L-function Lecture 5: PANTHers 2021
Comments on L-Function Talks: II

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https://web.williams.edu/Mathematics/sjmiller/public_html/

Part I (Classical RMT, Intro L-fns, Dirichlet): http://youtu.be/2PuUbk6gUMM (slides: part 1)
https://web.williams.edu/Mathematics/sjmiller/public_html/math/talks/Michigan2012Part1.pdf

Part II (Convolving families, cusp forms: slides here): http://youtu.be/vJz6W24tDik (slides part 2)
https://web.williams.edu/Mathematics/sjmiller/public_html/math/talks/Michigan2012Part2.pdf

More talks, as well as papers, available here:
https://web.williams.edu/Mathematics/sjmiller/public_html/

Dirichlet's Thm for primes in arithm progression

If $\gcd(a,m)=1$ Then $\exists$ ∞ many primes $p \equiv a \bmod m$

More: $\pi_{a,m}(x) = \frac{\pi(x)}{\varphi(m)}$ to first order

here $\pi(x) = \#\{p \le x\}$ and $\varphi(m) =$ # ints rel prime to m

Question: Prove if possible that $\frac{\pi_{a,m}(x)}{x} \to 0$

IF $\pi(x) = x/\log x$ then $\pi_{a,m}(x) \le x/\log x$ and done

Chebyshev $\exists$ $0 < A < 1 < B$ st $\forall x \ge x_0$

$A\, x/\log x \le \pi(x) \le B\, x/\log x$

Pascal's Identity

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$$

$\binom{1}{0} = 1$

$\binom{1}{1} = 1$

Proof: Brute force multiplication

Induction: $(x+y)^{n+1} = (x+y)^n (x+y)$

Story: Chocolate and Brocoli

need $k+1$ items and have n Choc and 1 piece Broc

$$\binom{n+1}{k+1} = \binom{1}{1}\binom{n}{k} + \binom{1}{0}\binom{n}{k+1}$$

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k}$$

Sneetches

n professors n students group of n people

if have k profs have n-k students

ans: $\sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$

Generalize: Study $\sum \binom{n}{k}^3$ or something else?