Teaching Experience and Teaching Statement

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1 Summary

If I had to describe my teaching philosophy in one word, it would be: *opportunities*. I believe my job as an educator is to provide opportunities for my students, noting that what is appropriate or needed for one is not necessarily what's right for another. I do this in a variety of ways, a few of which are listed below.

- My students collaborate with me in my research, both during the semester and in summers. Most projects end in publications (over 100 with students) and conference presentations, from undergraduate gatherings to international meetings. Many of my students begin working after their sophomore year, sometimes earlier. After over 20 years of guiding student research, I have an excellent sense of what interesting problems will be approachable for different students. In addition to thesis and research projects in number theory and probability, I have also supervised student work in operations research, sabermetrics, machine learning and graph theory, to name a few.

- I introduce by students to the mathematics community by having them co-organize conferences with me, referee papers for journals, and give math outreach talks. They have also written parts of books with me, including textbooks and an edited volume on Benford's law.

- My students assist in my Math Riddles page: http://mathriddles.williams.edu. In addition to the html coding and layout decisions, students are also providing input on the difficulty of the riddles and detailing their thought processes as they attack the riddles. The goal is to create a resource to help motivated students and teachers explore mathematics.

- I work with local and regional businesses and schools to incorporate problems from them in my classes, especially in Operations Research, Probability, and Advanced Applied Analysis. Examples include work with Berkshire Medical Center on lowering readmission rates, scheduling enrichment classes for schools, determining how to allocate state funds fairly in multi-school, multi-town districts, and determining marketing campaigns to help products go viral.
- Numerous graduate students and post-docs work with me at the Williams College SMALL REU program, where I mentor them in how to run research programs and work with students.
- I have strong ties with many of my former students and their current employers, and arrange for numerous campus recruitment visits and guest lectures for my classes and the campus, from financial analysts at Goldman-Sachs to analytics gurus at the NBA.

I work hard to make sure these opportunities are available to all. For the past several years both my REU students and thesis students have been approximately 40% women. I am a Math Alliance mentor, was the point person in their (National Alliance for Doctoral Studies in the Mathematical Sciences) successfully being awarded the prize for the 2017 AMS Mathematics Programs that Make a Difference, and am working with several REUs to increase the number of students from typically under-represented groups by establishing a mentoring program to work with them during the academic year so that they can begin the summer as an equal collaborator.

Additionally, I have recorded and freely posted on YouTube all of my lectures (and several of my talks) for the past few years. I have had students from other schools take my classes on-line, as well as some Williams students who wanted to take something else at the same time slot. I am working with LACOL (the Liberal Arts Consortium for Online Learning) to modify this program and expand it to include many classes at our schools. I led the first beta test, integrating new technology to have the remote students having as good an experience as possible (including live participation), and am currently the chair of the sub-group that is working on expansion.

In my standard classes I always provide advanced material and open research problems. Students respond to a challenge, and enjoy the opportunity to further explore the subject. Mathematics has been successfully applied to many problems in the real world, and these connections can be used to keep the students interested and motivated; this is especially important as I often have a diverse student body with enormous variations in motivation, ability and needs (ranging from some who are preparing for graduate school to others who have to satisfy the requirements of the major). Examples from my classes include cryptography (Abstract Algebra), scheduling problems for airlines (Linear Algebra / Linear Programming / Operations Research), art (Multivariable Calculus) and determining why certain baseball statistics predict a team's performance (Multivariable Calculus / Mathematical Statistics / Probability). While detailed descriptions of all the courses are provided later, one course I taught at Brown and one at Williams capture my teaching philosophy and style, and are briefly summarized below.

I taught Math 162 (Mathematical Statistics) for four years at Brown. This is a theory intensive statistics course. In addition to proving standard results such as the central limit theorem (assuming a standard result from complex analysis), we introduce numerous useful proof techniques and explore their applications. For example, we discuss differentiating identities and matching coefficients and show applications to formulas for tests based on runs. The course is peppered with numerous applied problems with interesting theoretical components (many of the examples come from student projects, a required and popular component of the class). Examples range from analysis of baseball games (where the fact that games cannot end in ties led us to the concept of structural zeros in $r \times c$ tables) to Benford's law and digit bias (which is related to important problems in equidistribution theory, and is currently being used by the IRS to detect corporate tax fraud). In my four years at Brown, this was the top rated mathematics course twice, and the second highest ranked course another year.

In Fall 2009 I taught Math 341 (Probability) at Williams (I have since taught it five more times at Williams and once at Mount Holyoke). In many respects this course is similar to the statistics course I taught at Brown; the differences, however, highlight my growth as an educator and are the result of what I have learned from years of teaching. We frequently encounter students with varied levels of background, interest, and time that they choose to devote to our courses. As such, we have to adjust our teaching to reach these different levels. I have added several innovations since arriving at Williams to further these goals; see the course homepage http://web.williams.edu/Mathematics/sjmiller/public_html/341Fa16/ for more details.

- After each class I spend 30 to 90 minutes typing up additional comments on the lecture. These range from summarizing key points to providing advanced theory and links for students who want to explore further. I am heartened by how many students at all levels and interests tell me they read these posts. I have also written many extensive handouts giving (almost) complete details of the advanced theory (for example, moment generating functions and the needed results from complex analysis).
- I post detailed beamer notes of the lectures online; during lecture I use these notes as a starting point for discussions of the theory, and then move to the blackboard for the detailed computations.
- I have started using clickers in my classes to get immediate feedback on how well the students understand the material. I design multiple choice questions whose answers advance the theory and class discussion, which we then vote on. Recent examples include questions on the rate of convergence to the standard normal to mathematical gambling to detecting fraud.
- I encourage all my students to do a project, which can count for 10% of their grade; many take me up on the offer and explore a subject in great detail.
- Occasionally my students are asked to create two homework problems on an assignment, one where they must include a solution and one where they pose a problem for me to solve. I have found these a wonderful way to tap into the abilities, interests and creativity of my students. They learn how difficult (but important) it is to formulate a problem clearly and unambiguously; further, this gives me opportunity for personalized feedback to them on how well they know the material. Other innovations in the homework assignments include the option of writing a scientific summary of a paper or giving a small presentation to me and part of the class instead of additional standard problems. At the end of the day the goal is not to train students to solve homework problems, but to apply what they've learned to the real world and to do original research; these projects help give students a flavor of what this is like.
- Starting in the 2012 iteration of the class, the textbook has been 'The Probability Lifesaver', a book I published with Princeton University Press in 2017 as a sequel of sorts to their successful 'The Calculus Lifesaver'. Several students have had significant roles in writing this book, ranging from reviewing the exposition to writing sections of the book. This has been a tremendous experience for the students, giving them a valuable perspective, and has helped keep the book clear and accessible.

Over the past 10 years, I have supervised over 300 undergraduate and graduate students on various projects. While many of these have led to publications, that is not the point (for undergraduate research!). There, for me the purpose is to show students what research is like, and to give them a sense of what problems we study, what techniques we use, and whether or not this is something they wish to pursue. This is not limited to my thesis and summer students, and is also a part of my classes.

I have served on three graduate dissertation committees, and been a major co-advisor to two other graduate theses. I have continued working and publishing with several of my undergraduate students after they've begun their graduate studies. Based on my prior experience with undergraduate and graduate students and post-docs, I am confident about supervising a PhD dissertation.

I have been the faculty advisor to the undergraduate math clubs at Ohio State and Brown University, and the Math Team at Williams. Duties range from giving lectures to helping them organize student conferences to running Putnam sessions to teaching them how to use LaTeX and programming environments such as Mathematica and Matlab to advising them on how to write papers and give talks (for example, in the past three years at least 30 of my undergraduates will give talks at research conferences).

Finally, I am very active in Mathematics Education. I write and review problems for the AMC competitions. For years I was a frequent participant at meetings at the Institute for Mathematics & Education at the University of Arizona. I have lectured at many of the top programs for talented high school students (the Ross program at Ohio State, the PROMYS program at BU, and Hampshire College), and in some years serve as a research mentor at PROMYS. I also maintain a math riddles webpage,

http://mathriddles.williams.edu/

(which is currently one of the top hits when googling 'math riddles'). The purpose of this site is to help students and teachers explore mathematics through fun riddles. It receives over 4000 distinct hits a month and is used in classrooms around the world; I am working on expanding the features with students and teachers (the password to the student/teacher corner is *corner*). Additionally, I have taught continuing education classes to high school and middle school teachers at the Teachers As Scholars program in Boston, and also individually (most recently at Wellesley High School). I am a writer in the Value of Computational Thinking across Grade Levels (VCTAL) Project, writing a unit for high school / junior high school classes on cryptography and another on streaming video, and am part of the new grant proposal submitted to continue this collaboration.

The sections below elaborate further on the items above.

2 Standard Classes

I've taught the following subjects (detailed descriptions below):

1. Complex Analysis: graduate level.

2. Advanced Applied Analysis: graduate level course on applications of Fourier series.

3. Number Theory: advanced senior seminar, introductory version, graduate level version.
4. Linear Programming / Operations Research: advanced senior seminar and introductory versions.
5. Linear Algebra: graduate linear algebra, honors linear algebra, linear algebra.
6. Probability.
7. Statistics: mathematical statistics.
8. Abstract Algebra.
9. Differential Equations.
10. Discrete Mathematics.
11. Calculus III: honors multivariable calculus, multivariable calculus.
12. Calculus I. (For some reason, have never done Calculus II!)
13. Seminars: freshmen seminar (from riddles to modern mathematics), reading classics working group (on Euler), fractal geometry working group.
14. Problem solving: honors problem solving, advanced honors problem solving.
15. Pre-calculus: fundamentals of college algebra.
16. Basic College Mathematics.

3 Independent Seminars and Winter Study

Independent Studies

Starting at Brown and continuing at Williams, I have almost always run an independent seminar on topics of either my or my students' interest. Topics include

- elliptic curves,
- cryptography,
- Benford's law,
- differential equations and mathematical modeling,
- advanced number theory,
- graph theory,
- Benford's law of digit bias,
- sabermetrics,
- problem solving,
- probability.

The sabermetrics class is indicative of what these courses are like. I've taught this multiple times in addition to my standard teaching load; in Fall of 2009 at Williams I had five very highly motivated students (who actually started working in the Spring of 2009!). The course was split between my giving standard lectures on advanced material (linear programming, difference equations, markov chains, et cetera) to discussion sessions about model formulation and progress reports by the students. They did a phenomenal job, putting in enormous amounts of effort and learning a great deal (about theory and applications) in the process. Numerous times our empirical studies served as a springboard to introduce advanced theory. By semester's end the students wrote up their projects. Some of their conclusions run counter to current baseball intuition, but the numerics support their analysis, and these were read with interest by a major league franchise.

Winter Study

At Williams College we teach a winter study class every two years (although I enjoy these so much I teach at least one a year). These are 3-4 week classes and can range from a standard course, a beta test for a course, or just something fun and different.

- Math 10: Lqwurgxfwlrq wr Fubswrjudskb (Introduction to Cryptography). This is a four week winter study exploration in cryptography. Students come from a variety of backgrounds, from English majors interested in the history to math majors excited about delving into the theory. This led to my cryptography book; the last time the class was taught it had 70 students.
- Math 12: The Mathematics of Lego Bricks. Since their introduction in 1949, LEGO bricks have challenged and entertained millions. In this course we explore some of the connections between LEGO bricks, mathematics and popular culture. We also have a construction challenge (we have built suspension bridges spanning great chasms on campus as part of the Martin Luther King celebrations for the past two years), and have an education outreach component by going in to the elementary school once a week for 3-4 weeks.
- Physics 12: The Science of Star Trek. Co-taught with my colleague Fred Strauch in physics, we look at the physics behind Star Trek, and use the show as a vehicle to discuss many scientific and cultural issues.

4 Undergraduate and Graduate Research

In addition to the research classes I have run, I typically have between 9 and 18 undergraduates working with me over the summer, and several writing theses with me during the year (23 at Williams in 9 years, two of which were sabbaticals). I have supervised 300+ undergraduate and graduate students over the past 20 years. My students have presented their work at numerous conferences. These range each year from undergraduate oriented conferences such as the Young Mathematicians Conference at Ohio State to research conferences such as AMS special sessions, CANT (Combinatorial and Additive Number Theory), and the Maine/Québec Number Theory Conference (where for the past five years my students were the only undergraduate presenters). Several of my students have spoken at other international gatherings (France, India, and Japan).

I see the purpose of undergraduate research as an opportunity to let students explore mathematics, and get a better sense of what we do and whether or not this is what they would like to do. Sometimes students ask me for a specific project; more often I tell them general areas I'm interested in, and we explore together until a project crystalizes.

It is challenging to provide a good research experience to summer students. One wants to have a problem worth doing, but at the same time make sure that it has a good chance of being done in just 9 weeks. I aim for a mix; I have small projects that almost surely can be done and lead to light publications, but possess numerous interesting offshoots which we can then explore. Most of my summer students work on several projects simultaneously (mostly in groups, but sometimes individually); this way they see more mathematics, get a better sense of what they want to do, have a greater chance of finding something that will lead to a strong publication, and most importantly are forced to learn excellent time management skills! We've recently explored topics in random matrix theory, L-functions, graph theory, Benford's law, number theory (analytic, algebraic, additive and combinatorial) and probability; I have over 100 accepted papers from work with students.

I have also mentored several graduate students over the years. This ranges supervising them in integrated research classes (with senior faculty and undergraduates) to collaborating with them on smaller projects related to their main research interests to helping supervising Duc Khiem Huynh's dissertation (University of Bristol). Duc Khiem's thesis continues a paper of mine on the behavior of zeros near the central point in families of elliptic curve L-functions. With his two thesis advisors and another colleague, we have written 3 papers; further, while a graduate student Duc Khiem stayed with me for a month here at Williams, with us splitting our time between lectures and research. Since then I have had six other graduate students and postdocs work with me in the SMALL REU program, as well as some junior faculty. I am thus confident of my ability to supervise PHD dissertations (and while at Brown served on three committees, two in number theory and one in analysis).

5 Mathematics Education and Outreach

I am especially proud of the following math outreach efforts.

- Serving on the Mt Greylock Regional School Committee (elected by members of the towns of Lanesborough and Williamstown). Successfully changed the formula for capital project assessments b/w Lanesborough and Williamstown, which helped get the high school building project passed. Also figured out a way to fairly divide state aid in our attempted full regionalization. Right now we have three school districts: an elementary school in each town, and the high school; interestingly this results in FOUR committees! We're trying to go down to 1, but there is fear that Lanesborough will want to spend less and force Williamstown to spend less on its elementary school than we want. Right now about $3.5 million of the $20 million budgets of the three schools come from Chapter 70 aid from the state. If we fully regionalize they will no longer tell us how much goes to each school, as they want us to treat all buildings equally. Doing this means, for the same level of service, Williamstown would pay about $360,000 more and Lanesborough $480,000 less; this would not pass (nor should it!). I figured out how to well-approximate the state's hidden formula; my predictions for 2018 were off by less than $10,000 per school, well within the $200,000 in additional savings we expect to get.
- For years I have recorded all my classes and posted on YouTube, and have occasionally had students from other schools, alumns, or even Williams students who wanted to take something else at the same time slot as well take my classes remotely. I am working with LACOL (the Liberal Arts Consortium for Online Learning) to modify this program and expand it to include many classes at our schools. I led the first beta test, integrating new technology to have the remote students having as good an experience as possible, and am currently the chair of the sub-group.

I am very active in Mathematics Education. I have given lectures at the Ross Program at Ohio State, PROMYS at BU and Hampshire College; additionally, I have been a research mentor for the past 2 years at PROMYS. I didn't know about programs like these when I was younger, but I wish I had! I frequently bring my current students with me when I lecture; it's a great experience for both them and the high school students.

I have been active in math competitions as well. I have written many of the exams for the annual Williams - Middlebury Math Competition, and help write and review problems for the AMC.

I have participated in several conferences at the Institute for Mathematics & Education, and have taught continuing education classes for teachers in cryptography, Benford's law of digit bias, and calculus. Along these lines, I'm participating in the Value of Computational Thinking across Grade Levels (VCTAL) Project, where I write modules for schools.

I am revamping my math riddles page http://mathriddles.williams.edu/ (with several students). The goal is to help students and teacher see that math can be both fun and useful. We have added a student / teacher's corner (the password is *corner*), where we have extensive discussions about how to think about / attack these riddles, as well as additional reading about the mathematics behind these problems. For example, one of my favorites is the chess problem posted here: http://mathriddles.williams.edu/?p=57. The solution (or perhaps its better to say a very good way to find the solution) to this riddle involves the same principle as airlines use to determine their optimal schedules! Building on these problems, I am working with OIT at Williams to create short web videos for general audiences. Two are online at

- http://www.youtube.com/watch?v=Esa2TYwDmwA&feature=g-upl and
- http://www.youtube.com/watch?v=aMorr1h4Egs&feature=g-upl.

6 VIGRE Undergraduate Mathematics Laboratory

6.1 Purpose

For many years I was involved in the design and implementation of a new advanced undergraduate class at Princeton, NYU, and Ohio State. Sponsored by a VIGRE grant from the NSF, Professor Sarnak and I created an Undergraduate Mathematics Laboratory at Princeton. We then brought the class to the Courant Institute, NYU (where we co-taught it with Alex Barnett), and I then brought the course to Ohio State. I have run a smaller version at Brown (by myself and with Jill Pipher in 2006), and then at Williams I taught a senior seminar from a book I wrote for the course.

The statements and simple cases of many interesting conjectures are accessible to undergraduates; further, there is often very little numerical support for the sweeping generalizations that are claimed in the conjectures. Undergraduates learn the necessary theory and quickly become involved in cutting edge research. They see what types of problems mathematicians study, and experience what it would be like to be a graduate student by doing original (guided) research.

While many graduate students and most postdocs have taught a standard class, few have mentored students or helped design research programs; additionally, many graduate students have yet to conduct original research, write papers or give research talks. Under the supervision of the faculty, these skills are built by having the graduate students help choose problems to investigate with the undergraduates they are mentoring. The integrated nature of the program provides valuable training and exposure not seen in a typical class or research group.

6.2 Course Structure

The courses run roughly as follows: the professors and postdocs lecture on various topics for a few weeks. Participants often come from very diverse backgrounds with different skills, and supplemental lectures are given on needed background material. For example, to investigate certain conjectures we rarely need an entire course on Probability or Complex Analysis, but rather a few key results. Students are quickly given problems to think about / experiment with; as the semester progresses, the students and staff break into smaller groups. The general lectures are replaced with more specific presentations. At the end of the semester, students submit a .tex version of their work, documented computer codes, and present their research to the class.

When possible, we try to choose an integrated series of problems, so that students will be able to converse with each other about their work. We have worked hard to maintain good faculty to student ratios (usually about 1:3 or 1:4). For example, at Ohio State we had two senior faculty, two post-docs, and eight undergraduate and graduate students. One group was constructing families of elliptic curves with moderate to high rank, which was used by another group looking at the effect of additional zeros at the central point on the first zero above the central point, both of which help a third group investigating family dependent lower order corrections to distribution of zeros near the central point.

Typically these classes have a few faculty members, several graduate students or postdocs, and an undergraduate computer assistant. These are mathematics classes, not computer classes. While many undergraduates have programming experience, not much is required to do good investigations, due to the power of packages such as Mathematica, Maple, Matlab, Magma, C/PARI, etc. Not all the staff needs to be expert in these systems; typically only a subset is, and an undergraduate with strong skills is sometimes hired to be point man for debugging. Also, after running these courses for years, we have assembled an extensive database of documented programs, which makes it easy for future investigators with little computer experience to continue these researches.

6.3 Topics and Results

Projects from previous years are available on-line:

- Princeton: `http://www.math.princeton.edu/~mathlab/index.html`
- NYU: `http://www.williams.edu/Mathematics/sjmiller/public_html/math/generalmath/uml@nyu`
- AIM: `http://www.williams.edu/Mathematics/sjmiller/public_html/math/generalmath/AIM`
- Ohio State: `http://www.williams.edu/Mathematics/sjmiller/public_html/math/generalmath/reu`

Some previous topics of investigation include

1. Hardy-Littlewood Circle Method (Varieties; Goldbach; Germain Primes)
2. Random Matrix Theory (band matrices; Toeplitz and Palindromic Toeplitz matrices; truncated Cauchy matrices; sparse matrices)
3. Ramanujan Graphs / Random Graphs / k-Regular Graphs
4. Elliptic Curves (Birch and Swinnerton-Dyer Conjecture; excess rank, points of low height, signs of functional equations, first zero above the central point, spacings between zeros; Sato-Tate, constructing one-parameter families with rank)
5. Primality Testing
6. Equidistribution of Roots of Polynomials mod p
7. Continued Fractions (distribution of digits; special families; closed form expressions; periodic continued fractions)
8. Poissonian Behavior (especially of $\{n^k\alpha\}$)
9. Dynamical Piston
10. Lone Runner
11. Interval Exchanges
12. $3x+1$ Problem
13. Benford's Law and Digit Bias

6.4 My Duties

My duties range from designing the class and choosing problems to giving background and advanced lectures to supervising undergraduate and graduate research and finally to mentoring graduate students and postdocs in how to advise and mentor undergraduates and design research programs. I am also responsible for teaching the participants how to use the software (C, PARI, Matlab, Maple, Mathematica, LaTeX), as well as providing help with programming and efficient algorithm design.

6.5 Summary

Designing and running the UML was instrumental in my becoming the educator I am. It taught me how to supervise undergraduate and graduate research. What matters most is not the background of the participants, but their enthusiasm and interest in mathematics. There is no dearth of accessible problems, and in fact some of the best projects were done by students with the weakest background. With other professors, I supervised a variety of simultaneous research projects (eight students the first year, eleven the second, twenty in the third, eight in the fourth and three in the fifth). The UML gave me two rare opportunities for a graduate student: I was an organizer / coordinator for a research group, and I participated in the design and creation of a non-standard class. I also learned how to budget my time so that my own research does not suffer.

It has been a very exciting and enjoyable class to teach. I helped students investigate unsolved conjectures, and helped to train / work with graduate students and postdocs. While it is time consuming, the problems are exciting and worth doing, and at the end of the year I have helped prove new results (or at least gather experimental evidence in their support). It does take significantly longer to solve and write-up a project than it would if I worked alone, but I am exposed to a lot more mathematics and I help train future colleagues, benefiting from their enthusiasm and viewpoints.

Results from these researches have been presented at conferences and accepted in research journals for publication. Also, Ramin Takloo-Bighash and I have written a book, *An Invitation to Modern Number Theory* (Princeton University Press, 2006). The purpose of our book is to provide an introduction to current problems in Number Theory. While capable of being used as a standard text, it is based on lectures, problems and results from the past few years of the UML. Several new results in the fields are contained, as well as extensive literature review and lists of additional topics for research.

Finally, the course has evolved to more heavily include graduate students. Building on our success with undergraduates, we are expanding the role of graduate students. Instead of solely involving upper level graduate students as mentors, we now have pre-Generals graduate students, who can explore an area of mathematics before having to choose a thesis topic. Both sets of graduate students are given assistance in mentoring and learning how to lecture on current research.