Math 313 / 331 : Spring 2016 : sjm1@williams.edu

In[211]:=
```
(*C cookies, 5 People function*)
Ccookies5people[numC_] := Module[{},
   (* numC is the number of cookies to divide among 5 people *)
   x = Sum[
     Sum[Sum[Sum[If[a1 + a2 + a3 + a4 + a5 == numC, 1, 0], {a5, 0, numC}],
        {a4, 0, numC}], {a3, 0, numC}], {a2, 0, numC}], {a1, 0, numC}];
   Return[x];
  ];
```

In[214]:=
```
Timing[Ccookies5people[40]]
```

Out[214]= {122.726, 135 751}

In[215]:=
```
Ccookies5peopleSlightlyFaster[numC_] := Module[{},
   x = Sum[
     Sum[Sum[If[a1 + a2 + a3 + a4 ≤ numC, 1, 0], {a4, 0, numC}],
       {a3, 0, numC}], {a2, 0, numC}], {a1, 0, numC}];
   Return[x];
  ];
```

In[216]:=
```
Timing[Ccookies5peopleSlightlyFaster[40]]
```

Out[216]= {2.66762, 135 751}

In[161]:=
```
Ccookies5peoplefaster[numC_] := Module[{},
   x = Sum[Sum[Sum[Sum[1, {a4, 0, numC - (a1 + a2 + a3)}], {a3, 0,
        numC - (a1 + a2)}], {a2, 0, numC - a1}], {a1, 0, numC}];
   Return[x];
  ];
```

In[218]:=
```
Timing[Ccookies5peoplefaster[200]]
```

Out[218]= {6.61444, 70 058 751}

In[219]:=
```
CcookiesPpeople[numC_, numP_] := Module[{},
   (*numC is number of cookies, numP is number of people *)
   count = 0; (* sets the number of successes to 0*)
   (* key
    idea: count from 0 to (numC+1)^(numP-1)-1 in base numC+1 *)
   (* the digits are going to be 0, 1, ..., numC *)
   (* if the sum is at most numC then
    can give the remaining cookies to new person*)
   (* IntegerDigits converts n to a list of numP-1 digits *)
   (* the second argument is the base,
   the third pads 0 digits in front *)
   (* this counts POORLY, but advantage is
    don't need to know number of people*)
   For[n = 0, n ≤ (numC + 1) ^ (numP - 1) - 1, n++,
    {
     x = IntegerDigits[n, numC + 1, numP - 1];
     If[Sum[x[[i]], {i, 1, numP - 1}] ≤ numC, count = count + 1];
    }];
   Print[count];
  ];
```

In[221]:=
```
Timing[CcookiesPpeople[40, 5]]
```

```
135751
```

Out[221]= {18.3301, Null}

In[222]:=
```
For[c = 1, c ≤ 20, c++, Print[Ccookies5peoplefaster[c]]]
```

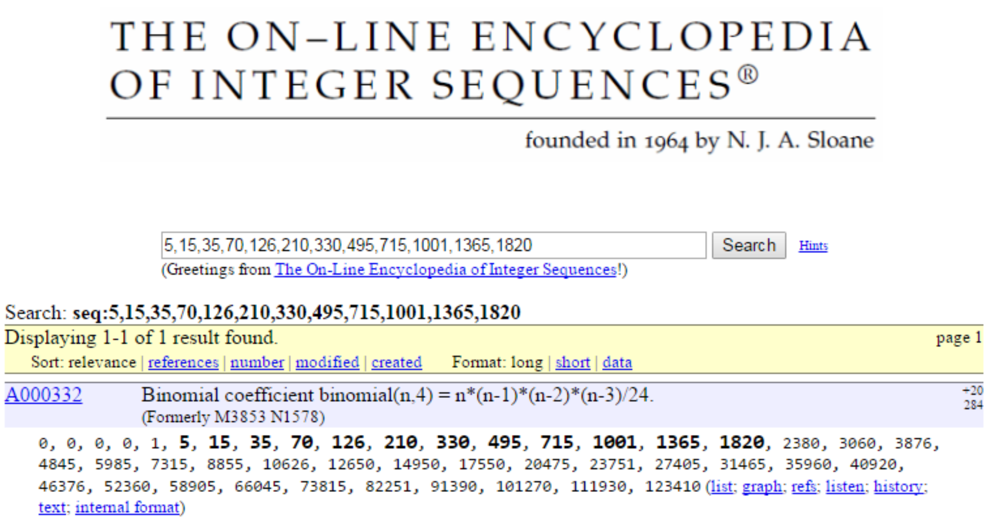
```
5
15
35
70
126
210
330
495
715
1001
1365
1820
2380
3060
3876
```

```
4845
5985
7315
8855
5
15
35
70
126
210
330
495
715
1001
1365
1820
2380
3060
3876
4845
5985
7315
8855
10 626
5
15
35
70
126
210
330
495
715
1001
1365
1820
```

```
2380

3060

3876

4845

5985

7315

8855

10 626

10 626
```

In[150]:=
```
Print[Hyperlink["http://oeis.org/A000332"]]
```

```
http://oeis.org/A000332
```



THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

5,15,35,70,126,210,330,495,715,1001,1365,1820 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **seq:5,15,35,70,126,210,330,495,715,1001,1365,1820**

Displaying 1-1 of 1 result found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A000332 Binomial coefficient binomial(n,4) = n*(n-1)*(n-2)*(n-3)/24. +20 284

(Formerly M3853 N1578)

0, 0, 0, 0, 1, **5, 15, 35, 70, 126, 210, 330, 495, 715, 1001, 1365, 1820**, 2380, 3060, 3876, 4845, 5985, 7315, 8855, 10626, 12650, 14950, 17550, 20475, 23751, 27405, 31465, 35960, 40920, 46376, 52360, 58905, 66045, 73815, 82251, 91390, 101270, 111930, 123410 (list; graph; refs; listen; history; text; internal format)

CODE TO LOOK AT FRACTIONS FROM 1 thru 9, each digit once

In[244]:=
```
list = {};
For[j = 1, j ≤ 9, j++, list = AppendTo[list, j]];
permlist = Permutations[list];
For[n = 1, n ≤ 9!, n++,
  {
   x = permlist[[n]];
   If[(x[[1]] / (10 x[[2]] + x[[3]])) +
      (x[[4]] / (10 x[[5]] + x[[6]])) +
      (x[[7]] / (10 x[[8]] + x[[9]])) == 2, Print[x]];
  }];
```

Subset problem : Set of N distinct elements Take any subset, A (N) Take a subset of A (N), B (A (N)).How many ways can you do this?

In[265]:=
```
Clear[n];
subsetfunction[n_] := Module[{},
  count = 0;
  list = {};
  For[j = 1, j ≤ n, j++, list = AppendTo[list, j]];
  subsetlist = Subsets[list, n];
  For[m = 1, m ≤ Length[subsetlist],
   m++, count = count + 2^Length[subsetlist[[m]]]];
   (*Print[count];*)
  Return[count];
 ];
```

In[267]:=
```
For[num = 0, num ≤ 10, num++, Print[num, " ", subsetfunction[num]]]
```

```
0 1
1 3
2 9
3 27
4 81
5 243
6 729
7 2187
8 6561
9 19683
10 59049
```

Math / Stat 341 : Fall 2015 :

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```
(* Computing a 5-0 trump split among two hands *)
deck = {}; (* initialize deck to empty *)
 (* assign five 1s to the deck; the 1s represent the trump suit *)
(* then we assign 21 0s, these are the non-trump *)
(* taking time and coding well can save you a LOT of trouble *)
For[n = 1, n ≤ 5, n++, deck = AppendTo[deck, 1]];
For[n = 6, n ≤ 26, n++, deck = AppendTo[deck, 0]];
Length[deck] (* makes sure got 26 cards *)
(* should have this in the program so we make sure we use the right deck,
and thus will paste it below! *)
```

```
26
```

```
trumpsplit[numdo_] := Module[{},
   count = 0;
   deck = {}; (* initialize deck to empty *)
   For[n = 1, n ≤ 5, n++, deck = AppendTo[deck, 1]];
   For[n = 6, n ≤ 26, n++, deck = AppendTo[deck, 0]];
   For[n = 1, n ≤ numdo, n++, (* main loop of code *)
    {
     hand = RandomSample[deck, 13]; (* randoml1y choose 13 cards *)
     numtrump = Sum[hand[[k]], {k, 1, 13}];
     (* note numtrump is 0 or 5 if we have a 5-0 split *)
     If[numtrump == 0 || numtrump == 5, count = count + 1];
     (* count is our counter, counts how often have 5-0 *)
     (* we use || for or; would use && for and use two equal signs for comparison*)
    }]; (* end of n loop *)
   Print["Two theories: 2(1/2)^5 gave ", 6.25, "%, other gave 3.913%."];
   Print["We observe ", 100. count / numdo, "."];
  ];

Timing[trumpsplit[1000000]]
```

```
Two theories: 2(1/2)^5 gave 6.25%, other gave 3.9%.

We observe 3.9166.

{11.2945, Null}
```

```
(* Getting exactly two kings *)
twokings[numdo_] := Module[{},
   deck = {}; (* initialize deck to empty *)
   (* 1 is a king, 0 non-king *)
   For[n = 1, n ≤ 4, n++, deck = AppendTo[deck, 1]];
   For[n = 5, n ≤ 52, n++, deck = AppendTo[deck, 0]];
   count = 0; (* initialize num of successes to 0 *)
   For[n  = 1, n ≤ numdo, n++,
    {
     hand = RandomSample[deck, 5]; (* 5 card hand *)
     numkings = Sum[hand[[k]], {k, 1, 5}];
     If[numkings == 2, count = count + 1];
    }]; (* end of n loop *)
   Print["Theory predicts prob exactly two kings is ",
    100.0 Binomial[4, 2] Binomial[48, 3] / Binomial[52, 5], "."];
   Print["Observed probability is ", 100.0 count / numdo, "."];
  ];

Timing[twokings[1000000]]
```

```
Theory predicts prob exactly two kings is 3.99298.

Observed probability is 3.9965.

{6.94204, Null}
```

```
Length[deck]
```

```
52
```

```
(* calculating probability of a full house, queens and kings *)
(* probability is VERY small so must do a lot of simulations! *)
(* sadly the more you want to compute, the worse Mathematica is *)
(* this is not a hard code, don't really need the special fns here *)
(* would want to shift to another language that is better *)
fullkingqueens[numdo_] := Module[{},
   deck = {}; (* initialize deck to empty *)
   (* 10 is a queen, 1 is a king, 0 non-king *)
   For[n = 1, n ≤ 4, n++, deck = AppendTo[deck, 1]];
   For[n = 5, n ≤ 8, n++, deck = AppendTo[deck, 10]];
   For[n = 9, n ≤ 52, n++, deck = AppendTo[deck, 0]];
   count = 0; (* initialize num of successes to 0 *)
   For[n  = 1, n ≤ numdo, n++,
    {
     hand = RandomSample[deck, 5]; (* 5 card hand *)
     numkings = Sum[hand[[k]], {k, 1, 5}];
     (* want full house of Qs and Ks *)
     (* sum is either 23 or 32! *)
     If[numkings == 32 || numkings == 23, count = count + 1];
    }]; (* end of n loop *)
   Print["Theory predicts prob full house (Qs and Ks) is ",
    100.0 Binomial[2, 1] Binomial[4, 3] Binomial[4, 2] / Binomial[52, 5], "."];
   Print["Observed probability is ", 100.0 count / numdo, "."];
  ];
```

```
Timing[fullkingqueens[10000000]]
```

```
Theory predicts prob full house (Qs and Ks) is 0.00184689.

Observed probability is 0.00168.

{71.9165, Null}
```

```
Timing[fullkingqueens[40000000]]
```

```
Theory predicts prob full house (Qs and Ks) is 0.00184689.

Observed probability is 0.0018925.

{298.945, Null}
```

```
fullhouse[numdo_] := Module[{},
   count = 0; (* this is our count variable, records successes *)
   deck = {}; (* initializing deck; cards are 1, 10, 100, 1000, ... *)
   For[n = 1, n ≤ 13, n++,
    For[j = 1, j ≤ 4, j++,
     deck = AppendTo[deck, 10^(n - 1)]
    ]]; (* end of for loops, deck created *)
   For[n = 1, n ≤ numdo, n++,
    {
     hand = RandomSample[deck, 5]; (* randomly chooses 5 cards *)
     value = Sum[hand[[k]], {k, 1, 5}]; (* sums the value of cards *)
     (* next few lines is a nice trick. if our hand is 10002011 this means *)
     (* we have one number repeated twice, the 1000; and have one 1, one 10, *)
     (* and one 10000000. notice how easy it is to check! the only way to have *)
     (* a full house is to have one 2 and one 3 among digits, or the product of *)
     (* non-zero digits is a 6. Easier to do MemberQ to check on 2, 3 but show both *)
     valuelist = IntegerDigits[value];
     If[
      MemberQ[valuelist, 2] == True && MemberQ[valuelist, 3] == True, count = count + 1];
     (* this is how to do as a product of digits *)
     (* ----------------- *)(*
     trimlist = {};
     For[j = 1, j ≤ Length[valuelist], j++,
      If[valuelist[[j]] > 1, trimlist = AppendTo[trimlist,valuelist[[j]]]
      ] ];
     If[Product[trimlist[[j]],{j,1,Length[trimlist]}] == 6, count = count+1];
     --------------- *)
    }]; (* end of n loop *)
   Print["Prob of a full house: ", 100.0 Binomial[13, 2]
     Binomial[2, 1] Binomial[4, 3] Binomial[4, 2] / Binomial[52, 5], "%."];
   Print["Observed prob: ", 100.0 count / numdo, "%."];
  ];

Timing[fullhouse[2 000 000]]
```

```
Prob of a full house: 0.144058%.

Observed prob: 0.14615%.

{69.4828, Null}
```