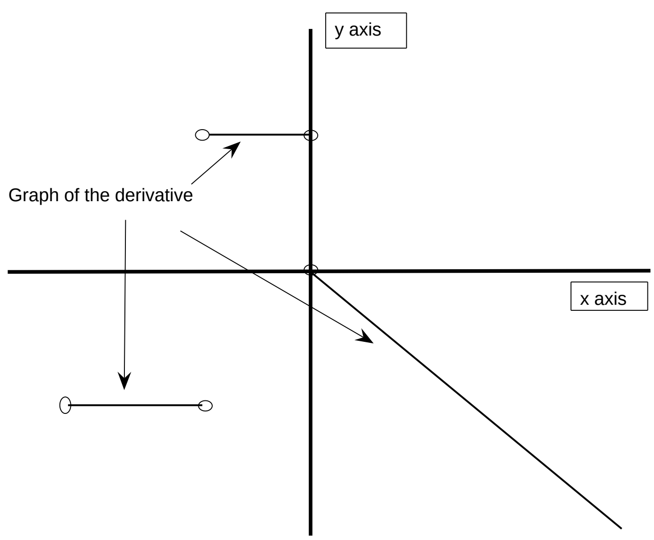
Math 101, Fall 2005, Exam 1 Answers

Problem 1. Part (a): $f'(x) = (10x^9 + 9x^8 + 8x^7 + 7x^6 + 6x^5) \cdot \tan(3x) + 3\sec^2(3x)(x^{10} + x^9 + x^8 + x^7 + x^6)$.
Part (b): $g'(x) = \dfrac{x - 2x\ln x}{x^4}$.
Part (c): $\dfrac{dy}{dx} = (2x\ln x + x)x^{x^2}$.
Problem 2. Part (a): $lim_{x\to 0}\dfrac{sin^2(x)}{x^2} = 1$. (hint: use the basic trig limit)
Part (b): $lim_{x\to 0}\dfrac{t^2-4}{t-2} = 4$. (hint: factor the numerator)

Problem 3. The proper graph looks like the following:



Problem 4. One computes the derivative, and evaluates this and the original function to get the necessary information to write down...

$$y - 1 = \frac{2+e}{2}(x-1).$$

Problem 5. The derivative of f at $x = a$ is the following limit

$$f'(a) = \lim_{h\to 0}\frac{f(a+h) - f(a)}{h},$$

provided it exists.
To find the derivative of $f(x) = 1/x$, we simply compute the limit as follows.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h} = \lim_{h\to 0} \frac{\dfrac{x-(x+h)}{x(x+h)}}{h} \\ &= \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2} \end{aligned}$$

Problem 6. One must maximize the function $A(x) = \sin(x)$, we check the endpoints of 0 and π along with the critical point $\pi/2$. The maximum area occurs when the rectangle is $\pi/2$ by $2/\pi$.
Problem 7. We check that the function is a polynomial, hence continuous, and that $f(-1) < 0$ and $f(0) > 0$. So the intermediate value theorem says that there is some point between -1 and 0 where f takes the value 0.