

HOMWORK ASSIGNMENT # 2

MATH 251, FALL 2006, WILLIAMS COLLEGE

ABSTRACT. This assignment has 5 problems on 2 pages. It is due on Thursday, September 21 in class. Talk with me if you have difficulty. Good luck!

1. PROBLEM ONE

Using Venn diagrams, investigate whether the following statements are true or false.

- (1) $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$
- (2) $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
- (3) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
- (4) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$
- (5) $A \cup (B \oplus C) = (A \cup B) \oplus (A \cup C)$

2. PROBLEM TWO

This problem has two parts.

- (1) List all the sets in the power set of the following sets:
 - (a) $\{a, b\}$
 - (b) $\{a, b, c\}$
 - (c) $\{\emptyset, 0, \{0\}\}$
- (2) List all partitions of the following sets:
 - (a) $\{a\}$
 - (b) $\{a, b\}$
 - (c) $\{a, b, c\}$

3. PROBLEM THREE

Let $X = \{a, b, c\}$ and S be the partial order defined on the powerset $\mathcal{P}(X)$ given by $S = \{(A, B) \mid A \subseteq B\}$.

- (1) List the elements of S .
- (2) Draw the Hasse diagram of S .

Date: September 14, 2006.

4. PROBLEM FOUR

Let $A = \{a, b, c, d\}$. Draw the digraph of each relation on A given below and decide if the relation is reflexive, symmetric, transitive or antisymmetric.

- (1) $R = \{(b, b), (b, c), (b, d), (c, b), (c, c), (c, d)\}$
- (2) $R = \{(a, b), (b, a)\}$
- (3) $R = \{(a, a), (b, b), (c, c), (d, d)\}$
- (4) $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a)\}$
- (5) $R = \{(a, c), (a, d), (b, c), (b, d), (c, a), (c, d)\}$
- (6) $R = \{(a, b), (b, c), (c, d)\}$

5. PROBLEM FIVE

Let $A = \{2, 3, 4, 6, 8, 12, 16, 24\}$, and let S be the partial order on S defined by $S = \{(a, b) \mid a \text{ divides } b\}$.

- (1) Find the minimal elements in A .
- (2) Find the maximal elements in A .
- (3) Find the least upper bound of the set $B = \{4, 6, 12\}$.
- (4) Draw the Hasse diagram of this partial order.