Second Exam

Math 211, Williams College

November 10, 2006

Directions: This Exam has 4 questions. Be sure to show all of your work and explain yourself carefully. You have two and a half hours to complete this exam, to be done in a single block of time. You may not use any references. Please do not discuss the exam with anyone until after class on Wednesday. Please sign the honor code pledge below and attach this cover sheet to your answers.

This exam is due in class on Wednesday, November 15th.

Pledge: On my honor, I have neither given nor received any aid on this examination.

Write pledge and sign name here:

Print your name here: ______________________________

Problem	*Possible*	*Score*
1	8	
2	4	
3	10	
4	8	
Total	30	

1. Problem One

Consider the following two linear subspaces W_1, W_2 of $\mathbb{R}^3$:

$$W_1 = \operatorname{span}\left\{w_1 = \begin{pmatrix}1 & 2 & 1\end{pmatrix}^t, w_2 = \begin{pmatrix}-1 & 1 & 1\end{pmatrix}^t\right\}$$

$$W_2 = \operatorname{span}\left\{v_1 = \begin{pmatrix}1 & 2 & 1\end{pmatrix}^t, v_2 = \begin{pmatrix}2 & 1 & 0\end{pmatrix}^t, v_3 = \begin{pmatrix}0 & 3 & 2\end{pmatrix}^t\right\}$$

(1) Are these two subspaces equal?
(2) Either show that $\{v_1, v_2, v_3\}$ is a basis for W_2, or find a subset of this set which is a basis of W_2.

2. Problem Two

Show that if two hyperplanes in $\mathbb{R}^6$ intersect, then they must do so in a set of dimension at least 4.

3. Problem Three

Let $M_2(\mathbb{R}) = \{\text{real } 2\times 2 \text{ matrices}\}$ and $S_2(\mathbb{R}) = \{\text{real } 2\times 2 \text{ symmetric matrices}\}$. Recall that a matrix A is *symmetric* when $A^t = A$. We have previously seen that $M_2(\mathbb{R})$ is a vector space.

(1) Show that $S_2(\mathbb{R})$ is a vector subspace of $M_2(\mathbb{R})$.
(2) Show that $T : M_2(\mathbb{R}) \to S_2(\mathbb{R})$ defined by $T(A) = \dfrac{1}{2}\left(A^t + A\right)$ is a linear transformation.
(3) Describe the kernel and image of T and verify the result of the rank-nullity theorem for this example.

4. Problem Four

Let $T : U \to V$ and $S : V \to W$ be linear transformations

(1) Prove that the composition $S \circ T$ is a linear transformation.
(2) Is it always true that $\operatorname{nullity}(S\circ T) = \operatorname{nullity}(S) + \operatorname{nullity}(T)$? If so, give a proof. If not, give a counterexample.