HOMEWORK ASSIGNMENT # 4

MATH 211, FALL 2006, WILLIAMS COLLEGE

Abstract. This homework assignment has three problems on 1 page. It is due on Friday, October 6 in class. Please ask for help if you are stuck. Start this one early. Good Luck!

1. Problem: Cofactors and Cramer's rule

(1) Use the classical adjoint method to compute the inverse of the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 6 & -3 & 4 \end{pmatrix}.$$

(2) Use Cramer's rule to solve the following system.

$$\begin{cases} x_1 & -2x_2 & +x_3 & +x_4 & = & 12 \\ -x_1 & +3x_2 & +x_3 & +2x_4 & = & 12 \\ & +x_2 & +x_3 & +3x_4 & = & 0 \\ x_1 & +2x_2 & +5x_3 & +x_4 & = & 96 \end{cases}$$

2. On elementary matrices

Show that any invertible square matrix can be written as a product of elementary matrices.

3. On singular matrices

Consider the system $Ax = b$ for

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \text{ and } b = \begin{pmatrix} 4 \\ 4 \end{pmatrix}.$$

Show that the matrix A is singular. One one set of axes, draw a picture of the column space of A in $\mathbb{R}^2$. On another set of axes, draw a picture of the null space of A in $\mathbb{R}^2$ and the solution set to $Ax = b$. Reasoning from the pictures, find a vector b' so that the system $Ax = b'$ has no solution. (Which set of axes should b' live in?)

Can you describe what is happening from the viewpoint of intersecting hyperplanes?