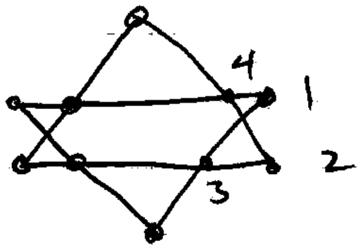
MATH 251 HOMEWORK 9 SOLUTIONS

10.2 #2

a)

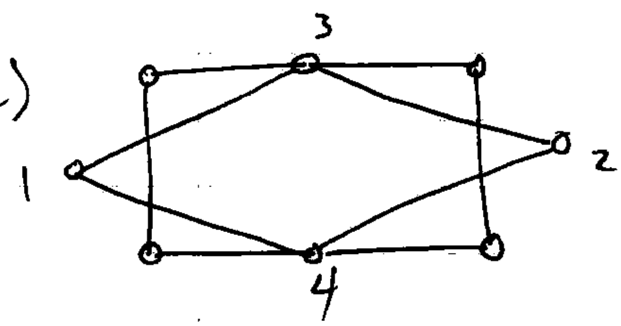


Not Hamiltonian.
vertices 1, 2 have degree 2
⇒ all edges adjacent must lie in any Hamiltonian path.

But then you've already used the two possible edges from 3, 4 allowed and made a ~~circuit~~ cycle, while missing many vertices.

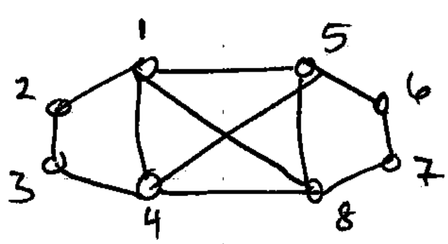
b) Not Hamiltonian: its disconnected!

c)



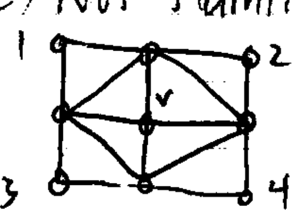
similar argument as in (a)

d) Hamiltonian! follow vertices in this order



1 2 3 4 5 6 7 8 1

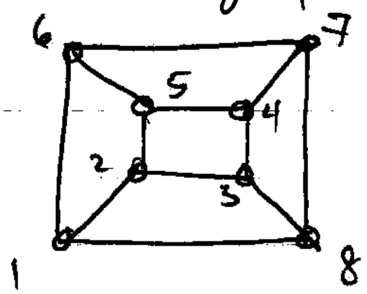
e) Not Hamiltonian : 1,2,3,4 are degree 2
→ must use all ~~with~~ edges adjacent this closes a cycle without visting V



10.2#2 (cont)

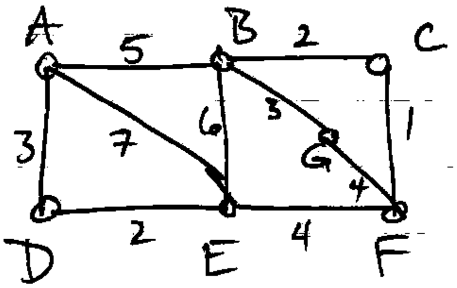
(f) Hamiltonian. go around the ring!

10.2#12 The graph is as below.

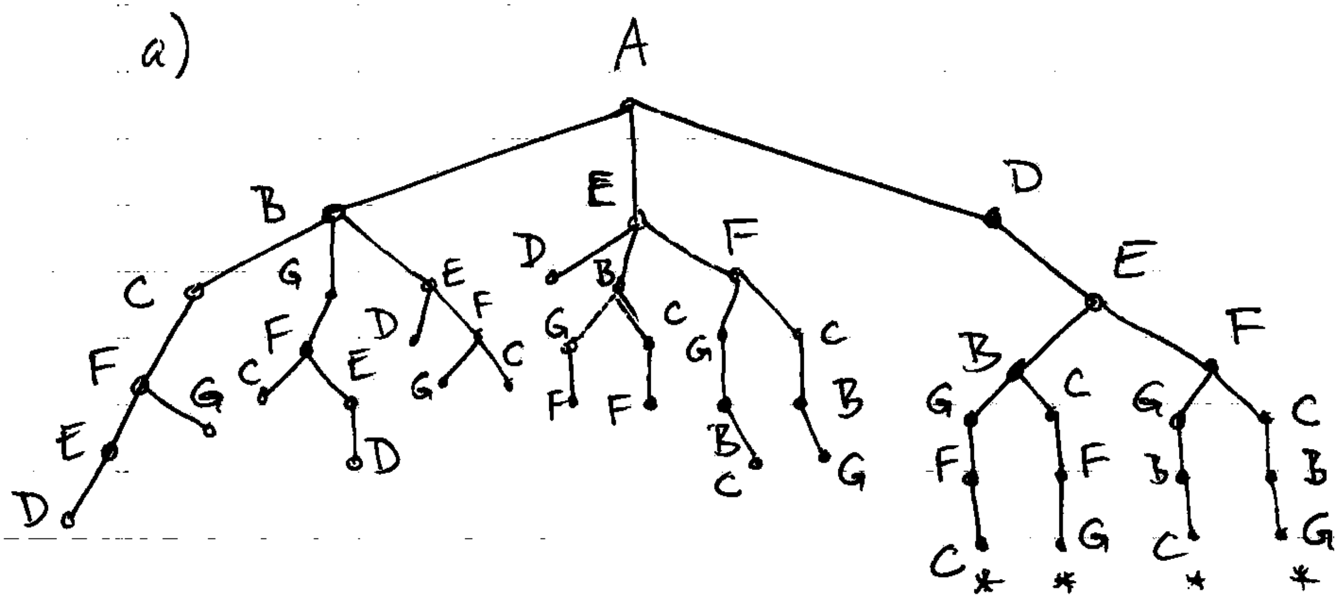


follow vertices in order to see a Hamiltonian ~~path~~ cycle

12.1#9



a)



Paths from A to C: ABC, ABGFC, ABEFC, AEBC, AEFC, AEFGBC, ADEFC, ADEFGBC, ADEBC, ADEBGFC

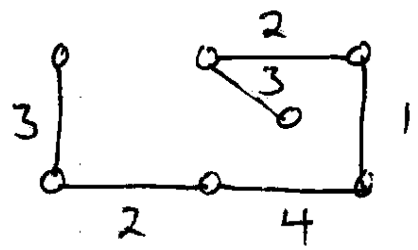
Hamiltonian paths are *'d

12.1#9 (cont.)

(b) The graph is not Hamiltonian. Its 4 Hamiltonian paths (from A) end at vertices not adjacent to A so it is impossible to close them.

Alternately, G, C are degree 2 $\rightarrow$ these 4 edges must be used. But this closes the path BGFC, which is insufficient.

(c) Using either Prim or Kruskal one can construct the following minimal spanning tree



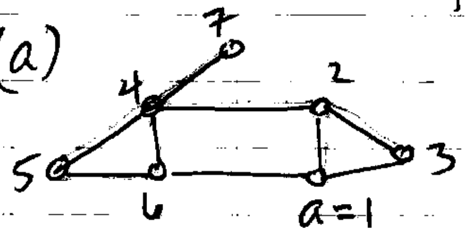
which has cost $= 15$.

12.3#8 (a) In the weighted case, change the weight of e so that $W(e) < W(e')$ for all e'. Then Kruskal will start by choosing e.
In the unweighted case, treat it as a weighted graph with all edges of weight one, then apply the above.

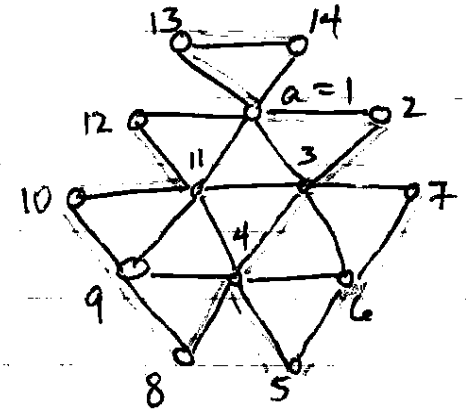
b) Do this similarly. Choose a number much smaller than the given weights (in the weighted case) or than one (in the unweighted case). Now Kruskal's algorithm will grow the given subgraph at the initial stages of building a spanning tree.

12.5 #1 I've labelled the vertices in my search.
the spanning tree is in red.

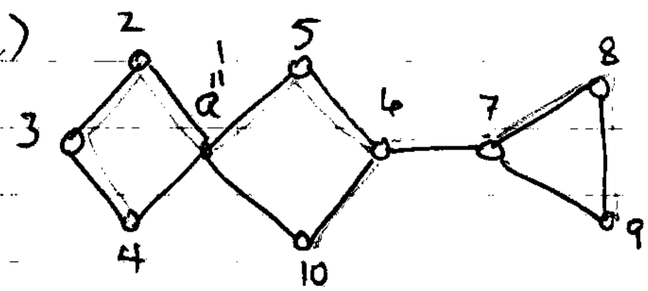
(a)



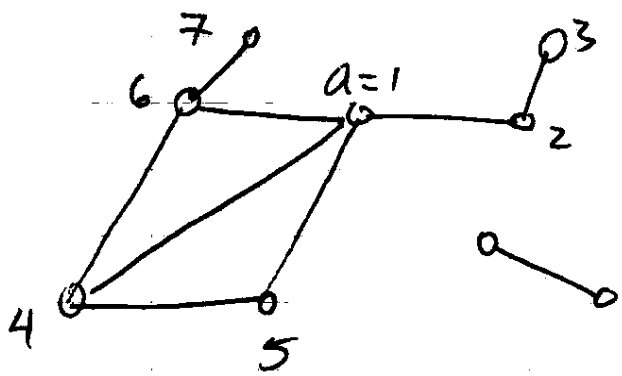
b)



(c)

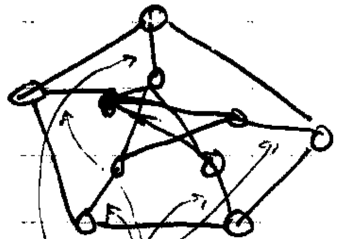


(d)



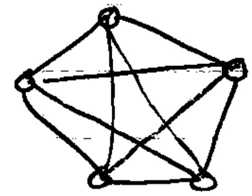
No spanning tree
→ Not connected.

$\boxed{13.1 \# 4}$ The Petersen graph is not planar.
If we collapse the "radial edges"



"radial" edges

we get



which is K_5.

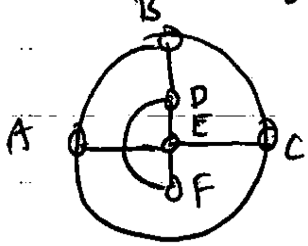
Alternate Argument: $V = 10$, $E = 15$
If it can be realized as a plane graph we must have $R = 2 - V + E = 2 - 10 + 15 = 7$.

the graph contains no triangles ~~so~~ and no 4-cycles

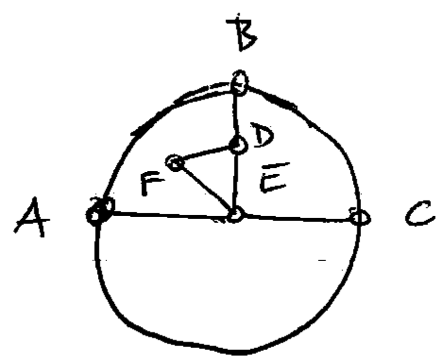
$$2E \geq N = \sum_{\text{regions}} (\#\text{edges bounding the region}) \geq 5R$$

so $35 = 5R \leq 2E = 30$
a contradiction.

The other graph is planar



can be be redrawn isomorphically as



Which is a plane graph.