HOMEWORK ASSIGNMENT 5

MATH 251, WILLIAMS COLLEGE, FALL 2006

ABSTRACT. This assignment has 13 problems on 3 pages and is due on November 2 in class. Each problem will be worth 2 points instead of the usual 3. Good luck!

1. BIG BROTHER

The social security number of a person is a sequence of nine digits that are not necessarily distinct. If X is the set of all social security numbers, find the number of elements of X.

2. CARD SHARKS

Find the number of ways of picking the following subsets out of a standard deck of cards:

(1) A king and a queen,
(2) a king or a queen,
(3) a king and a red card,
(4) a king or a red card.

3. MEMORY HUNGRY

A sequence of digits which are either 0 or 1 is called a binary number. A binary number with eight digits is called a *byte*.

(1) Find the number of bytes.
(2) Find the number of bytes that begin with 10 and end with 01.
(3) Find the number of bytes that begin with 10 but do not end with 01.
(4) Find the number of bytes that begin with 10 or end with 01.

4. ARE WE THERE YET?

There are three bridges connecting two towns, A and B. Between towns B and C there are four bridges. A salesperson has to travel from A to C via B. Find

(1) the number of possible choices for bridges from A to C,
(2) the number of choices for round-trip travel from A to C,
(3) the number of choices for round-trip travel from A to C if no bridge is repeated.

5. ON PERMUTATIONS

(1) Show that $P(n, r+1) = (n-r) \cdot P(n, r)$ and use this result to find the value of n if $P(n, 9) = 15P(n, 8)$.
(2) Compute $P(17; 4, 3, 2)$,
(3) Compute $P(17; 2, 2, 2)$

6. ON COMBINATIONS

Compute the following.

(1) $C(9, 4)$,
(2) $C(10, 7)$,
(3) $C(8, 4)$.

7. WHY NOT *West Alabama*?

Find the number of ways to rearrange the letters of $MISSISSIPPI$

(1) without restriction,
(2) so that all four S's stay together,

8. THANKSGIVING IS COMING!

Find the number of ways of seating 14 people such that 8 of them are around a round table and the rest are on a bench.

9. I HOPE IT'S NOT FOR DODGEBALL

A discrete math class consists of 10 math majors and 12 computer science majors. A team of 12 has to be selected from this class. Find the number of ways of selecting a team if

(1) the team has 6 from each discipline,
(2) the team has a majority of computer science majors.

10. THIS GUY AGAIN?

Use a combinatorial argument to prove *Newton's identity*:

$$C(n, r) \cdot C(r, k) = C(n, k) \cdot C(n-k, r-k).$$

11. SOMETHING FAMILIAR

Prove that

$$C(n, 0) + C(n, 1) + \cdots + C(n, n-1) + C(n, n) = 2^n.$$

12. SOMETHING NEW

Use a combinatorial argument to prove the following identity:

$$[C(n, 0)]^2 + [C(n, 1)]^2 + \cdots + [C(n, n-1)]^2 + [C(n, n)]^2 = C(2n, n).$$

13. One Mississippi, Two Mississippi...

Find the number of ways to rearrange the letters of $MISSISSIPPI$ so that no two S's are adjacent.