HOMEWORK ASSIGNMENT # 3

MATH 211, FALL 2006, WILLIAMS COLLEGE

Abstract. This homework assignment has four problems on 3 pages. It is due on Monday, October 2 in class. Please ask for help if you are stuck. Start this one early. Good Luck!

1. Problem: On LU decompositions

This problem will lead you through understanding the LU decomposition of a matrix. Your answer for this problem should show all of the work indicated in detail. Consider the matrix

$$A = \begin{pmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{pmatrix}.$$

- Working left to right and top to bottom, apply multiplications by elementary matrices to put the matrix into row echelon form, U. Keep careful track of the matrices you use, and the order you apply them.
- Multiply the elementary matrices you used in the last step in the correct order to write a matrix E, and write out the matrix equation $EA = U$.
- In each elimination step from the first part of the problem, the number $m_{ji} = -a_{ji}/a_{ii}$ you used to scale the coefficients is called a *multiplier*. Write down the matrix

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & -m_{32} & 1 \end{pmatrix}.$$

- Show that $E^{-1} = L$.
- Conclude that $A = L \cdot U$. Check this by performing the matrix multiplication.
- Use the LU decomposition you just found to solve the system $Ax = b$ for $b = \begin{pmatrix} 1 & 1 & 2 \end{pmatrix}^t$ by the "two backsolvings" method we discussed in class.

Note: not every matrix has an LU decomposition. If one must switch rows when doing the forward elimination, then on must keep track of these. This results in a PLU decomposition, where the P is a permutation matrix, which keeps track of the row switching.

2. Problem: Homogeneous and inhomogeneous equations

Consider the system of equations

$$\begin{cases} x_1 & -2x_2 & +x_3 & +x_4 & +2x_5 & = & 2 \\ -x_1 & +3x_2 & & +2x_4 & -2x_5 & = & 2 \\ & +x_2 & +x_3 & +3x_4 & +4x_5 & = & 4 \\ x_1 & +2x_2 & +5x_3 & +13x_4 & +5x_5 & = & 18 \end{cases}$$

- Write the matrix version of this equation as $Ax = b$.
- Write the associated homogeneous equation in matrix form.
- Compute the null space of A.
- Show that b lies in the column space of A. To what matrix equation does this correspond? What particular solution of the original system do you obtain?
- Write the vector form of the general solution to the original system.

3. Problem: On linear dependence

Are the following sets of vectors linearly dependent or linearly independent?

(1) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}.$

(2) $\begin{pmatrix} -3 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix}, \begin{pmatrix} -3 \\ 4 \\ 18 \end{pmatrix}.$

(3) $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}.$

4. Problem: Determinants

Evaluate the following determinants.

(1) By passing to row echelon form:

$$\begin{vmatrix} 2 & -1 & 3 \\ -1 & 2 & -2 \\ 1 & 4 & 0 \end{vmatrix}$$

(2) By expanding along a row or column and using cofactors:

$$\begin{vmatrix} 3 & 3 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

(3) By whatever method (or combination) seems appropriate:

$$\begin{vmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 2 & 2 \\ -1 & -1 & -1 & 2 \end{vmatrix}$$