

HOMEWORK ASSIGNMENT # 10

MATH 211, FALL 2006, WILLIAMS COLLEGE

ABSTRACT. This assignment has three problems on two pages. It is due on Tuesday, December 12 by 5pm. Good Luck!

1. ON EIGENVALUES AND THE TRANSPOSE

Let A be an $n \times n$ square matrix. Show that the matrices A and A^t have the same eigenvalues.

For the next three problems, as far as you can, compute the characteristic polynomial, find the eigenvalues, and compute a basis of the corresponding eigenspaces for each matrix. Then use this information to decompose the matrix in the form $A = PDP^{-1}$, where D is diagonal and P is invertible, if possible. If it is not possible, point out where the program fails. If the matrix is symmetric, be sure to arrange it so that P is orthogonal. The whole problem should be done with real scalars.

2. SOME 2×2 's

$$A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 8 & 1 \\ -1 & 6 \end{pmatrix}$$

3. SOME 3×3 's

$$A = \begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 & 1 \\ 3 & -7 & 0 \\ 1 & 3 & 1 \end{pmatrix}$$

4. SOME 4×4 's

$$A = \begin{pmatrix} 8 & 1 & 1 & 1 \\ -1 & 6 & 1 & 0 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & -1 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & -4 \\ 2 & 1 & 4 & -3 \end{pmatrix}$$