FIRST EXAM–STUDY COPY

These are the problems from the first midterm exam.

1. PROBLEM ONE

Compute the determinant of the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 5 \\ 2 & 0 & -1 & 0 \\ 0 & 7 & 0 & 1 \end{pmatrix}.$$

2. PROBLEM TWO

Consider the following system of linear equations.

$$\begin{array}{rrrcr} 4x & +3y & +z & = & 8 \\ 2x & +y & -z & = & 2 \\ x & -y & +z & = & 1 \end{array}$$

- Write the matrix form of this system.
- Find the set of solutions to the following system of linear equations by any method involving matrices.

3. PROBLEM THREE

- Add the matrices $C = \begin{pmatrix} 0 & -5 \\ 3 & 7 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 1 \\ -3 & 6 \end{pmatrix}$.
- Compute the inner product of the vectors $v_1 = \begin{pmatrix} 3 & 4 \end{pmatrix}^t$ and $v_2 = \begin{pmatrix} -1 & 2 \end{pmatrix}^t$
- Find the inverse of the matrix $E = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 0 & 5 \\ 0 & -1 & 1 \end{pmatrix}$, if it exists.

4. PROBLEM FOUR

Suppose that A is a square matrix. Show that $\text{null}(A) = \{0\}$ if and only if the columns of A are linearly independent.

5. PROBLEM FIVE

Write down a system of two equations in three unknowns that fails to have a solution. Draw a picture that explains why your system fails to have a solution and explain the relevance of your picture.