HOMEWORK ASSIGNMENT # 5

MATH 211, FALL 2006, WILLIAMS COLLEGE

Abstract. This assignment has four problems on two pages. It is due on Wednesday, October 25 in class. Talk with me if you have difficulty. Good Luck!

1. Vector Spaces and Subspaces

(1) Show directly that the set V of real valued continuous functions on the interval $[0, 1]$ is a vector space under the pointwise operations we discussed in class.

(2) Decide if the following two subsets of V are subspaces. If a subspace, prove it. If not a subspace, say why explicitly.
   - W^{even} is the set of real polynomials of even degree.
   - $W^{\text{even power}}$ is the set of real polynomials in which every term has even degree.

2. Spans

(1) Find the span of the subset $S = \{w_1, w_2, w_3, w_4\}$ in $\mathbb{R}^3$, where

$$w_1 = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \qquad w_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \qquad w_3 = \begin{pmatrix} 10 \\ 10 \\ 25 \end{pmatrix}, \qquad w_4 = \begin{pmatrix} 5 \\ -1 \\ 8 \end{pmatrix}.$$

Can you describe this set in a simple, compact way?

(2) Show that the set $S = \{v_1, v_2, v_3\}$ is a linearly independent subset of $\mathbb{R}^4$, where

$$v_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \\ 4 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 5 \\ 10 \\ 3 \\ 25 \end{pmatrix}.$$

3. Bases and dimension

Find the dimension of the intersection of the following collection of hyperplanes in $\mathbb{R}^4$. (Note that they all pass through the origin, so they are subspaces and so is their intersection.) Write down a basis for this intersection.

$$\mathcal{H}_1 = \{w + 2x - y + 3z = 0\}$$
$$\mathcal{H}_2 = \{w - 2x + 10y - 4z = 0\}$$
$$\mathcal{H}_3 = \{5w + 2x + 17y + z = 0\}$$

Date: October 23, 2006.

4. Bases and Coordinates

Consider the vector space $W_3 = \{$real polynomials of degree less than or equal to 3$\}$. Show that the set $B = \{1, t, \frac{1}{2}(3t^2 - 1), \frac{1}{2}(5t^3 - 3t)\}$ is a basis of W_3. Write the coordinates of the vector $v = 1 + t + t^2 + t^3$ with respect to this ordered basis.