MATH 211 HOMEWORK ASSIGNMENT 7

FALL 2006, WILLIAMS COLLEGE

Abstract. This assignment has 4 problems on 1 page. It is due Wednesday, November 8 in class. Don't hesitate to ask for help.

1. Nonsingularity

Show that the composition of two nonsingular linear transformations is also nonsingular.

2. Rank-Nullity

Find bases of the kernel and image and verify the Rank-Nullity theorem for the linear transformation $T_A : \mathbb{R}^5 \to \mathbb{R}^4$ associated to the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 & 2 & -2 \\ 1 & 0 & 3 & 5 & 1 \\ 6 & 2 & 1 & 0 & -9 \\ -3 & 3 & -1 & 7 & 8 \end{pmatrix}.$$

3. Hyperplanes

Is it possible to find a family of 4 hyperplanes in $\mathbb{R}^4$ so that any subset of three hyperplanes must intersect in at least a line, but that the common intersection of all four hyperplanes is empty? If so, give a concrete example. If not, explain why it is not possible in terms of some of the tools we have developed.

4. An interesting new space

Let $\hom(\mathbb{R}^3, \mathbb{R}^4)$ be the collection of all linear mappings from $\mathbb{R}^3$ into $\mathbb{R}^4$.

- What should addition and scalar multiplication be in this space?
- Show that $\hom(\mathbb{R}^3, \mathbb{R}^4)$ is a vector space under these operations.
- Find a basis for $\hom(\mathbb{R}^3, \mathbb{R}^4)$ and use it to compute the dimension of this space.
- Describe two different more easily understood spaces which are isomorphic to $\hom(\mathbb{R}^3, \mathbb{R}^4)$. Write down explicit isomorphisms between $\hom(\mathbb{R}^3, \mathbb{R}^4)$ and the two spaces.