HOMEWORK ASSIGNMENT 4

MATH 251, WILLIAMS COLLEGE, FALL 2006

ABSTRACT. This assignment has 4 problems on 1 pages.

1. A theorem on sets

Prove that for any sets $A, B_1, \ldots, B_n$,

$$A \cap \left(\bigcup_{i=1}^{n} B_i \right) = \bigcup_{i=1}^{n} (A \cap B_i).$$

2. A geometric theorem

Prove that for every integer $n \geq 2$ the number of lines obtained by joining n distinct points in the plane, no three of which are collinear, is $\frac{1}{2} n(n-1)$.

3. A new form of induction

Use either form of induction to prove that the following form of induction is also valid:

> Suppose that $P(n)$ is a statement about the natural number n such that
> (1) $P(1)$ is true,
> (2) for any $k \geq 1$, $P(k)$ true implies $P(2k)$ is also true, and
> (3) for any $k \geq 2$, $P(k)$ true implies that $P(k-1)$ is also true.
>
> Then $P(n)$ is true for all n.

4. The arithmetic-geometric mean inequality

Use the last problem to prove the *arithmetic-geometric mean inequality*: For any $n \geq 1$ and any n nonnegative real numbers $a_1, \ldots, a_n$, we have that

$$\frac{a_1 + \cdots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \ldots a_n}\,.$$