FIRST EXAM–STUDY COPY

MATH 251, FALL 2006, WILLIAMS COLLEGE
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These are the problems from the first midterm exam.

1. Problem One

Show that the following argument is valid.

$$\begin{array}{c} p \\ \neg q \to \neg p \\ \hline q \end{array}$$

2. Problem Two

(1) Give an example of a relation which is both symmetric and antisymmetric.
(2) Give an example of a relation which is neither symmetric nor antisymmetric.

3. Problem Three

(1) Suppose that $f : A \to B$ is a function between finite sets A and B with the same cardinality. Prove that f is an injection if and only if f is a surjection.
(2) Is this necessarily true for infinite sets?

4. Problem Four

(1) Prove that, for any set X, the relation of set containment is a partial order on the power set $\mathcal{P}(X)$ of X.
(2) Does $\mathcal{P}(X)$ contain a maximal or a minimal element?

5. Problem Five

Let X be a set with n elements. Prove that the power set of X has 2^n elements.

6. Problem Six

Recall that the symmetric difference $A \triangle B$ of two sets A and B is the set $(A \cup B) \setminus (A \cap B)$. We can recursively define the symmetric difference of n sets as

$$A_1 \triangle A_2 \triangle \cdots \triangle A_n = (A_1 \triangle A_2 \triangle \cdots \triangle A_{n-1}) \triangle A_n.$$

Consider the claim that $A_1 \triangle A_2 \triangle \cdots \triangle A_n$ is the set of things which belong to exactly an odd number of the sets $A_1, A_2, \ldots, A_n$. If this is true, give a proof. If it is false, give a counterexample.