Second Exam

Math 251, Williams College

November 9, 2006

Directions: This Exam has 7 questions. Be sure to show all of your work and explain yourself carefully. You have three hours to complete this exam, to be done in a single block of time. You may not use any references. Please do not discuss the exam with anyone until after class on Tuesday. Please sign the honor code pledge below and attach this cover sheet to your answers.

This exam is due in class on Tuesday, November 14th.

Pledge: On my honor, I have neither given nor received any aid on this examination.

Write pledge and sign name here:

Print your name here: ____

Problem	*Possible*	*Score*
1	5	
2	4	
3	3	
4	4	
5	5	
6	5	
7	4	
Total	30	

1. Problem One

Find the number of solutions of the linear equation $a+b+c+d+e=10$ if

(1) all the variables are nonnegative integers,
(2) all the variables are positive integers,
(3) all the variables are positive integers and the variable a is odd.

2. Problem Two

If X and Y are two sets with n elements each and if there are no elements common to the two sets, find the number of ways of arranging the $2n$ elements of these two sets in a circular pattern so that on either side of an element of X there is an element of Y, and vice versa.

3. Problem Three

What kind of statement can be "vacuously true"? What does it mean for such a statement to be vacuously true? Give an example of a statement which is vacuously true.

4. Problem Four

Give a combinatorial proof of the following identity:

$$C(m,0)\cdot C(n,0)+C(m,1)\cdot C(n,1)+C(m,2)\cdot C(n,2)+\dots$$
$$\dots+C(m,n)\cdot C(n,n)=C(m+n,n)$$

5. Problem Five

Prove that a function $f: A \to B$ is invertible if and only if it is bijective.

6. Problem Six

Let x be a (finite length) string of distinct symbols. A *derangement* of x is a permutation of the symbols in which every symbol must move away from its original position. For example: 1243 is not a derangement of 1234, but 4312 is a derangement.

How many derangements of a string of length n are there?

7. Problem Seven

In a gathering of 30 people, there are 104 different pairs of people who know each other.

(1) Show that some person must have at least seven acquaintances.
(2) Show that some person must have fewer than seven acquaintances.