

MATH 251 HOMEWORK 6

FALL 2006, WILLIAMS COLLEGE

ABSTRACT. This assignment has 9 problems on 2 pages. It is due on Thursday, November 9 in class.

1. COLLECTIONS

Find the number of r -collections that can be formed using the elements of the set $X = \{A, B, C, D, E, F, G\}$ if

- $r = 4$ and the elements are distinct,
- $r = 4$,
- $r = 9$.

2. EQUATIONS, PART ONE

Find the number of distinct solutions in nonnegative integers of the equation $a + b + c + d + e = 24$.

3. DON'T MULTIPLY

Find the number of terms in the multinomial expansion of $(a + b + c + d + e)^{24}$.

4. AN IDENTITY

Establish the following identity:

$$C(n, n) + C(n + 1, n) + C(n + 2, n) + \cdots + C(n + r, n) = C(n + r + 1, n + 1)$$

5. AN ALLOCATION PROBLEM

Find the number of ways of allocating r identical objects to n distinct locations such that the location i gets at least p_i objects, where $i = 1, 2, \dots, n$.

6. EQUATIONS, PART TWO

Find the number of solutions in nonnegative integers of the strict inequality $a + b + c + d + e < 11$.

7. EQUATIONS, PART THREE

Find the number of solutions in integers of the linear equation $p + q + r = 25$ where p is at least 2 and at most 4, q is at least 3 and at most 6, and r is at least 4 and at most 8.

8. THE FIRST YEAR STUDENT MIXER

Prove that in any group of 10 people either there is a subgroup of 3 strangers or a subgroup of 4 people known to one another.

9. PERMUTATIONS AGAIN

Find the number of permutations of the digits 1, 2, ..., 9 such that

- The blocks 12, 34 and 567 do not appear,
- The blocks 12, 23 and 415 do not appear.