MATH 251 HOMEWORK 7

FALL 2006, WILLIAMS COLLEGE

Abstract. This assignment has 4 problems on 1 page. It is due on Tuesday, November 21 in class.

1. The Towers of Hanoi

Three vertical cylindrical poles of equal radius and height are place along a line on top of a table and n circular disks of decreasing radius, each with a hole at its center, are attached to the first pole such that the largest one is at the bottom, the next largest is just above that, and so on, until the smallest is on the top of the stack. The distance between the feet of any two poles is not less than the diameter of the largest disk. A legal move is defined as a transfer of the top disk from any one of the poles to another pole as long as no disk is placed upon a smaller disk.

Find and describe an algorithm for moving all of the disks from the first pole to one of the other two poles (a single new stack). Let $f(n)$ be the number of moves required to transfer all the disks as required. Obtain a recurrence relation for $f(n)$ and solve it.

Note: This is a simple way that recurrence relations get used. An algorithm often has recursive parts (as should yours here), so to describe its complexity one can use recurrence relations.

2. Recurrence Relations

Solve the recurrence relation

$$\begin{cases} a_n &= a_{n-2} + 4n \\ a_0 &= 3 \\ a_1 &= 2 \end{cases}$$

by using a generating function.

3. Growth of functions

Prove that for any $b > 1$,

$$\log_b(\log_b(n)) \prec \log_b(n) \prec n.$$

4. Growth of functions again

Show that $f \asymp g$ is an equivalence relation on the set of functions

$$\{f : \mathbb{N} \to \mathbb{R}_{\geq 0}\}.$$