HOMEWORK ASSIGNMENT # 3

MATH 251, FALL 2006, WILLIAMS COLLEGE

Abstract. This assignment has 5 problems on 2 pages. It is due on Friday, September 29 by noon. Talk with me if you have difficulty. Good luck!

1. Problem One: On injections

Let $f : A \to B$ and $g : B \to C$ be functions.

(1) Prove that if f and g are injective, then $g \circ f$ is injective.
(2) Suppose that $g \circ f$ is injective. Is it necessarily true that f is injective? Or that g is injective? Give a proof or counterexample to back up your answers.

2. Problem Two: On surjections

Let $f : A \to B$ and $g : B \to C$ be functions.

(1) Prove that if f and g are surjective, then $g \circ f$ is surjective.
(2) Suppose that $g \circ f$ is surjective. Is it necessarily true that f is surjective? Or that g is surjective? Give a proof or counterexample to back up your answers.

3. Problem Three: On functions and set operations

Suppose that $f : X \to Y$ is a function and that A and B are subsets of X. Prove the following.

(1) $f(A \cup B) = f(A) \cup f(B)$,
(2) $f(A \cap B) \subseteq f(A) \cap f(B)$,
(3) if f is an injection, then $f(A \cap B) = f(A) \cap f(B)$.

Date: September 22, 2006.

4. Problem Four: On algebraic numbers

A real number α is called *algebraic* when it is the root of some polynomial with integral coefficients. What is the cardinality of the set of algebraic integers? (Back up your answer with a proof.) A number which is not algebraic is called *transcendental*. What is the cardinality of the set of transcendental numbers?

This is not part of the assignment. How many transcendental numbers do you know? My guess is that you know two right off the top of your head. (let them come to you...) Try looking for some others. (Ask around, look into the literature, etc.) Think about this in the context of the answers to this problem for some perspective on your experiences with real numbers. I'll give a small prize to any student who finds a transcendental number I haven't heard about, yet.

5. Problem Five: On Brevity

Prove that there are real numbers that cannot be defined uniquely in a finite number of words.