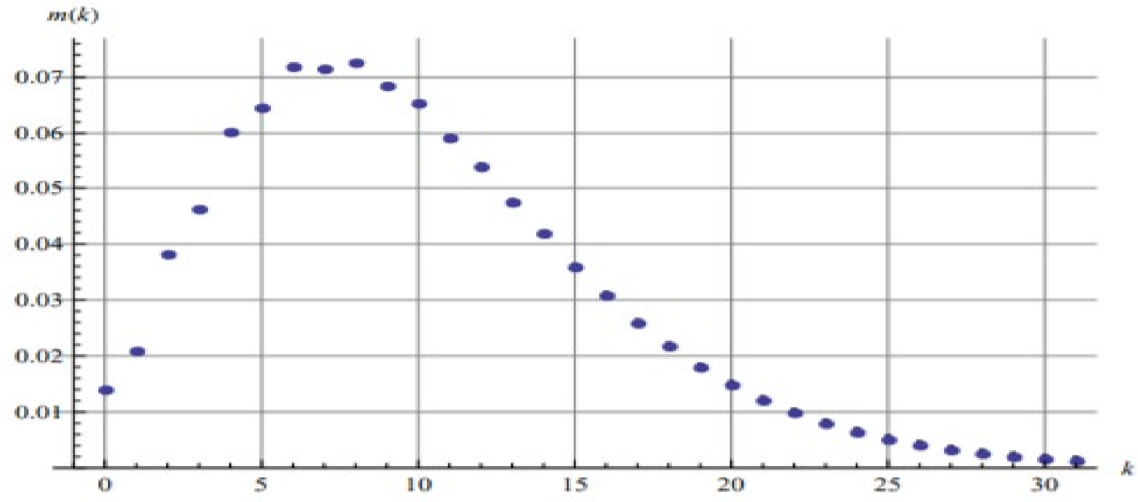
- Goldwater Scholars
- Other students: 400+ high school and college students
- Faculty Colleagues: Especially those at Williams and Michigan
- Fellowship Office: Especially Lynn Chick, Katya King
- Other Programs: Graduate School, Industry, Churchill (8), NSF, ….

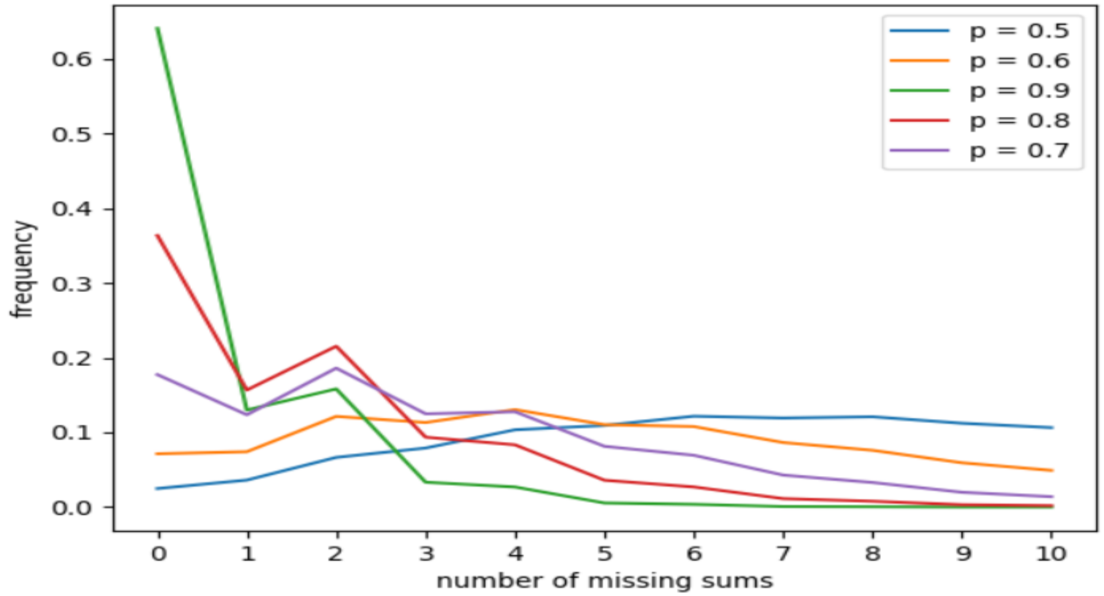
(Only a subset – my colleagues, advisors and family….)



Joys of Undergraduate Research

- **Opens students to possibilities.**





Joys of Undergraduate Research

- Opens students to possibilities.

- **Valuable skills for all walks of life.**

$$\mathcal{D}_d(A, \mathbf{B}) = \begin{bmatrix} \begin{matrix} \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} & B_2 \\ B_2 & \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} \end{matrix} & \cdots & B_d \\ \vdots & \ddots & \vdots \\ B_d & \cdots & \begin{matrix} \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} & B_2 \\ B_2 & \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} \end{matrix} \end{bmatrix}.$$

Joys of Undergraduate Research

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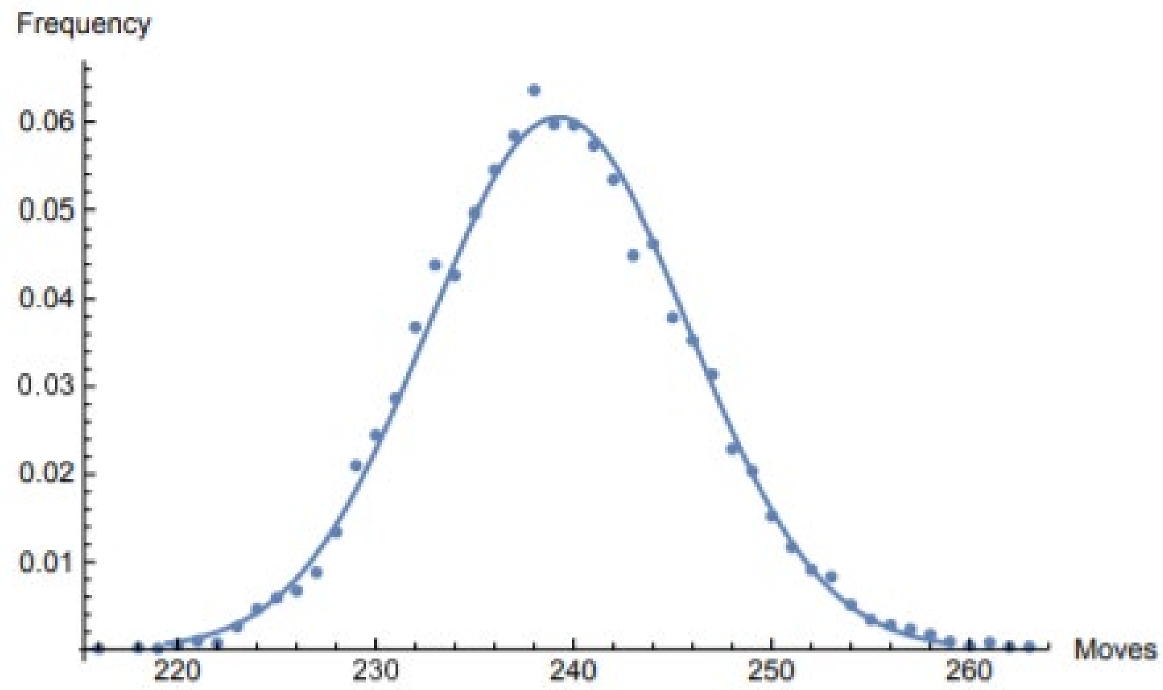
- Valuable skills for all walks of life.
- **Rising tide lifts all boats.**



Joys of Undergraduate Research

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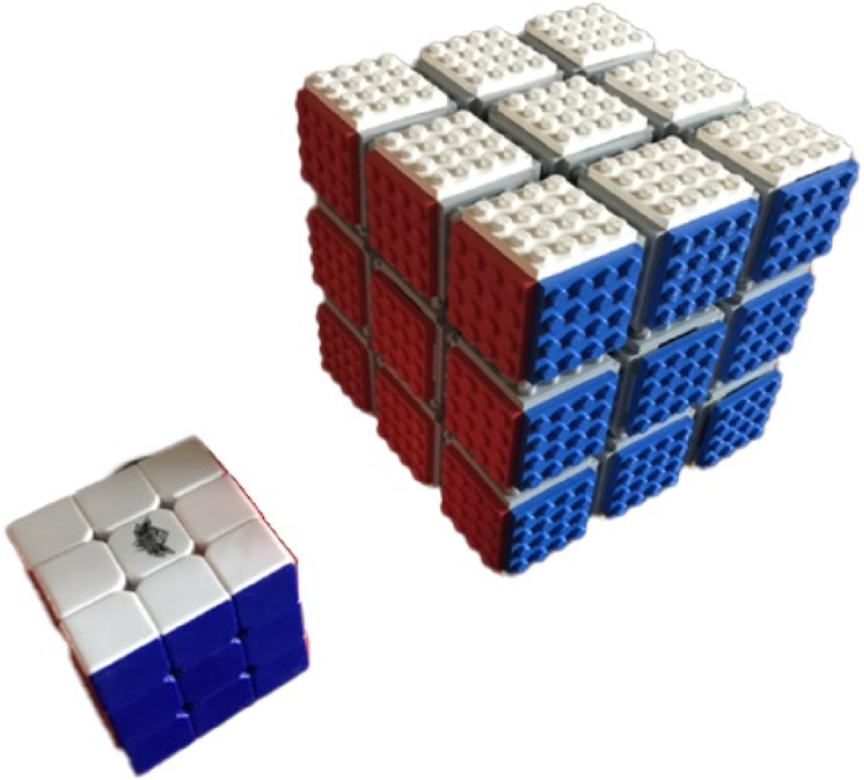
- **Pay it forward (service, next scholars).**

MATH CORPS AT U(M)

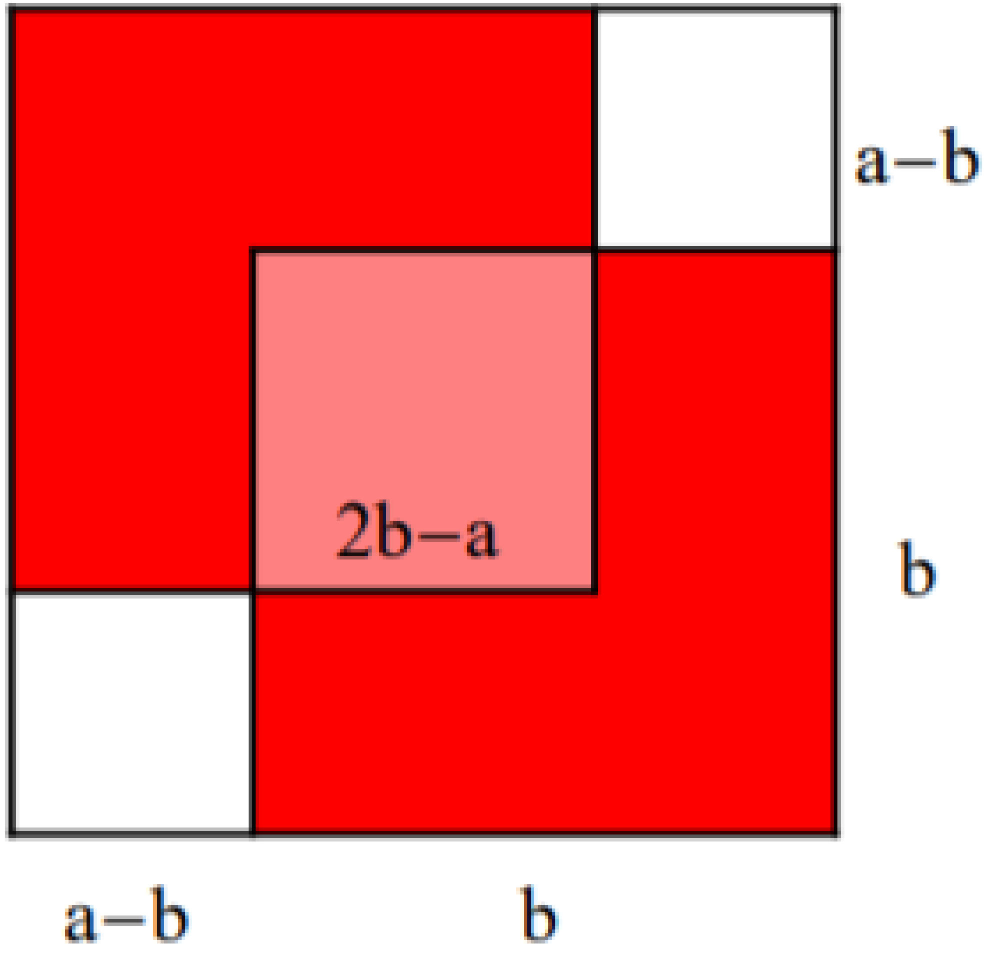


Joys of Undergraduate Research

- Opens students to possibilities.
- Valuable skills for all walks of life.
- Rising tide lifts all boats.
- Pay it forward (service, next scholars).
- **Springboard to the future.**

Problems across the years: Irrationality

Good problem is interesting, explainable, and generalizable.



Stanley Tennenbaum:

Geometric proof of the irrationality of $\sqrt{2}$
(not a/b for integers a, b).

If rational, $2 = \ a^2 \ / \ b^2$ and assume b smallest such integer.

Double counted pink square equals the two white squares.

Get $2 = (2b - a)^2 \ / \ (a - b)^2,$ and a-b < b.

Problems across the years : Irrationality

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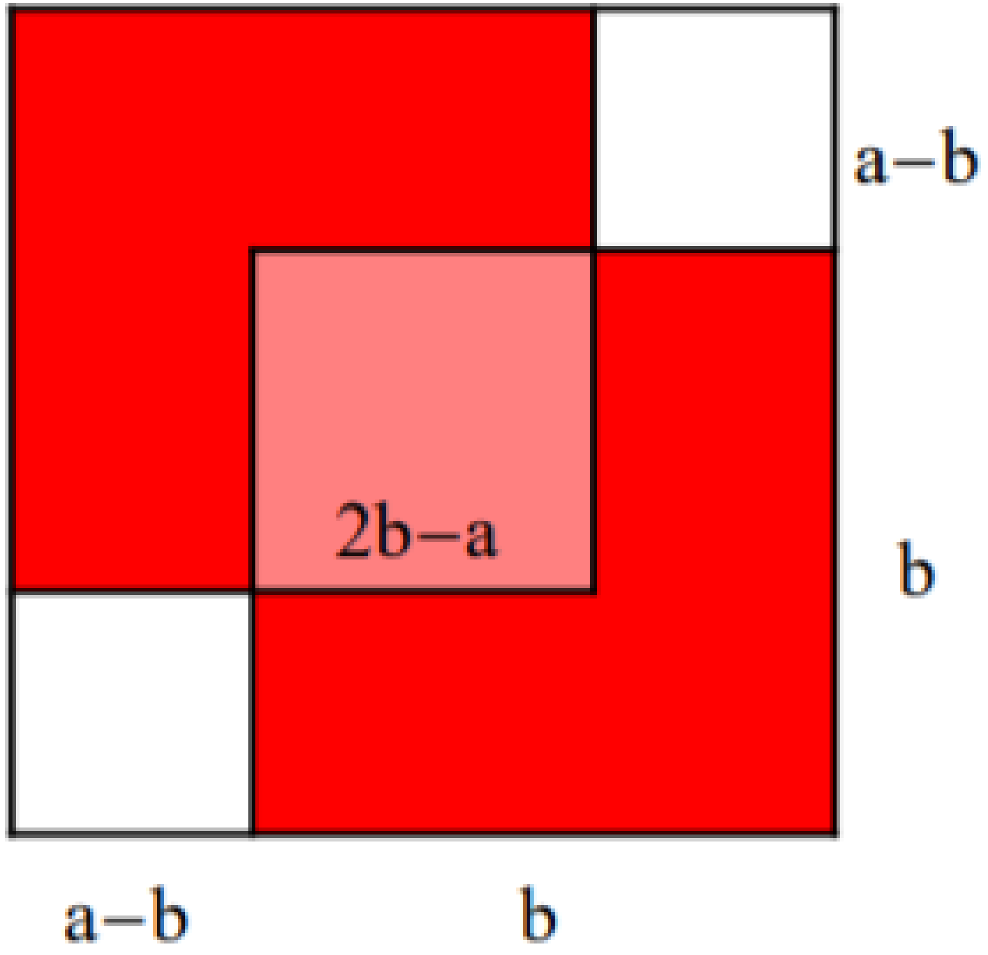
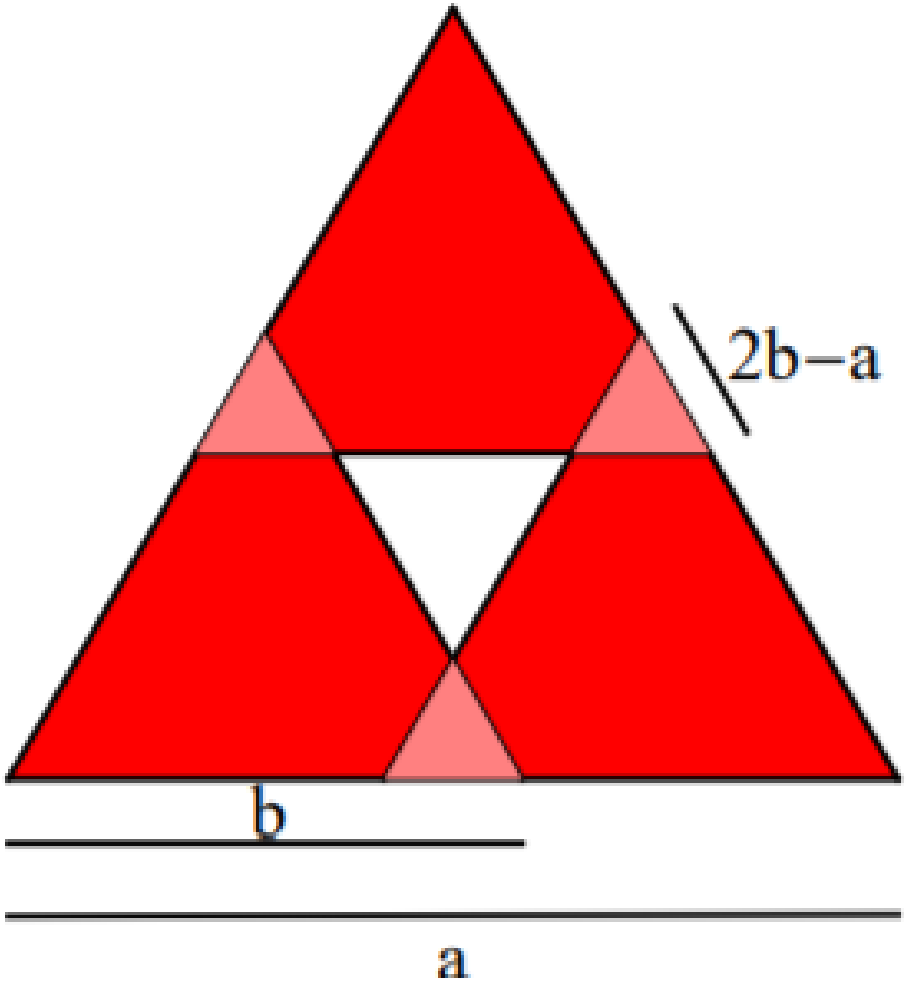
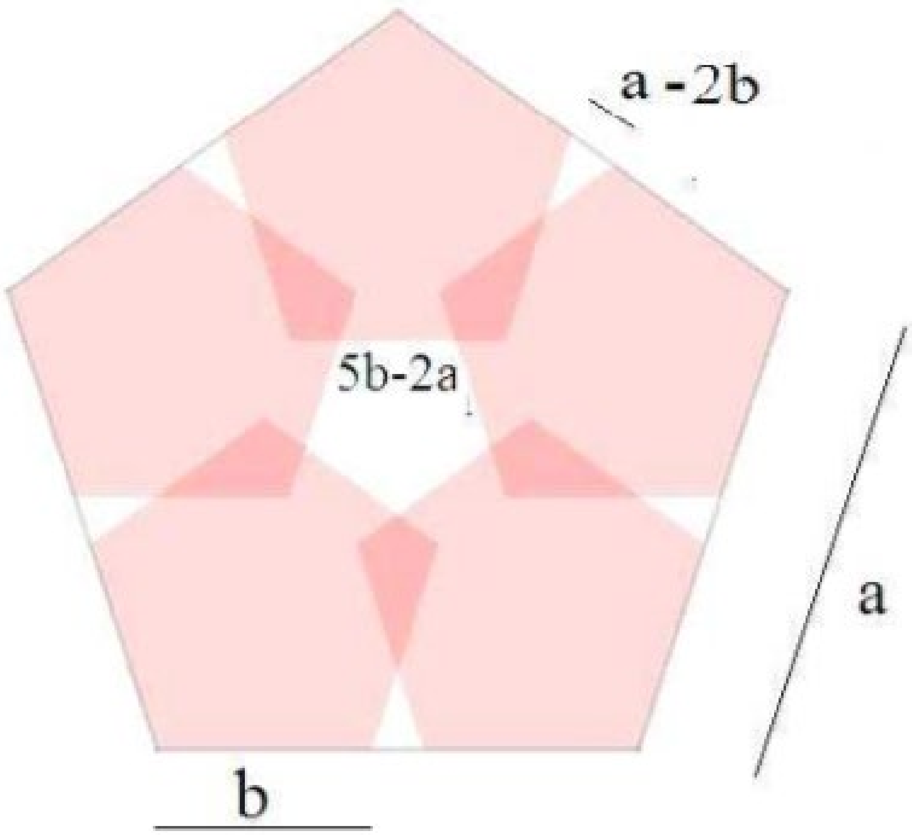
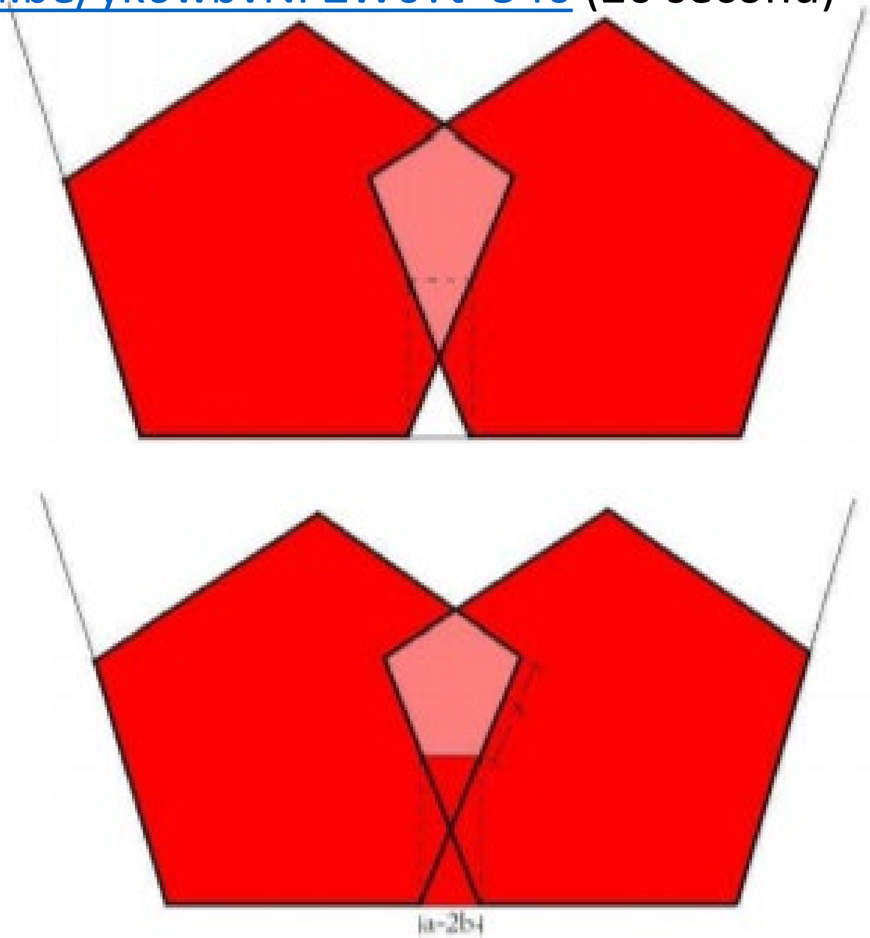




Problems across the years : Irrationality

Rational irrationality proofs (with David Montague), Mathematics Magazine 85 (2012), no. 2, 110--114. https://web.williams.edu/Mathematics/sjmiller/public_html/math/papers/irrationality50.pdf
(expanded version): Mathologer's animation: https://youtu.be/yk6wbvNPZW0?t=540 (20 second)





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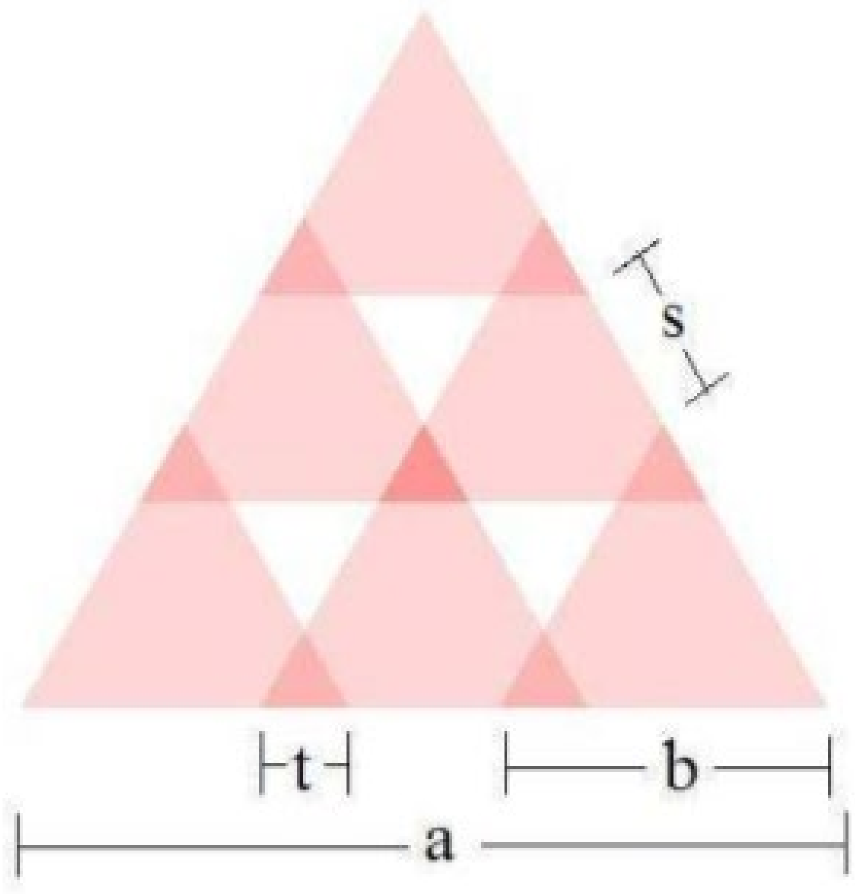
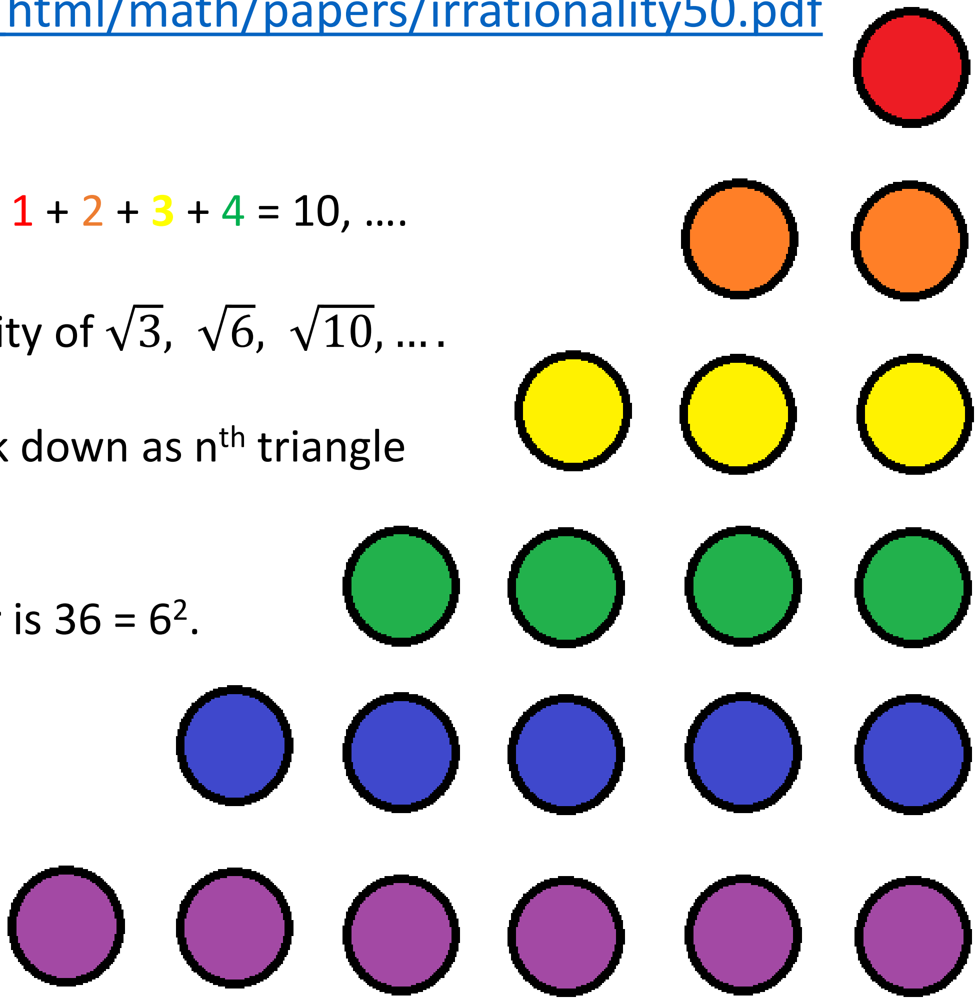


Triangle numbers:
1, 1 + 2 = 3, 1 + 2 + 3 = 6, 1 + 2 + 3 + 4 = 10, ….

We proved the irrationality of $\sqrt{3}$, $\sqrt{6}$, $\sqrt{10}$, … .

But the proof ***must*** break down as n^{th} triangle number is n(n+1)/2.

Note 8^{th} triangle number is $36 = 6^2$.



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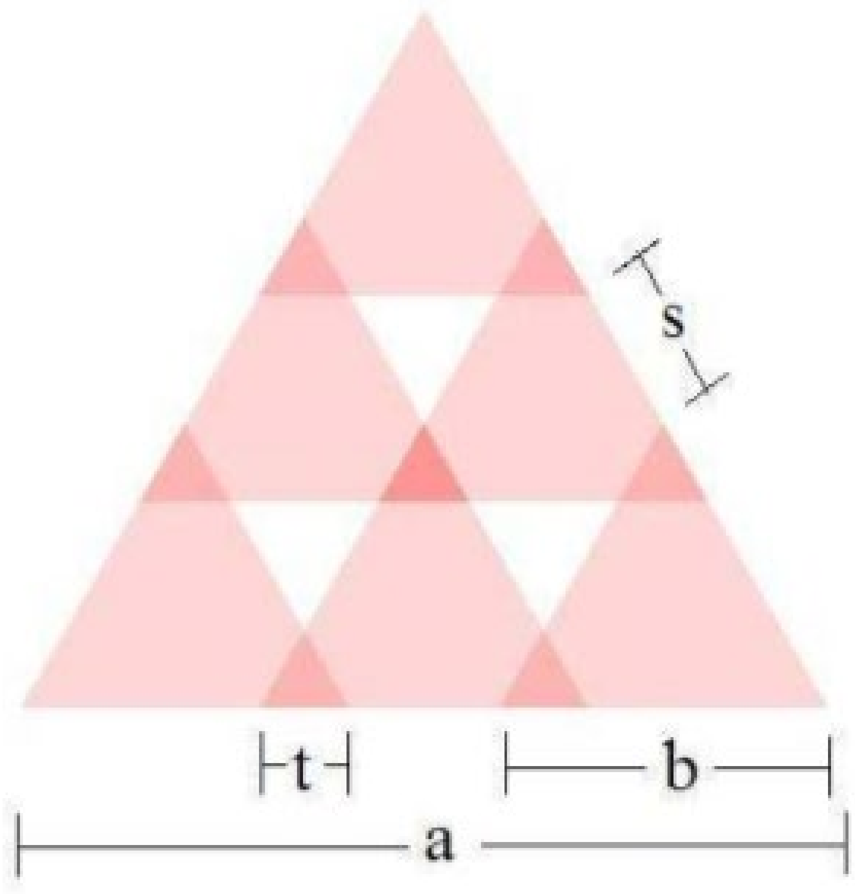

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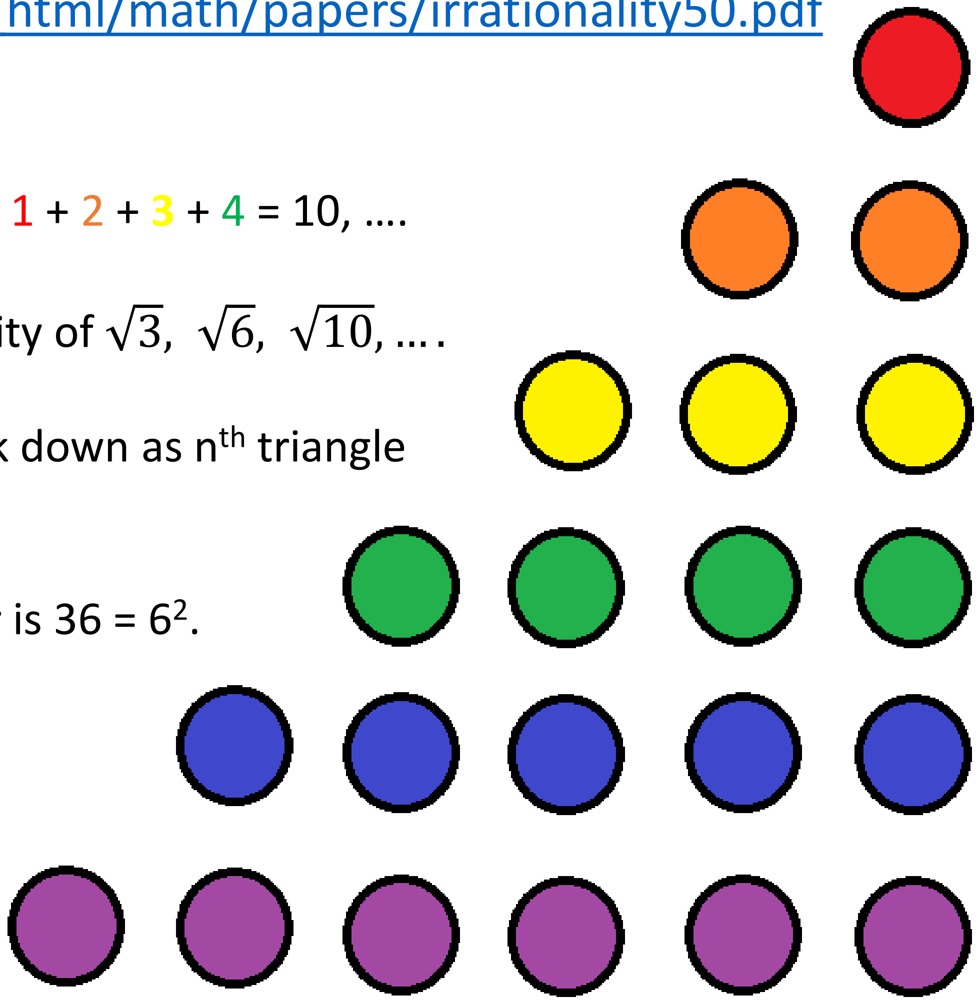
OPEN:
$\sqrt{7}, \sqrt[3]{2}$



Thank you!

https://web.williams.edu/Mathematics/sjmiller/public_html/

Problems across the years: Fibonacci Games

Fibonacci numbers: 1, 2, 3, 5, 8, 13, 21, 34, 89, 144, 233, 377, 610, 987, 1597, 2584, …, in general $F_{n+1} = F_n + F_{n-1}$.

Zeckendorf's Theorem: Every integer has a unique representation as a sum of non-consecutive Fibonacci numbers.

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Example: 2021 = 1597 + 377 + 34 + 13.

Created game on this….

Problems across the years: Fibonacci Games

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- Two player game, alternate turns, last to move wins.
- Bins $F_1, F_2, F_3, \ldots,$ start with N pieces in F_1 and others empty.
- A turn is one of the following moves:
  - ◇ If have two pieces on F_k can remove and put one piece at F_{k+1} and one at F_{k-2}
    (if $k = 1$ then $2F_1$ becomes $1F_2$)
  - ◇ If pieces at F_k and F_{k+1} remove and add one at F_{k+2}.

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Start with 10 pieces at F_1, rest empty.

10	0	0	0	0
$[F_1 = 1]$	$[F_2 = 2]$	$[F_3 = 3]$	$[F_4 = 5]$	$[F_5 = 8]$

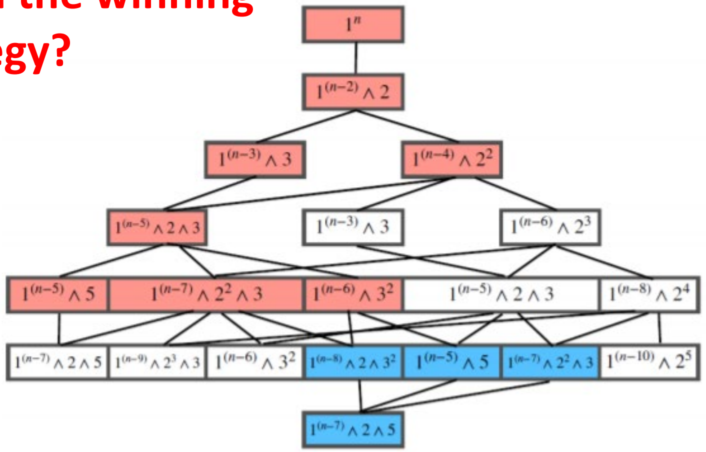
Next move: Player 1: $F_1 + F_1 = F_2$

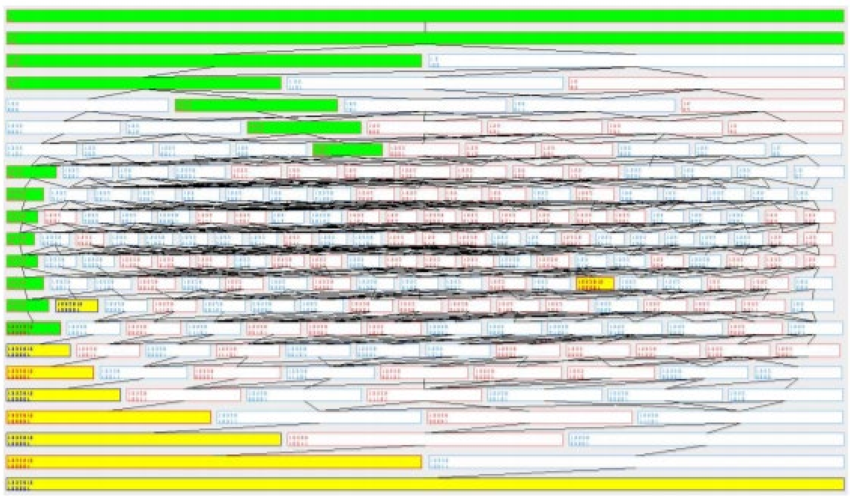
Problems across the years: Fibonacci Games

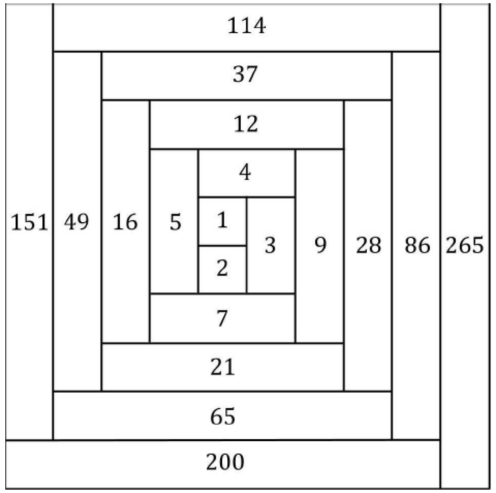
Fibonacci numbers: 1, 2, 3, 5, 8, 13, 21, 34, 89, 144, 233, 377, 610, 987, 1597, 2584, …, in general $F_{n+1} = F_n + F_{n-1}$.

1	2	0	1	0
$[F_1 = 1]$	$[F_2 = 2]$	$[F_3 = 3]$	$[F_4 = 5]$	$[F_5 = 8]$

- **Theorem: If n > 2 then Player 2 has a winning strategy, but it is a non-constructive proof!**
- Lots of papers with students, most recently with John Haviland: Extending Zeckendorf's Theorem to a Non-constant Recurrence and the Zeckendorf Game on this Non-constant Recurrence Relation} (with Ela Boldyriew, Anna Cusenza, Linglong Dai, Pei Ding, Aidan Dunkelberg, John Haviland, Kate Huffman, Dianhui Ke, Daniel Kleber, Jason Kuretski, John Lentfer, Tianhao Luo, Clayton Mizgerd, Vashisth Tiwari, Jingkai Ye, Yunhao Zhang, Xiaoyan Zheng, and Weiduo Zhu), Fibonacci Quarterly **58** (2020), no. 5, 55--76. https://web.williams.edu/Mathematics/sjmiller/public_html/math/papers/ZeckExtendingZeckNonConstantGame40.pdf
- **Can you find the winning strategy?**







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