# HOMEWORK ASSIGNMENT # 6

MATH 211, FALL 2006, WILLIAMS COLLEGE

ABSTRACT. This assignment has five problems on two pages. It is due on Wednesday, November 1 in class. Talk with me if you have difficulty. Good Luck!

## 1. Rank

Find the rank of the following matrices

$$A = \begin{pmatrix} 1 & 3 & -2 & 5 & 4 \\ 1 & 4 & 1 & 3 & 5 \\ 1 & 4 & 2 & 4 & 3 \\ 2 & 7 & -3 & 6 & 13 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 & 2 \\ 4 & 5 & 5 \\ 5 & 8 & 1 \\ -1 & -2 & 2 \end{pmatrix}.$$

## 2. Reverse engineering

Find a homogeneous system of linear equations whose solution set is spanned by the vectors

$$u_1 = \begin{pmatrix} 1 \\ -2 \\ 0 \\ 3 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 4 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 5 \end{pmatrix}.$$

## 3. Rows and Columns

Find a basis for the row space and a basis for the column space for each of these matrices.

$$A = \begin{pmatrix} 0 & 0 & 3 & 1 & 4 \\ 1 & 3 & 1 & 2 & 1 \\ 3 & 9 & 4 & 5 & 2 \\ 4 & 12 & 8 & 8 & 7 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 1 & 0 & 1 \\ 1 & 2 & 2 & 1 & 3 \\ 3 & 6 & 5 & 2 & 7 \\ 2 & 4 & 1 & -1 & 0 \end{pmatrix}$$

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*Date*: October 31, 2006.

## 4. General functions

Suppose that $f : A \to B$ and $g : B \to C$ are functions and that $g \circ f : A \to C$ is surjective. Is it necessary that $f$ is surjective? Is it necessary that $g$ is surjective? If either function must be surjective, give a proof that this is true. If not, give an example of a pair of functions $f$ and $g$ for which the relevant function is not surjective, but the composition still is.

## 5. Linear functions

Let $V$ be the vector space of all smooth (i.e. infinitely many times differentiable) functions $\mathbb{R} \to \mathbb{R}$, $W$ be the vector space of all polynomials with real coefficients in the variable $t$, and $W_3$ the vector space of all polynomials of degree at most three in the variable $t$.

Show that the following mappings are linear maps, and find their kernels and ranges.

(1) $T : W \to W$ defined by $T(p) = t^2 \cdot p$.
(2) $S : W_3 \to W_3$ defined by $S(p) = t \cdot p'$, where $p'$ is the derivative of $p$ with respect to $t$,
(3) $T_{3,\pi} : V \to W_3$ defined by setting $T_3(f)$ equal to the third degree Taylor polynomial of $f$ centered at the point $a = \pi$.