Math 101 Fall 2005 **Exam #1**

This exam consists of 7 questions. You must show all your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. Pledge your exam when finished, and include your name and section number on the front of the exam. Good luck!

1. Find the derivative (with respect to x) for each of the following functions:

   (a) $f(x) = \left(x^{10} + x^9 + x^7 + x^6\right) \tan(3x)$

   (b) $g(x) = \dfrac{\ln x}{x^2}$

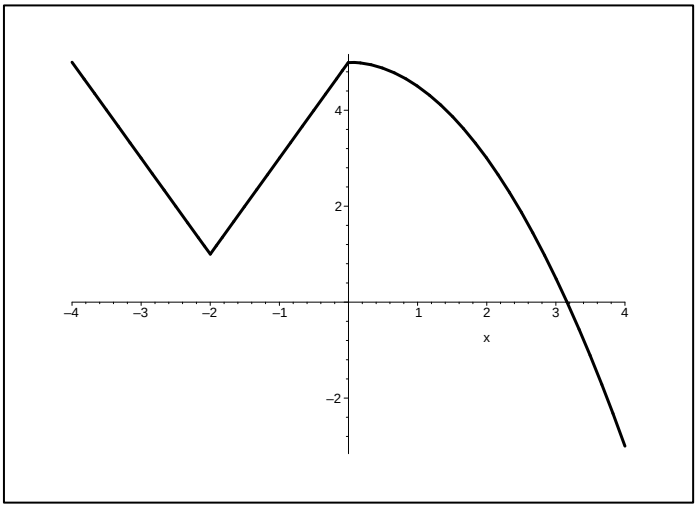
   (c) $y = x^{x^2}$

2. Evaluate the following limits:

   (a) $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{x^2}$

   (b) $\lim\limits_{t \to 2} \dfrac{t^2 - 4}{t - 2}$

3. Given the graph of $f(x)$ shown here, sketch a graph of $f'(x)$. At which points is $f(x)$ differentiable?



4. Find the equation of the line tangent to $f(x)$ at $x = 1$, where

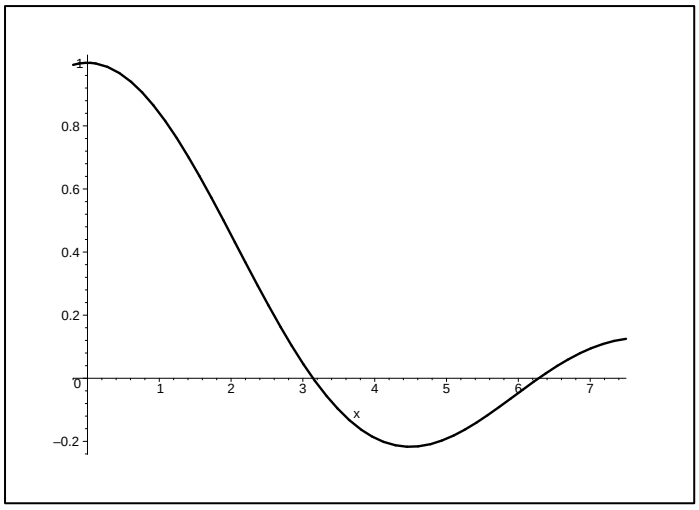
$$f(x) = \sqrt{x^2 + e^x - e}$$

5. State the limit definition of the derivative of $f(x)$. Use this to find the derivative of $f(x) = \frac{1}{x}$.

6. Let

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ \\ 1 & x = 0 \end{cases}$$

A graph of $f(x)$ is shown below. What are the dimensions of the largest rectangle that lies entirely underneath $f(x)$, whose lower left corner is at the origin, and whose upper right corner lies on the graph of $f(x)$?



7. Use the Intermediate Value Theorem to show that

$$f(x) = x^3 + x + 1$$

has at least one root in the interval $[-1, 0]$. What conditions on $f(x)$ allow you to use the Intermediate Value Theorem?