HOMEWORK ASSIGNMENT # 2

MATH 211, FALL 2006, WILLIAMS COLLEGE

Abstract. This assignment has five problems on two pages. It is due on Monday, September 25 in class. Talk with me if you have difficulty. Be prepared to deal with some messy numbers: be careful and have a four button calculator nearby. Good Luck!

1. Problem One

Use the elimination/back-solving algorithm to solve the following systems. (i.e. put the system in row echelon form and use back-solving.)

(1)

$$\begin{cases} x_1 & +x_2 & +x_3 & +x_4 & +x_5 & = & 7 \\ 3x_1 & +2x_2 & +x_3 & +x_4 & -3x_5 & = & -2 \\ & +x_2 & +2x_3 & +2x_4 & +6x_5 & = & 23 \\ 5x_1 & +4x_2 & +3x_3 & +3x_4 & -x_5 & = & 12 \end{cases}$$

(2)

$$\begin{cases} -2x_1 & +x_2 & +6x_3 & = & 18 \\ 5x_1 & & +8x_3 & = & -16 \\ 3x_1 & +2x_2 & -10x_3 & = & -3 \end{cases}$$

2. Problem Two

(1) Consider the hyperplane $\mathcal{H} = \{a_1x_1 + \ldots + a_nx_n = b\}$. Show that the vector $n = (a_1, \ldots, a_n)$ is normal to $\mathcal{H}$.
(2) Show that two planes are parallel if and only if their normal vectors are proportional.

Date: September 21, 2006.

3. Problem Three

Use Gaussian elimination (i.e. go to reduced row echelon form) to find the solution set to the following systems.

(1)

$$\begin{cases} x_1 & +x_2 & = & 4 \\ 2x_1 & -3x_2 & = & 7 \\ 3x_1 & -2x_2 & = & 11 \end{cases}$$

(2)

$$\begin{cases} x_1 & +x_2 & = & 4 \\ 2x_1 & -3x_2 & = & 7 \\ 3x_1 & +2x_2 & = & 8 \end{cases}$$

(3)

$$\begin{cases} 3x_1 & +x_2 & -x_3 & +2x_4 & = & 1 \\ 2x_1 & -x_2 & +x_3 & +x_4 & = & 4 \\ 8x_1 & +x_2 & +x_3 & +5x_4 & = & 6 \end{cases}$$

4. Problem Four

Use the augmented matrix method to

(1) solve the system

$$\begin{cases} x_1 & +2x_2 & +4x_3 & = & 1 \\ 2x_1 & +x_2 & +5x_3 & = & 0 \\ 3x_1 & -x_2 & +6x_3 & = & 1 \end{cases}$$

(2) find the inverse to the matrix

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 5 & 2 \\ 1 & 0 & 0 \end{pmatrix}.$$

Show both of your answers are correct using matrix mulitiplication.

5. Problem Five

What is the "expected dimension" of the solution set to a system of n linear equations in m unknowns? That is, what happens most often? Be sure to discuss the possible cases of $m < n$, $m = n$, and $m > n$. (I don't expect a proof, but I want some discussion as to why that is the expected answer.)